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CENTRALIZING PROPERTIES OF $(\alpha, 1)$-REVERSE DERIVATIONS IN SEMIPRIME RINGS

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ABSTRACT: Let R be a semiprime ring with center Z, S be a non-empty subset of R, α be an endomorphism on R and d be an $(\alpha, 1)$ reverse derivation of R. A mapping $d: R \to R$ is called centralizing derivation of S if $[d(x), x] \in Z$, for all $x \in S$. In the present paper, we study some centralizing properties of $(\alpha, 1)$ Reverse derivations in semiprime rings one of the following conditions holds: (i)$d([x, y]) = [x, y]_{\alpha,1}$, for all x, y ∈ R. (ii)$d([x, y]) = -[x, y]_{\alpha,1}$, for all x, y ∈ R. (iii)$d(x)d(y) \mp xy \in Z$, for all x, y ∈ R. (iv)$d(xoy) = (xoy)_{\alpha,1}$, for all x, y ∈ R. (v)$d(xoy) = -(xoy)_{\alpha,1}$, for all x, y ∈ R. Also we prove that d is centralizing on R if d acts as a homomorphism on R and d is centralizing on S if d acts as an anti-homomorphism on R.

Key words: Semiprime ring, Reverse derivation, $(\alpha, 1)$- Reverse derivation, Centralizing mappings, homomorphism and anti-homomorphism.

AMS Subject Classification: 16U10, 16D60, 16N60.

I. INTRODUCTION

The study of centralizing mappings was initiated by E.C.Posner [14]. Bresar and Vukman [4] have introduced the notion of a reverse derivation. Samman and Alyamani [15] and Jaya Subba Reddy [5-6] have studied some properties of prime or semiprime rings with reverse derivations. Merva Ozdeir and Neset Aydin [11] have studied prime and semiprime rings with (α, β) reverse derivations. G.Shobhalatha and et.al [13] have studied centralizing properties of (α, 1)- derivations in semiprime rings. Many authors have established commutativity theorems for prime rings or semiprime rings admitting auto-morphisms or reverse derivations which are centralizing or commuting on appropriate subsets of R (See [1, 2, 17]). The purpose of this paper is to study some centralizing properties of (α, 1) reverse derivations in semiprime rings. Also we prove that d is centralizing on R if d acts as a homomorphism on R and d is centralizing on S if d acts as an anti-homomorphism on R.

II. PRELIMINARIES

Throughout this paper, R will represent an associative ring with center Z. A ring R is said to be prime if $xRy = 0$ implies that either $x = 0$ or $y = 0$ and semiprime if $xRx = 0$ implies that $x = 0$, where $x, y \in R$. A prime ring is obviously semiprime for any $x, y \in R$, the symbol $[x, y]$ stands for the commutator $xy - yx$ and the symbol (x, y) stands for the anti-commutator $xy + yx$. A reverse derivation d on R is determined to be an additive endomorphism satisfying the product rule $d(xy) = d(y)x + yd(x)$, $x, y \in R$. Let α be an endomorphism on R. An additive mapping from R into itself to be an $(\alpha, 1)$ reverse derivation if $d(xy) = d(y)\alpha(x) + yd(x)$, for all x, y $\in$ R. Let S be a non-empty subset of R. A mapping f from R into itself is called centralizing on S if $[f(x), x] \in Z$, for all $x \in S$ and is called commuting on S if $[f(x), x] = 0$, for all $x \in S$. If $d(xy) = d(x)d(y)$ or $d(xy) = d(y)d(x)$, for all x, y $\in$ R, then d is said to act as homomorphism or anti-homomorphism on R respectively. Throughout the present paper, we will make extensive use of the following basic commutator identities [12]:

$$[x, yz] = y[x, z] + [x, y]z;$$

$$[xy, z] = [x, z]y + x[y, z];$$

$$[xy, z]_{\alpha,1} = x[y, z]_{\alpha,1} + [x, z]y = x[y, \alpha(z)] + [x, z]_{\alpha,1}y;$$

$$[x, yz]_{\alpha,1} = y[x, z]_{\alpha,1} + [x, y]_{\alpha,1}\alpha(z);$$

$$xo(yz) = (xoy)z - y[x, z] = y(xoz) + [x, y]z;$$

$$(xy)oz = x(yoz) - [x, z]y = (xoz)y + x[y, z];$$

$$(xo(yz))_{\alpha,1} = (xoy)_{\alpha,1}\alpha(z) - y[x, z]_{\alpha,1} = y(xoz)_{\alpha,1} + [x, y]_{\alpha,1}\alpha(z);$$

$$((xy)oz)_{\alpha,1} = x(yoz)_{\alpha,1} - [x, z]y = (xoz)_{\alpha,1}y + x[y, \alpha(z)].$$

III. MAIN RESULTS

Theorem 3.1: Let R be a semiprime ring and d be an $(\alpha, 1)$ reverse derivation of R. If d satisfies one of the following conditions, then d is centralizing.

(i) $d([x, y]) = [x, y]_{\alpha,1}$, for all $x, y \in R$.

(ii) $d([x, y]) = -[x, y]_{\alpha,1}$, for all $x, y \in R$.

Proof: (i) Assume that $d([x, y]) = [x, y]_{\alpha,1}$, for all $x, y \in R$. (3.1)

Replacing y by xy, we get $d([x, xy]) = [x, xy]_{\alpha,1}$, for all $x, y \in R$.

$d(y)\alpha([x, x]) + yd([x, x]) + d([x, y])\alpha(x) + [x, y]d(x) = y[x, x]_{\alpha,1} + [x, y]_{\alpha,1}\alpha(x)$.

Using (3.1), we obtain $[x, y]d(x) = 0$, for all x, y $\in$ R. (3.2)

Substituting $d(x)y$ for y in (3.2) and using (3.2), we have $[x, d(x)]yd(x) = 0$. (3.3)

Replacing y by yx in (3.3) we get $[x, d(x)]yxd(x) = 0$, for all x, y $\in$ R. (3.4)

Multiplying (3.3) on the right of x, we have $[x, d(x)]yd(x)x = 0$. (3.5)

Subtracting (3.5) from (3.4), we arrive at $[x, d(x)]y[x, d(x)] = 0$, for x, y $\in$ R.

By the semiprimeness of R, we find that $[x, d(x)] = 0$, for all $x \in R$ and so $[x, d(x)] \in Z$. Hence d is commuting and so centralizing.

(ii) If d is an $(\alpha, 1)$ reverse derivation satisfying the property $d([x, y]) = -[x, y]_{\alpha,1}$, for all $x, y \in R$, then

(-*d*) satisfies the condition $(-d)([x,y]) = -[x,y]_{\alpha,1}$, for all $x,y \in R$.

Hence *d* is centralizing by condition (i).

Corollary 3.2: Let *R* be a prime ring and *d* be an $(\alpha,1)$ reverse derivation of *R*. If *d* satisfies one of the following conditions, then *d* is centralizing.

(i) $d([x,y]) = [x,y]_{\alpha,1}$, for all $x,y \in R$.

(ii) $d([x,y]) = -[x,y]_{\alpha,1}$, for all $x,y \in R$.

Theorem 3.3: Let *R* be a semiprime ring and *d* be an $(\alpha,1)$ reverse derivation of *R*. If *d* acts as a homomorphism on *R*, then *d* is centralizing.

Proof: Assume that *d* acts as a homomorphism on *R*.

Now we have d(xy) = d(x)d(y), for all x, y∈ R. (3.6)

$d(y)\alpha(x) + yd(x) = d(x)d(y)$, for all x, y ∈R.

Replacing x by zx, $z \in R$ in the above equation, we get

$d(y)\alpha(z)\alpha(x) + yd(x)(z) + yxd(z) = d(y)d(x)\alpha(z) + d(y)xd(z)$, for all x, y ∈R.

Using the hypothesis and *d* is derivation on *R* in the last relation gives $xyd(z) = d(y)xd(z)$, and so $(d(y) - y)xd(z) = 0$, for all x, y ∈R. (3.7)

Writing x by *d*(*x*) in (3.7) we get $(d(y) - y)\,d(x)\,d(z) = 0$, for all $x,y,z \in R$.

By the hypothesis, we obtain $(d(y) - y)d(xz) = (d(y) - y)d(z)\alpha(x) + (d(y) - y)zd(x) = 0$. (3.8)

Using (3.7), we have $(d(y) - y)\,d(z)\,\alpha(x) = 0$, and so $d(y)\,d(z)\,\alpha(x) = yd(z)\,\alpha(x)$,

$d(zy)\,\alpha(x) = d(y)\,\alpha(z)\,\alpha(x) + yd(z)\,\alpha(x) = yd(z)\,\alpha(x)$. *i.e.*, $d(y)\alpha(z)\alpha(x) = 0$, for all x, y ∈R. (3.9)

Replacing y by x in (3.9), we get $d(x)\alpha(z)\alpha(x) = 0$. (3.10)

Writing $\alpha(x)$ by $d(x)$ in (3.10), we get $d(x)\alpha(z)d(x) = 0$. (3.11)

Replacing *d(x)* by *xd(x)* in (3.11), we get $xd(x)\alpha(z)xd(x) = 0$, for all x, y ∈R. (3.12)

Again replacing *d(x)* by *d(x)x* in (3.11), we get $d(x)x\alpha(z)d(x)x = 0$, for all x, z ∈R. (3.13)

Subtracting (3.13) from (3.12) and replace z by y, we arrive at $[x,d(x)]\,\alpha(y)\,[x,d(x)] = 0$, for all x, y∈R. By the semiprimeness of *R*, we find that $[x,d(x)] = 0$, for all $x \in R$ and so $[x,d(x)] \in Z$. Hence *d* is commuting and so *d* is centralizing.

Corollary 3.4: Let *R* be a prime ring and *d* be an $(\alpha,1)$ reverse derivation of *R*. If *d* acts as a homomorphism on *R*, then *d* is centralizing.

Theorem 3.5: Let *R* be a semiprime ring and *S* be a non-empty subset of *R*. Let *d* be an $(\alpha,1)$ reverse derivation of *R* such that $\alpha(x) = x$, for all x ∈ S. If *d* acts as an anti-homomorphism on *R*, then *d* is centralizing on *S*.

Proof: Assume that *d* acts as an anti- homomorphism on R. Now by the hypothesis we have

$d(xy) = d(y)\alpha(x) + yd(x) = d(y)\,d(x)$, for all x, y ∈R.

Replacing x by xy in the last relation and using *d* is an $(\alpha,1)$ reverse derivation of *R*, we arrive at $d(y)\alpha(x)\alpha(y) = d(y)\alpha(x)d(y)$, for all x, y ∈R. (3.14)

That is $d(y)\alpha(x)d(y) = d(y)\alpha(x)d(y)$, for all x, y ∈R. (3.15)

And writing xy by x in (3.15), we have $d(y)\alpha(x)(d(y)\alpha(y) - \alpha(y)d(y)) = 0$ and so $d(y)\alpha(x)[d(y),\alpha(y)] = 0$, for all x, y ∈R.

Interchange x and y places in the last relation, we get $d(x)\alpha(x)[d(x),\alpha(x)] = 0$, for all $x \in R$.

Using the same arguments in the proof of Theorem 3.1 (i), we obtain $[d(x),\alpha(x)] = 0$.

Since $\alpha(x) = x$, for all $x \in S$, then $[d(x), x] = 0$, for all $x \in S$.

Hence *d* is commuting on *S*, and *d* is centralizing on *S*.

Corollary 3.6: Let *R* be a prime ring and *d* be an $(\alpha,1)$ reverse derivation of *R*. If *d* acts as anti-homomorphism on *R*, then *R* is commutative integral domain.

Theorem 3.7: Let *R* be a semiprime ring and *d* be an $(\alpha,1)$ reverse derivation of *R*. If *R* admits an $(\alpha,1)$ reverse derivation such that $d(x)d(y) - xy \in Z$, for all $x, y \in R$, then *d* is centralizing.

Proof: Given hypothesis $d(x)d(y) - xy \in Z$, for all $x, y \in R$.

Replacing x by zx in the hypothesis, we get

$d(x)\alpha(z)d(y) + x(d(z)d(y) - zy) \in Z$, for all $x, y, z \in R$. (3.16)

Commuting (3.16) with x, we have $[d(x)\alpha(z)d(y), x] = 0$, for all $x, y, z \in R$ and so

$[d(x)\alpha(z), x]d(y) + d(x)\alpha(z)[d(y), x] = 0$, for all $x, y, z \in R$.

Writing $\alpha(z)$ by $zd(t)$, $t \in R$ in this equation and using this equation yields that

$[d(x)zd(t), x]d(y) + d(x)zd(t)[d(y), x] = 0$.

That is, $[d(x)zd(t)[d(y), x] = 0$, for all $t, x, y, z \in R$.

Taking x instead of y in the above equation, we find that

$d(x)zd(t)[d(x), x] = 0$, for all $t, x, z \in R$. (3.17)

Multiplying (3.17) on the left by x, we have $xd(x)zd(t)[d(x), x] = 0$, for all $t, x, z \in R$. (3.18)

Again replacing z by xz in (3.18), we obtain $d(x)xzd(t)[d(x), x] = 0$, for all $t, x, z \in R$. (3.19)

Subtracting (3.18) from (3.19), we see that $[d(x),x]zd(t)[d(x),x] = 0$. (3.20)

Again multiplying (3.20) on the left by d(t), we have $d(t)[d(x),x]zd(t)[d(x),x] = 0$, for $t, x, z \in R$.

Since R is semiprime ring, we get $d(t)[d(x),x] = 0$, for all $t, x \in R$.

Substituting tx for t in the last equation and using the last equation, we obtain $d(x)\alpha(t)[d(x),x] = 0$, for all $t, x \in R$.

Using the same arguments in the proof of Theorem 3.1(i), we conclude that $[d(x),x]\alpha(t)[d(x),x] = 0$, for all $t, x \in R$.Again using the semiprimeness of *R*, we get $[d(x),x] = 0$, for all $x \in R$.

This yields that *d* is commuting, and so *d* is centralizing.

Corollary 3.8: Let *R* be a prime ring and *d* be an $(\alpha,1)$ reverse derivation of *R*. If *R* admits an $(\alpha,1)$ reverse derivation such that $d(x)d(y) - xy \in Z$, for all $x, y \in R$, then *d* is centralizing.

In the similar manner of Theorem 4, we obtain the following theorem.

Theorem 3.9: Let *R* be a semiprime ring and *d* be an $(\alpha,1)$ reverse derivation of *R*. If *R* admits an $(\alpha,1)$ reverse derivation such that $d(x)d(y) + xy \in Z$, for all $x, y \in R$, then *d* is centralizing.

Corollary 3.10: Let *R* be a prime ring and *d* be an $(\alpha,1)$ reverse derivation of *R*. If *R* admits an $(\alpha,1)$ reverse derivation such that $d(x)d(y) + xy \in Z$, for all $x, y \in R$, then *d* is centralizing.

Theorem 3.11: Let *R* be a semiprime ring and *d* be an $(\alpha,1)$ reverse derivation of *R*. If *d* satisfies one of the following conditions, then *d* is centralizing.

(i) $d(xoy) = (xoy)_{\alpha,1}$, for all $x, y \in R$.

(ii) $d(xoy) = -(xoy)_{\alpha,1}$, for all $x, y \in R$.

Proof: (i) Assume that $d(xoy) = (xoy)_{\alpha,1}$, for all $x, y \in R$.

Replacing x by yx, we get $d\big((yx)oy\big) = ((yx)oy)_{\alpha,1}$, for all $x, y \in R$.

$d\,(\mathrm{y(xoy)} - [y,y]x) = y(xoy)_{\alpha,1} - [y,y]\,x$, for all $x, y \in R$.

$d(xoy)\alpha\,(y) + (xoy)\mathrm{d(y)} - xd([y,y]) = y(xoy)_{\alpha,1}$, for all $x, y \in R$.

Using hypothesis, we obtain $(xoy)_{\alpha,1}\alpha\,(y) + (xoy)\mathrm{d(y)} - x[y,y]_{\alpha,1} = y(xoy)_{\alpha,1}$, for all $x, y \in R$.

Implies that $(xoy)\mathrm{d(y)} = 0$, for all $x, y \in R$. (3.21)

Interchange x and y place in (3.21), we have $(yox)\mathrm{d(x)} = 0$, for $x, y \in R$. (3.22)

Replacing y by zy in (3.22) and using (3.21), we get $[x,z]yd(x) = 0$, for all x, y, z $\in$R.

Again replacing z by $d(x)$ in the above equation, we have $[x,d(x)]yd(x) = 0$, for all x, y $\in$R. (3.23)

Replacing y by yx in (3.23), we get $[x,d(x)]\,yxd(x) = 0$. (3.24)

Multiplying (3.23) on the right of x, we have $[x,d(x)]yd(x)\mathrm{x} = 0$, for all x, y $\in$R. (3.25)

Subtracting (3.25) from (3.24), we arrive at $[x,d\,(x)]\,y\,[x,d\,(x)] = 0$, for all x, y $\in$R.

By the semiprimeness of R, we find that $[x,d(x)] = 0$, for all $x \in R$ and so $[x,d(x)] \in Z$.

Hence d is commuting and so centralizing.

(ii) In the similar manner, we can prove that $d(xoy) = -(xoy)_{\alpha,1}$, for all $x, y \in R$.

Corollary 3.12: Let R be a prime ring and d be an $(\alpha, 1)$ reverse derivation of R. If d satisfies one of the following conditions, then d is centralizing.

(i) $d(xoy) = (xoy)_{\alpha,1}$, for all $x, y \in R$.

(ii) $d(xoy) = -(xoy)_{\alpha,1}$, for all $x, y \in R$.

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