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Some Properties via GS_β-Preopen sets in Grill Topological Spaces

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Abstract

This paper introduces and investigate the notion of GS_β-disconnected sets, GS_β-Connected set, GS_β-Precompact set and separation axioms via GS_β-Preopen sets. In this paper I have introduced Grill Topological Space by using GS_β-Preopen sets

Keywords

Preopen sets; GS_β-Precompact; Grill Topological Space

1. Introduction

The idea of Grill on a Topological Space, is given by Choquet[1], goes as follows. A non empty collection G of subsets of a topological space X is said to be a Grill on X if the following conditions holds

1) $A \in G$ and $A \subseteq B \Longrightarrow B \in G$
2) $A, B \subset X$ and $A \cup B \in G \Longrightarrow A \in G$ or $B \in G$

Let (X, τ) be a Topological Space along with G be a grl upon Y. We define a mapping $\phi: P(X) \rightarrow P(X)$, defined by $\phi_G(A, \tau)$ or $\phi_G(A)$ or simply $\phi(A)$ forenamed the operator combined with the Grill G along with the topology τ, and is defined by $\phi_G(A) = \{y \in Y: B \cap V \in G, \forall V \in \tau(x)\}$ where $\tau(x)$ erect the compilation based on all open neighborhood based on x. We also entitle a mapping $\tau: P(X) \rightarrow P(X)$ by $\psi(A) = A \cup \phi(A)$, for every $A \in P(X)$, which is a Kuratowski closure operator([3], [4],[5]) along with consequently induced a topology θ_G upon X. Then the Triplet (X, τ, G) is called Grill Topological Space.

In 1982 Mashhour [7], introduced the notion of a Precontinuous function. In 2009 Al-Omari and Noiri [8], introduced the notion of $\mathcal{N}$-Precontinuous function. In this paper I have investigated about GS_β-disconnected sets, GS_β-Precompact sets and Seperation axioms.

2. Preliminaries

For a Topological Space (X, τ) and $A \subseteq X$, throughout this paper, we mean $Cl(A)$ and $Int(A)$ the Closure set and Interior set respectively.

Definition 2.1(Pre open and Pre Closed set)

A subset A of a Topological Space (X, τ) is called Preopen set if $A \subseteq Int(Cl(A))$. The Complement of Preopen set is called Pre closed set.

Definition 2.2 (GS_β-Preopen and GS_β-Preclosed sets)

A subset A of a Topological Space (X, τ, G) is called GS_β-Preopen set if $A \subseteq Int(Cl(\psi(A))$. The complement of GS_β-Preopen set is called GS_β-Pre closed set.

Definition 2.3 (GS_β-Pre continuous function)

A function $f:(X, \tau, G) \to (Y, \sigma, H)$ is called GS_β-Pre continuous function if $f^{-1}(U)$ is GS_β-Preopen set in X for every GS_β-Preopen set U in Y.

Definition 2.4 (Disconnected space)

A Topological Space (X, τ) is disconnected Space if it is union of two nonempty subsets A and B such that $Cl(A) \cap B = \emptyset$ and $A \cap Cl(B) = \emptyset$.

Definition 2.5 (Compact Space)

A Topological Space (X, τ) is called a Compact Space if every open cover of A has a finite sub cover

Definition 2.6 (Strongly Compact Set)

A subset A of a topological space (X, τ) is called Strongly Compact Space in X if every Preopen cover of A has a finite subcover.

3. GS_β-Connected Sets

Definition 3.1

Let (X, τ, G) be a grill topological space and A, B be two nonempty subsets of X. The sets A and B are called a GS_β-Seperated sets if $GS_\beta\ Cl(A) \cap B = \emptyset$ and $A \cap GS_\beta\ Cl(B) = \emptyset$.

Definition 3.2

A grill topological space (X, τ, G) is called GS_β-disconnected space if it is the union of two GS_β-Seperated sets. Otherwise, a grill topological space (X, τ, G) is called a GS_β-Connected space.

Theorem 3.3

Any grill topological space (X, τ, G) with a finite set X is a GS_β-disconnected space if X has more than one point

Proof:

The Proof of the Theorem is obvious

Theorem 3.4

Every disconnected space is a GS_β-disconnected space

Proof:

The Proof of the Theorem is clear since $GS_\beta\ Cl(A) \subset Cl(A)$

Theorem 3.5

A grill topological space (X, τ, G) is GS_β-disconnected space if and only if it is the union of two disjoint nonempty GS_β-Preopen sets

Proof:

Suppose that (X, τ, G) is GS_β-disconnected space. Then X is the union of two GS_β-Seperated sets, that is, there are two non empty subsets A and B of X such that $GS_\beta\ Cl(A) \cap B = \emptyset$, $A \cap GS_\beta\ Cl(B) = \emptyset$ and

$A \cup B = X$. Take $G = X - GS_\beta\ Cl(A)$ and $H = X - GS_\beta\ Cl(B)$. Then G and H are GS_β Preopen sets. That is, $G \cap H = \emptyset$ and

$$G \cup H = \left(X - GS_\beta\ Cl(A)\right) \cap \left(X - GS_\beta\ Cl(B)\right)$$
$$= X - (GS_\beta\ Cl(A) \cap GS_\beta\ Cl(B))$$
$$\subseteq X - (GS_\beta\ Cl(A) \cap B)$$
$$= X - \emptyset = X$$

Conversely, suppose that (X, τ, G) is the union of disjoint nonempty GS_β Preopen subsets, say G and H. Take $A = X - G$ and $B = X - H$. Then A and B are GS_β Preopen sets, that is $GS_\beta\ Cl(A) = A$ and $GS_\beta\ Cl(B) = B$. Since $H \neq \emptyset$ and $H \cap G = \emptyset$, then $H \subseteq X - G = A$, that is $A \neq \emptyset$. So $GS_\beta\ Cl(A) \cap B = A \cap B = (X - G) \cap (X - H)$

$$= X - (G \cup H) = X - X = \emptyset$$

Similar, $A \cap GS_\beta\ Cl(B) = \emptyset$. Note that

$$A \cup B = (X - G) \cup (X - H) = X - (G \cap H) = X - \emptyset = X$$

That is, (X, τ, G) is a GS_β- disconnected space.

4. GS_β-Pre compact Spaces

Definition 4.1

Let (X, τ, G) be a grill topological space and $A \subseteq X$. A is called GS_β-Precompact set in a grill topological space (X, τ, G) if for every GS_β-Preopen cover $\{G_\lambda : \lambda \epsilon I\}$ of A has finite GS_β-Preopen sub cover $\{G_{\lambda_K} : K = 1, 2, \dots n\}$ of A such that $A \subseteq \bigcup_{K=1}^{n} G_{\lambda_K}$. Similarly , X is called a GS_β-Pre compact space if $X = \bigcup_{K=1}^{n} G_{\lambda_K}$

Theorem 4.2

Every GS_β-Pre compact set is compact set.

Proof:

The Proof of the theorem is clear, since every open set is GS_β-Preopen set.

Theorem 4.3

Every strongly Compact space is GS_β-Pre compact space

Proof:

Let $\{G_\lambda : \lambda \epsilon I\}$ be a GS_β-Preopen cover of A be a grill topological space and $\{G_\lambda : \lambda \epsilon I\}$ be a GS_β-Preopen cover of X and $A \subseteq \bigcup_{\lambda \epsilon I} G_\lambda$. By the definition of Strongly Compact space, $\{G_\lambda : \lambda \epsilon I\}$ is a Preopen cover of X. Since (X, τ) is Strongly Compact Space, then $\{G_\lambda : \lambda \epsilon I\}$ has finite GS_β-Preopen subcover. Hence by the above theorem, X is a GS_β-Pre compact Space.

Theorem 4.4

Every GS_β-Pre compact subset of GS_β-Pre compact space is GS_β-Pre compact set.

Proof:

Suppose that F is a GS_β-Pre closed subset of GS_β-Pre compact space (X, τ, G). Let $\{V_\lambda : \lambda \epsilon I\}$ be any GS_β-Preopen cover of F, where I is an index set. Since F is a GS_β-Pre closed set in (X, τ, G) then $X - F$ is a GS_β-Preopen in (X, τ, G). Then $\{(X - F), V_\lambda : \lambda \epsilon I\}$ is GS_β Preopen cover of (X, τ, G). Since (X, τ, G) is a GS_β-Pre compact space then there is a finite sub cover $\{(X - F), V_{\lambda K} : K = 1, 2, \dots . m\}$. Hence $F \subseteq \bigcup_{k=1}^{m} V_{\lambda k}$. Hence F is a GS_β-Pre compact space.

5. GS_β- Seperation Axioms

Definition 5.1

A Grill Topological Space (X, τ, G) is called GS_β-T_2 if for any two points $x \neq y \in X$, there are two GS_β-Preopen sets G and U in X such that $x \in G$ and $y \in U$ and $U \cap G = \emptyset$

Definition 5.2

A Grill Topological Space (X, τ, G) is called GS_β-regular space if for each closed set F in (X, τ, G) and each $x \notin F$, there exists two GS_β-Preopen sets G and U in (X, τ, G) such that $F \subseteq G$, $x \in U$ and $U \cap G = \emptyset$. A Grill Topological Space (X, τ, G) is called GS_β-T_3 space if it is GS_β-regular space and T_1 space.

Definition 5.3

A Grill Topological Space (X, τ, G) is called GS_β-normal space if for each two disjoint closed set F and M in (X, τ, G), there are two GS_β-Preopen sets G and U in (X, τ, G) such that $F \subseteq G$, $M \subseteq U$ and $U \cap G = \emptyset$. A Grill Topological Space (X, τ, G) is called GS_β-T_4 space if it is GS_β-normal space and T_1 space.

Theorem 5.4

If (X, τ) is T_i space then the grill topological space (X, τ, G) is GS_β-T_i space for all $i = 2,3,4$.

Proof:

The Proof of the theorem is clear since every open set is GS_β-Preopen set.

Theorem 5.5

Every GS_β-T_3 space is a GS_β-T_2 space

Proof:

Let X be a GS_β-T_3 space and $x \neq y \in X$ be any points in (X, τ, G). Since (X, τ) is a T_1 space then by known theorem,$\{x\}$ is a closed set in (X, τ) and $y \notin \{x\}$. Since (X, τ, G) is a GS_β regular space then there are two GS_β Preopen sets G and U in (X, τ, G) such that $x \in \{x\} \subseteq G$ and $U \cap G = \emptyset$. Hence (X, τ, G) is a GS_β-T_2 space.

Theorem 5.6

Every GS_β-T_4 space is a GS_β-T_3 space.

Proof:

Let X be a GS_β-T_4 space. Let F be any closed set in (X, τ, G) and $x \notin F$ be any point in (X, τ, G). Since (X, τ) is a T_1 space then by known theorem,$\{x\}$ is a closed set in (X, τ) and $F \cap \{x\} = \emptyset$. Since (X, τ, G) is GS_β normal space then there are two GS_β Preopen sets G and U in (X, τ, G) such that $x \in \{x\} \subseteq G$ and $U \cap G = \emptyset$. Hence (X, τ, G) is a GS_β-T_3 space.

References

[1] Choquet, G., **1947.** Sur les notions de filter et grill. *Comptes Rendus Acad. Sci. Paris*, *224*, pp 171-173.

[2] Thron, W.J., **1973.** Proximity structures and grills. *Mathematische Annalen*, *206*(1), pp 35-62.

[3] Corson, H.H. and Michael, E., **1964.** Metrizability of certain countable unions. *Illinois Journal of Mathematics*, *8*(2), pp 351-360.

[4] Dontchev, J., **1995.** On generalizing semi-preopen sets. *Mem. Fac. Sci. Kochi Univ. Ser. A, (Math.)*, *16*, pp. 35-48.

[5] Al-Omari, A., and Noiri, T., **2011.** Decomposition of continuity via grilles. *Jordan J. Math and Stat*, *4*(1), pp. 33-46.

[6] Janaki, C. and Arockiarani, I., **2011.** γ-open sets and decomposition of continuity Via grills. *Int. J. of Math. Archive-2* (7) pp. 1087-1093.

[7] Njåstad, O., **1965.** On some classes of nearly open sets. *Pacific journal of mathematics*, *15*(3), pp. 961-970.

[8] Karthikeyan, N. and Rajesh, N., **2015.** Faint continuous via topological grills. *Int. Jour. of Math. Archive*, *6*, pp. 24-28.

[9] Reilly, I.L. and Vamanamurthy, M.K., **1985.** On α-continuity in topological spaces. *Acta Mathematica Hungarica*, *45*(1-2), pp. 27-32.

[10] Aluja, J.G. and Lafuente, A.M.G., **2012.** Towards an advanced modelling of complex economic phenomena. *Studies in fuzziness and soft computing*, 276.