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# ON DECOMPOSITIONS OF NEW TYPES OF $\tilde{g}$ (1,2)*-CONTINUOUS MAPS IN BITOPOLOGICAL SPACES

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**Abstract:** In this paper, we first introduce $\tilde{g}$ (1,2)*-continuous maps and study their relations with various generalized (1,2)*-continuous maps. We also discuss some decompositions of $\tilde{g}$ (1,2)*-continuous maps.

**Key words and Phrases:** $\tilde{g}$ (1,2)*-open set, $\tilde{g}$ (1,2)*-continuous maps, $T_{\tilde{g}\,(1,2)^*}$-space.

## 1.INTRODUCTION

Kelly [6] introduced the concepts of bitopological spaces. Recently Sheik John [18] introduced and studied another form of generalized continuous maps called $\omega$-continuous maps respectively. Levine [7], introduced the generalized closed sets in topology. Abd El-Monsef and et al. [1], introduced the β-open sets and β-continuous mapping, Andrijevic [2], introduced semi-preopen sets. Arya and et al. [3], introduced the characterization of s-normal spaces. Bhattacharya [4], introduced semi-generalized closed sets in topology. Duszynski [5], introduced a new generalization of closed sets in bitopology. Rajamani and et al. [9], introduced on αgs-closed sets in topological spaces. Ravi and et al. [13], on stronger forms of (1,2)*-quotient mappings in bitopological spaces.

In this paper, we first introduce $\tilde{g}$ (1,2)*-continuous maps and study their relations with various generalized (1,2)*-continuous maps. We also discuss some decompositions of $\tilde{g}$ (1,2)*-continuous maps.

## 2.PRELIMINARIES

Throughout this paper, $(X, \tau_1, \tau_2)$ (briefly, X) will denote bitopological space (briefly, BTPS).

**Definition 2.1** Let H be a subset of X. Then H is said to be $\tau_{1,2}$-open [11] if $H = P \cup Q$ where $P \in \tau_1$ and $Q \in \tau_2$.

The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed.

Notice that $\tau_{1,2}$-open sets need not necessarily form a topology.

**Definition 2.2 [11]** Let H be a subset of a bitopological space X. Then

(i) the $\tau_{1,2}$-closure of H, denoted by $\tau_{1,2}$-cl(H), is defined as $\cap \{F : H \subseteq F$ and F is $\tau_{1,2}$-closed$\}$.

(ii) the $\tau_{1,2}$-interior of H, denoted by $\tau_{1,2}$-int(H), is defined as $\cup \{F : F \subseteq H$ and F is $\tau_{1,2}$-open$\}$.

**Definition 2.3** A subset H of a BTPS X is called:
(i) (1,2)*-semi-open set [14] if $H \subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int(H));
(ii) (1,2)*-preopen set [10] if $H \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl(H));
(iii) (1,2)*-$\alpha$ -open set [8] if $H \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(H)));
(iv) regular (1,2)*-open set [10] if $H = \tau_{1,2}$-int($\tau_{1,2}$-cl(H)).

The complements of the above-mentioned open sets are called their respective closed sets.

**Definition 2.4**

A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

(i) (1,2)*-ĝ-continuous [15] if $f^{-1}(V)$ is a (1,2)*-ĝ-closed set of X for every $\sigma_{1,2}$-closed set V of Y.

(ii) (1,2)*-g-continuous [16] if $f^{-1}(V)$ is a (1,2)*-g-closed set of X for every $\sigma_{1,2}$-closed set V of Y.

(iii) (1,2)*- $\ddot{g}$ -continuous [17] if $f^{-1}(V)$ is an (1,2)*- $\ddot{g}$ -closed set of X for every $\sigma_{1,2}$-closed set V of Y.

(iv) (1,2)*-semi-continuous [11] if $f^{-1}(V)$ is a (1,2)*-semi-open set of X for every $\sigma_{1,2}$-open set V of Y.

(v) (1,2)*- $\alpha$ -continuous [10] if $f^{-1}(V)$ is an (1,2)*- $\alpha$ -closed set of X for every $\sigma_{1,2}$-closed set V of Y.

(vi) (1,2)*-continuous [11] if $f^{-1}(V)$ is a $\tau_{1,2}$-closed set of X for every $\sigma_{1,2}$-closed set V of Y.

## 3. $\tilde{g}$ (1,2)*-CONTINUOUS MAPS

We introduce the following definitions:

**Definition 3.1**

(i) A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called a $\tilde{g}$ (1,2)*-continuous if the inverse image of every $\sigma_{1,2}$-closed set in Y is $\tilde{g}$ (1,2)*-closed set in X.

(ii) A bitopological space X is called a $T_{\tilde{g}(1,2)*}$-space if every $\tilde{g}$ (1,2)*-closed subset of X is $\tau_{1,2}$-closed in X.

**Example 3.2**

(i) Let X = Y = {a, b, c}, $\tau_1 = \{\phi, X, \{c\}\}$ and $\tau_2 = \{\phi, X, \{a, c\}\}$. Then the sets in {ϕ, X, {c}, {a, c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b}, {a, b}} are called $\tau_{1,2}$-closed. Let $\sigma_1 = \{\phi, Y, \{b\}\}$ and $\sigma_2 = \{\phi, Y, \{a, b\}\}$. Then the sets in {ϕ, Y, {b}, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}, {a, c}} are called $\sigma_{1,2}$-closed. We have (1,2)*-$\ddot{G}$C(X) = {ϕ, {b}, {a, b}, X}. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is (1,2)*- $\ddot{g}$ -continuous.

(ii) Let X = Y = {a, b, c}, $\tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{b, c\}\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$ and $\sigma_2 = \{\phi, Y, \{b, c\}\}$. Then the identity function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $\tilde{g}$ (1,2)*-continuous.

**Proposition 3.3**

A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $\tilde{g}$ (1,2)*-continuous if and only if $f^{-1}(U)$ is $\tilde{g}$ (1,2)*-open in X for every $\sigma_{1,2}$-open set U in Y.

**Proof**

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be $\tilde{g}$ (1,2)*-continuous and U be an $\sigma_{1,2}$-open set in Y. Then $U^c$ is $\sigma_{1,2}$-closed in Y and since f is $\tilde{g}$ (1,2)*-continuous, $f^{-1}(U^c)$ is $\tilde{g}$ (1,2)*-closed in X. But $f^{-1}(U^c) = (f^{-1}(U))^c$ and so $f^{-1}(U)$ is $\tilde{g}$ (1,2)*-open in X.

Conversely, assume that $f^{-1}(U)$ is $\tilde{g}$ (1,2)*-open in X for each $\sigma_{1,2}$-open set U in Y. Let F be a $\sigma_{1,2}$-closed set in Y. Then $F^c$ is $\sigma_{1,2}$-open in Y and by assumption, $f^{-1}(F^c)$ is $\tilde{g}$ (1,2)*-open in X. Since $f^{-1}(F^c) = (f^{-1}(F))^c$, we have $f^{-1}(F)$ is $\tilde{g}$ (1,2)*-closed in X and so f is $\tilde{g}$ (1,2)*-continuous.

**Remark 3.4**

The composition of two $\tilde{g}$ (1,2)*-continuous maps need not be $\tilde{g}$ (1,2)*-continuous and this is shown by the following example.

**Example 3.5**

Let X = Y = Z = {a, b, c}, $\tau_1 = \{\phi, X, \{a\}\}$ and $\tau_2 = \{\phi, X, \{b, c\}\}$. Then the sets in $\{\phi, X, \{a\}, \{b, c\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, X, \{a\}, \{b, c\}\}$ are called $\tau_{1,2}$-closed. Let $\sigma_1 = \{\phi, Y, \{a, b\}\}$ and $\sigma_2 = \{\phi, Y\}$. Then the sets in $\{\phi, Y, \{a, b\}\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, Y, \{c\}\}$ are called $\sigma_{1,2}$-closed. Let $\eta_1 = \{\phi, Z, \{b\}\}$ and $\eta_2 = \{\phi, Z, \{a, b\}\}$. Then the sets in $\{\phi, Z, \{b\}, \{a, b\}\}$ are called $\eta_{1,2}$-open and the sets in $\{\phi, Z, \{c\}, \{a, c\}\}$ are called $\eta_{1,2}$-closed. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be the identity maps. Then f and g are $\tilde{g}$ (1,2)*-continuous but their $g \circ f : (X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$ is not $\tilde{g}$ (1,2)*-continuous, because V = {a, c} is $\eta_{1,2}$-closed in Z but $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V)) = f^{-1}(g^{-1}(\{a, c\}) = f^{-1}(\{a, c\}) = \{a, c\}$, which is not $\tilde{g}$ (1,2)*-closed in X.

**Proposition 3.6**

Let $(X, \tau_1, \tau_2)$ and $(Z, \eta_1, \eta_2)$ be two bitopological spaces and $(Y, \sigma_1, \sigma_2)$ be a $T_{\tilde{g}(1,2)*}$-space. Then the composition $g \circ f : (X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$ of the $\tilde{g}$ (1,2)*-continuous maps $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ is $\tilde{g}$ (1,2)*-continuous.

**Proof**

Let F be any $\eta_{1,2}$-closed set of $(Z, \eta_1, \eta_2)$. Then $g^{-1}(F)$ is $\tilde{g}$ (1,2)*-closed in $(Y, \sigma_1, \sigma_2)$, since g is $\tilde{g}$ (1,2)*-continuous. Since Y is a $T_{\tilde{g}(1,2)*}$-space, $g^{-1}(F)$ is $\sigma_{1,2}$-closed in Y. Since f is $\tilde{g}$ (1,2)*-continuous, $f^{-1}(g^{-1}(F))$ is $\tilde{g}$ (1,2)*-closed in X. But $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ and so $g \circ f$ is $\tilde{g}$ (1,2)*-continuous.

**Proposition 3.7**

If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $\tilde{g}$ (1,2)*-continuous and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ is (1,2)*-continuous, then their composition $g \circ f : (X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$ is $\tilde{g}$ (1,2)*-continuous.

**Proof**

Let F be any $\eta_{1,2}$-closed set in $(Z, \eta_1, \eta_2)$. Since $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ is (1,2)*-continuous, $g^{-1}(F)$ is $\sigma_{1,2}$-closed in $(Y, \sigma_1, \sigma_2)$. Since $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $\tilde{g}$ (1,2)*-continuous, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is $\tilde{g}$ (1,2)*-closed in X and so $g \circ f$ is $\tilde{g}$ (1,2)*-continuous.

**Proposition 3.8**

If A is $\tilde{g}$ (1,2)*-closed in X and if $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1,2)*-$\hat{g}$-irresolute and (1,2)*-$\alpha$-closed, then f(A) is $\tilde{g}$ (1,2)*-closed in Y.

**Proof**

Let U be any (1,2)*- $\hat{g}$ -open in Y such that f(A) $\subseteq$ U. Then A $\subseteq$ $f^{-1}$(U) and by hypothesis, (1,2)*-αcl(A) $\subseteq$ $f^{-1}$(U). Thus f((1,2)*-αcl(A)) $\subseteq$ U and f((1,2)*-αcl(A)) is a (1,2)*-α-closed set. Now, (1,2)*-αcl(f(A)) $\subseteq$ (1,2)*-αcl(f((1,2)*-αcl(A))) = f((1,2)*-αcl(A)) $\subseteq$ U. That is (1,2)*-αcl(f(A)) $\subseteq$ U and so f(A) is $\tilde{g}$ (1,2)*-closed.

**Theorem 3.9**

Let f : (X, $\tau_1$, $\tau_2$) → (Y, $\sigma_1$, $\sigma_2$) be a pre-(1,2)*- $\hat{g}$ -closed and (1,2)*-open bijection and if X is a $T_{\tilde{g}(1,2)*}$-space, then Y is also a $T_{\tilde{g}(1,2)*}$-space.

**Proof**

Let y ∈ Y. Since f is bijective, y = f(x) for some x ∈ X. Since X is a $T_{\tilde{g}(1,2)*}$-space, {x} is (1,2)*- $\hat{g}$ -closed or $\tau_{1,2}$-open. If {x} is (1,2)*- $\hat{g}$ -closed then {y} = f({x}) is (1,2)*- $\hat{g}$ -closed, since f is pre-(1,2)*- $\hat{g}$ -closed. Also {y} is $\sigma_{1,2}$-open if {x} is $\tau_{1,2}$-open since f is (1,2)*-open. Therefore Y is a $T_{\tilde{g}(1,2)*}$-space.

**Theorem 3.10**

If f : (X, $\tau_1$, $\tau_2$) → (Y, $\sigma_1$, $\sigma_2$) is $\tilde{g}$ (1,2)*-continuous and pre-(1,2)*- $\hat{g}$ -closed and if A is an $\tilde{g}$ (1,2)*-open (or $\tilde{g}$ (1,2)*-closed) subset of Y, then $f^{-1}$(A) is $\tilde{g}$ (1,2)*-open (or $\tilde{g}$ (1,2)*-closed) in X.

**Proof**

Let A be an $\tilde{g}$ (1,2)*-open set in Y and F be any (1,2)*- $\hat{g}$ -closed set in X such that F $\subseteq$ $f^{-1}$(A). Then f(F) $\subseteq$ A. By hypothesis, f(F) is (1,2)*- $\hat{g}$ -closed and A is $\tilde{g}$ (1,2)*-open in Y. Therefore, f(F) $\subseteq$ (1,2)*-α-int(A) and so F $\subseteq$ $f^{-1}$((1,2)*-α-int(A)). Since f is $\tilde{g}$ (1,2)-continuous and $\sigma_{1,2}$-int(A) is $\sigma_{1,2}$-open in Y, $f^{-1}$($\sigma_{1,2}$-int(A)) is $\tilde{g}$ (1,2)*-open in X. Thus F $\subseteq$ (1,2)*-α-int($f^{-1}$($\sigma_{1,2}$-int(A))) $\subseteq$ (1,2)*-α-int($f^{-1}$(A)). That is F $\subseteq$ (1,2)*-α-int($f^{-1}$(A)) and $f^{-1}$(A) is $\tilde{g}$ (1,2)*-open in X. By taking complements, we can show that if A is $\tilde{g}$ (1,2)*-closed in Y, $f^{-1}$(A) is $\tilde{g}$ (1,2)*-closed in X.

**Corollary 3.11**

Let (X, $\tau_1$, $\tau_2$), (Y, $\sigma_1$, $\sigma_2$) and (Z, $\eta_1$, $\eta_2$) be any three bitopological spaces. If f : (X, $\tau_1$, $\tau_2$) → (Y, $\sigma_1$, $\sigma_2$) is $\tilde{g}$ (1,2)*-continuous and pre-(1,2)*- $\hat{g}$ -closed and g : (Y, $\sigma_1$, $\sigma_2$) → (Z, $\eta_1$, $\eta_2$) is $\tilde{g}$ (1,2)*-continuous, then their composition g o f : (X, $\tau_1$, $\tau_2$)→ (Z, $\eta_1$, $\eta_2$) is $\tilde{g}$ (1,2)*-continuous.

**Proof**

Let F be any $\eta_{1,2}$-closed set in (Z, $\eta_1$, $\eta_2$). Since g: (Y, $\sigma_1$, $\sigma_2$) → (Z, $\eta_1$, $\eta_2$) is $\tilde{g}$ (1,2)*-continuous, $g^{-1}$(F) is $\tilde{g}$ (1,2)*-closed in Y. Since f : (X, $\tau_1$, $\tau_2$) → (Y, $\sigma_1$, $\sigma_2$) is $\tilde{g}$ (1,2)*-continuous and pre-(1,2)*- $\hat{g}$ -closed, then $f^{-1}$($g^{-1}$(F)) = $(g \circ f)^{-1}$(F) is $\tilde{g}$ (1,2)*-closed in X and so g o f is $\tilde{g}$ (1,2)*-continuous.

**Definition 3.12**

Let x be a point of X and A be a subset of a bitopological space X. Then A is called an $\tilde{g}$ (1,2)*-neighbourhood of x (briefly, $\tilde{g}$ (1,2)*-nbhd of x) in X if there exists an $\tilde{g}$ (1,2)*-open set U of X such that x ∈ U ⊂ A.

**Proposition 3.13**

Let A be a subset of a bitopological space X. Then x ∈ $\tilde{g}$ (1,2)*-cl(A) if and only if for any $\tilde{g}$ (1,2)*-nbhd $G_x$ of x in X, A ∩ $G_x$ ≠ ϕ.

**Proof**

Necessity. Assume x ∈ $\tilde{g}$ (1,2)*-cl(A). Suppose that there is an $\tilde{g}$ (1,2)*-nbhd G of the point x in X such that G ∩ A = ϕ. Since G is $\tilde{g}$ (1,2)*-nbhd of x in X, by Definition 3.12, there exists an $\tilde{g}$ (1,2)*-open set $U_x$ such that x ∈ $U_x$ $\subseteq$ G. Therefore, we have $U_x$ ∩ A = ϕ and so A $\subseteq$ $(U_x)^c$. Since $(U_x)^c$ is an $\tilde{g}$ (1,2)*-

closed set containing A, we have $\tilde{g}$ (1,2)*-cl(A) $\subseteq$ $(U_x)^c$ and therefore x $\notin$ $\tilde{g}$ (1,2)*-cl(A), which is a contradiction.

Sufficiency. Assume for each $\tilde{g}$ (1,2)*-nbhd $G_x$ of x in X, A $\cap$ $G_x \neq \phi$. Suppose x $\notin$ $\tilde{g}$ (1,2)*-cl(A). Then, there exists a $\tilde{g}$ (1,2)*-closed set F of X such that A $\subseteq$ F and x $\notin$ F. Thus x $\in$ $F^c$ and $F^c$ is $\tilde{g}$ (1,2)*-open in X and hence $F^c$ is a $\tilde{g}$ (1,2)*-nbhd of x in X. But A $\cap$ $F^c = \phi$, which is a contradiction.

In the next theorem we explore certain characterizations of $\tilde{g}$ (1,2)*-continuous functions.

**Theorem 3.14**

Suppose the collection of all $\tilde{g}$ (1,2)*-open sets of X is closed under arbitrary unions. Let f : (X, $\tau_1$, $\tau_2$ )→ (Y, $\sigma_1$, $\sigma_2$) be a map from a bitopological space (X, $\tau_1$, $\tau_2$) into a bitopological space (Y, $\sigma_1$, $\sigma_2$). Then the following statements are equivalent.

(i) The function f is $\tilde{g}$ (1,2)*-continuous.

(ii) The inverse of each $\sigma_{1,2}$-open set is $\tilde{g}$ (1,2)*-open.

(iii) For each point x in X and each $\sigma_{1,2}$-open set V in Y with f(x) $\in$ V, there is an $\tilde{g}$ (1,2)*-open set U in X such that x $\in$ U, f(U) $\subseteq$ V.

(iv) The inverse of each $\sigma_{1,2}$-closed set is $\tilde{g}$ (1,2)*-closed.

(v) For each x in X, the inverse of every neighborhood of f(x) is an $\tilde{g}$ (1,2)*-nbhd of x.

(vi) For each x in X and each neighborhood N of f(x), there is an $\tilde{g}$ (1,2)*-nbhd G of x such that f(G) $\subseteq$ N.

(vii) For each subset A of X, f( $\tilde{g}$ (1,2)*-cl(A)) $\subseteq$ $\sigma_{1,2}$-cl(f(A)).

(viii) For each subset B of Y, $\tilde{g}$ (1,2)*-cl($f^{-1}$(B)) $\subseteq$ $f^{-1}$($\sigma_{1,2}$-cl(B)).

**Proof**

(i) ⇔ (ii). It is trivial.

(i) ⇔ (iii). Suppose that (iii) holds and let V be an $\sigma_{1,2}$-open set in Y and let x $\in$ $f^{-1}$(V). Then f(x) $\in$ V and thus there exists an $\tilde{g}$ (1,2)*-open set $U_x$ such that x $\in$ $U_x$ and f($U_x$) $\subseteq$ V. Now, x $\in$ $U_x \subseteq f^{-1}$(V) and $f^{-1}(V) = \cup_{x \in f^{-1}(V)} U_x$. Then $f^{-1}$(V) is $\tilde{g}$ (1,2)*-open in X and therefore f is $\tilde{g}$ (1,2)*-continuous.

Conversely, suppose that (i) holds and let f(x) $\in$ V where V is $\sigma_{1,2}$-open in Y. Then $f^{-1}$(V) $\in$ (1,2)*-$\tilde{G}$O(X), since f is $\tilde{g}$ (1,2)*-continuous. Let U = $f^{-1}$(V). Then x $\in$ U and f(U) $\subseteq$ V.

(ii) ⇔ (iv). This result follows from the fact if A is a subset of Y, then $f^{-1}(A^c) = (f^{-1}(A))^c$.

(ii) ⇒ (v). For x in X, let N be a neighborhood of f(x). Then there exists an $\sigma_{1,2}$-open set U in Y such that f(x) $\in$ U $\subseteq$ N. Consequently, $f^{-1}$(U) is an $\tilde{g}$ (1,2)*-open set in X and x $\in$ $f^{-1}(U) \subseteq f^{-1}(N)$. Thus $f^{-1}$(N) is an $\tilde{g}$ (1,2)*-nbhd of x.

(v) ⇒ (vi). Let x $\in$ X and let N be a neighborhood of f(x). Then by assumption, G = $f^{-1}$(N) is an $\tilde{g}$ (1,2)*-nbhd of x and f(G) = f($f^{-1}$(N)) $\subseteq$ N.

(vi) ⇒ (iii). For x in X, let V be an $\sigma_{1,2}$-open set containing f(x). Then V is a neighborhood of f(x). So by assumption, there exists an $\tilde{g}$ (1,2)*-nbhd G of x such that f(G) $\subseteq$ V. Hence there exists an $\tilde{g}$ (1,2)*-open set U in X such that x $\in$ U $\subseteq$ G and so f(U) $\subseteq$ f(G) $\subseteq$ V.

(vii) ⇔ (iv). Suppose that (iv) holds and let A be a subset of X. Since A $\subseteq$ $f^{-1}$(f(A)), we have A $\subseteq$ $f^{-1}$($\sigma_{1,2}$-cl(f(A))). Since $\sigma_{1,2}$-cl(f(A)) is a $\sigma_{1,2}$-closed set in Y, by assumption $f^{-1}$($\sigma_{1,2}$-cl(f(A))) is an $\tilde{g}$ (1,2)*-closed set containing A. Consequently, $\tilde{g}$ (1,2)*-cl(A) $\subseteq$ $f^{-1}$($\sigma_{1,2}$-cl(f(A))). Thus f( $\tilde{g}$ (1,2)*-cl(A)) $\subseteq$ f($f^{-1}$($\sigma_{1,2}$-cl(f(A)))) $\subseteq$ $\sigma_{1,2}$-cl(f(A)).

Conversely, suppose that (vii) holds for any subset A of X. Let F be a $\sigma_{1,2}$-closed subset of Y. Then by assumption, f( $\tilde{g}$ (1,2)*-cl($f^{-1}$(F))) $\subseteq$ $\sigma_{1,2}$-cl(f($f^{-1}$(F))) $\subseteq$ $\sigma_{1,2}$-cl(F) = F. That is $\tilde{g}$ (1,2)*-cl($f^{-1}$(F)) $\subseteq$ $f^{-1}$(F) and so $f^{-1}$(F) is $\tilde{g}$ (1,2)*-closed.

(vii) ⇔ (viii). Suppose that (vii) holds and B be any subset of Y. Then replacing A by $f^{-1}(B)$ in (vii), we obtain $f(\tilde{g}$ (1,2)*-cl($f^{-1}(B))) \subseteq \sigma_{1,2}$-cl($f(f^{-1}(B))) \subseteq \sigma_{1,2}$-cl(B). That is $\tilde{g}$ (1,2)*-cl($f^{-1}(B)) \subseteq f^{-1}(\sigma_{1,2}$-cl(B)).

Conversely, suppose that (viii) holds. Let B = f(A) where A is a subset of X. Then we have, $\tilde{g}$ (1,2)*-cl(A) $\subseteq \tilde{g}$ (1,2)*-cl($f^{-1}(B)) \subseteq f^{-1}(\sigma_{1,2}$-cl(f(A))) and so $f(\tilde{g}$ (1,2)*-cl(A)) $\subseteq \sigma_{1,2}$-cl(f(A)). This completes the proof of the theorem.

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