Stimulus conduct of the property of common limit in the range of 'g' for fuzzy metric

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Abstract.

Fuzzy set theory has very wide assortment applications owed to perception of fuzzy set and fuzzy logic. More frequently to attest the fixed-point theorem for approximately maps is easy to prove for continuity, closeness and completeness for the space. In this paper we confer the conduct of the property of common limit in the range of 'g' for fuzzy metric. Our result encompasses and generalizes some result in fuzzy metric spaces. We also equip an example which satisfies our main result in this paper.

Key Words: Fuzzy set theory, Fuzzy Logic, Fuzzy Metric, range of 'g'

1. Introduction

The study on uncertainty and on randomness began to explore with the concept of fuzziness in mathematics. Fuzzy set is used in fuzzy metric space, which is initiated by Lofti. A. Zadeh [1]. Compatible mapping is generalized from commutatively mappings by Jungck [2]. After that Jungck and Rhodes [3] initiated the notion of weak compatible and proved that compatible maps are weakly compatible but converse is not true. Jungck and Rhoades defined the concepts of d-compatible and weakly compatible mappings as postponements of the perception of companionable mapping for single-valued mappings on metric spaces. Numerous authors hand-me-down these perceptions to attest some common fixed point theorems. Jungck prolonged the class of non-commuting mappings by compatible mappings and prolonged the class of non-commuting mappings by compatible mappings.

Grabic [4] extended the fixed point theorems of Banach and Edelstein to complete and compact fuzzy metric spaces respectively. Subramanyam[5] generalized the concept of Grabic [4] for a pair of commuting mapping. Pant[6,7,8] brought the concept of R-weakly commuting maps in metric spaces. Pathak [9] generalized R-weakly commuting maps and introduced the concept of R-weakly commuting maps of type (A_g).

2. Preliminaries

Definition-1: The 3-tuple $(X, M, ')$ is called a fuzzy metric space (Shortly, FM-space) if X is an arbitrary set, $(')$is a continuous t-norm and M is a fuzzy set in X^2 x [0,1]satisfying following conditions: for all $x, y, z \in X$and$s, t > 0$,

(AM-1)$M(x, y, 0) = 0$

(AM-2)$M(x, y, t) = 1$, for all t>0 if and only if $x = y$

(AM-3)$M(x, y, t) = M(y, x, t)$

(AM-4)$M(x, y, t)'M(y, z, s) \leq M(x, z, t + s)$

(AM-5)$M(x, y, .): [0,1) \rightarrow [0,1]$ is left continuous.

Definition-2: Two self- maps $P$and $Q$of fuzzy metric space$(X, M,*)$ into itself is said to satisfy the E.A Property if there exist a sequences $\{x_n\}$ in X such that

$\lim_{n\to\infty} M(Px_n, Qx_n, t) = 1, t > 0$

Definition-3: A function $\phi: [0, \infty) \rightarrow [0, \infty)$ is said to be contractive modulus if $\phi: [0, \infty) \rightarrow [0, \infty)$ and $\phi(t) < t$ for $t > 0$

Definition-4: [10] A 3-tuple (X, M, T) is said to be a fuzzy metric space if X is an arbitrary set, T is a continuous t-norm, and M is fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions for all $x, y, z \in X$ and $s, t > 0$

(i) $M(x, y, 0) > 0$;

(ii) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$;

(iii) $M(x, y, t) = M(y, x, t)$;

(iv) $T(M(x, y, t) , M(y, z, s)) \leq M(x, z, t + s)$;

(v) $M(x, y, .) : [0, \infty) \rightarrow [0, 1]$ is left continuous.

Then (X, M, T) is called a fuzzy metric space on X. The function $M(x, y, t)$ denote the degree of nearness between x and y w.r.t. t respectively.

Illustration-1: Let $X = \{1/n : n \in N \} \cup \{0\}$ and let T be the continuous t-norm defined by $T(a, b) = ab$ (or $T(a, b) = \min\{a, b\}$) respectively,

for all $a, b \in [0,1]$. For each $t > 0$ and $x, y \in X$, define (X, M, T) by

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|}, t > 0, \\ 0t = 0 \end{cases}$$

Clearly, (X, M, T) is a complete fuzzy metric space.

3. **Main Result**

Theorem : Let p and q are weakly compatible mappings of a fuzzy metric space (X, M, T) satisfying Common limit in the range of 'g' property satisfying inequality:

1. $M(px, py, kt) \geq M(qx, qy, t)$,k>0
2. $M(px, ppx, t) \geq max\{M(qx, qpx, t), M(px, qx, t), M(ppx, qpx, t), M(px, qpx, t), M(qx, ppx, t)\}$

   whenever $px \neq ppx$ if the range of p and q are subspace of X, then p and q have a common fixed point.

Proof: since p and q satisfy our property there exists a sequence $\{x_n\}$ in X such that $lim_{n\to\infty} p\, x_n = lim_{n\to\infty} q\, x_n = qx$ for some x in X.

Taking $x = y_n, y = x_n$, we have

$$M(Ay_n, Bx_n, kt) * [M(Cy_n, Ay_n, kt) \times M(Dx_n, Bx_n, kt)]$$

$$\geq \begin{Bmatrix} M(Cy_n, Ay_n, t) * M(Cy_n, Dx_n, t) \\ * M(Cy_n, Bx_n, t) \end{Bmatrix}$$

As taking$n \to \infty$, we obtain

$$M(Ay_n, p, kt) * [M(p, Ay_n, kt) \times M(p, p, kt)]$$

$$\geq \begin{Bmatrix} M(p, Ay_n, t) * M(p, p, t) \\ * M(p, p, t) \end{Bmatrix}$$

$\Rightarrow M(Ay_n, p, kt) \geq \{M(Ay_n, p, t)$ By using lemma, we get

$$\lim_{n\to\infty} Ay_n = p \text{ and } \lim_{n\to\infty} Ay_n = p = \lim_{n\to\infty} Dy_n$$

Let suppose $C(X)$ is a complete subspace of X , then$p = C(q)$, for some$q \in X$

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Cy_n = \lim_{n\to\infty} Bx_n$$

$$= \lim_{n\to\infty} Dx_n = p = C(q)$$

We will claim that $A(q) = C(q)$

Taking $x = q, y = x_n$

$M(Aq, Bx_n, kt) * [M(Cq, Aq, kt) \times M(Dx_n, Bx_n, kt)]$

$$\geq \begin{Bmatrix} M(Cq, Aq, t) * M(Cq, Dx_n, t) \\ * M(Cq, Bx_n, t) \end{Bmatrix}$$

As $n \to \infty$, we get

$M(Aq, q, kt) * [M(Cq, Aq, kt) \times M(p, p, kt)]$

$$\geq \begin{Bmatrix} M(Cq, Aq, t) * M(Cq, p, t) \\ * M(Cq, p, t) \end{Bmatrix} \Rightarrow M(Aq, Cq, kt) \geq \{M(Aq, Cq, t)$$

By using lemma

$\Rightarrow A(q) = C(q)$. This implies (A, C) have coincident point $q \in X$. By using conditions, the weak compatibility of (A, C) implies that $AC(q) = CA(q)$

$\Rightarrow AA(q) = AC(q) = CA(q) = CC(q)$

$\Rightarrow Cq = Aq = Dr$

We claim that $D(r) = B(r)$

Taking $x = q, y = r$in , we get

$M(Aq, Br, kt) * [M(Cq, Aq, kt) \times M(Dr, Br, kt)]$

$$\geq \begin{Bmatrix} M(Cq, Aq, t) * M(Cq, Dr, t) \\ * M(Cq, Br, t) \end{Bmatrix}$$

$D(r) = B(r)$

we get

$\Rightarrow Cq = Aq = Dr = Br$

Again by using the definition of weak compatibility of (B, D) implies that $BDr = DBr$

$\Rightarrow BDr = DBr = BBr = DDr$ We will prove that Aq is the common fixed point of A, B, C and D.

by taking $x = Aq, y = r$

$$M(AAq, Br, kt) * [M(CAq, AAq, kt) \times M(Dr, Br, kt)]$$

$$\geq \begin{Bmatrix} M(CAq, AAq, t) * M(CAq, Dr, t) \\ * M(CAq, Br, t) \end{Bmatrix}$$

Now we have $AAq = Br = Aq$. This implies $Aq = AAq = CAq$ is common fixed point of A and C.

Similarly, we prove that Br is the common fixed point of B and D.

$Aq = Br$, Aq is the fixed point of A, B, C and D.

Finally, we show the uniqueness of the common fixed point. If possible, let x' and y' be two fixed point of A, B, C and D. Then by taking $x = x', y = y'$

$$M(Ax', By', kt) * [M(Cx', Ax', kt) \times M(Dy', By', kt)]$$

$$\geq \begin{Bmatrix} M(Cx', Ax', t) * \\ M(Cx', Dy', t) * \\ M(Cx', By', t) \end{Bmatrix}$$

By using definition of fixed point and fuzzy metric spaces, we get $x' = y'$.

Thus, the mapping A, B, C and D have a unique common fixed point.

There exists some $u \in X$ p$u = qx$

Now we claim that p$u = qu$

If p$u \neq qu$ then $M(px_n, pu, kt) \geq M(qx_n, qu, t)$

Letting $n \to \infty$ we have $M(qx, pu, kt) \geq M(qx, qu, t)$ which implies p$u = qu$

Since p and q are weakly compatible then they must commute at their coincidence points which implies $pqu = qpu$

Now $M(pu, ppu, t) \geq max\{M(qu, qpu, t), M(pu, pu, t), M(ppu, pqu, t), M(pu, pqu, t), M(pu, qqu, t)\}$

$$M(pu, ppu, t) \geq max\{1, M(pu, pu, t), M(ppu, pqu, t), M(pu, pqu, t), M(pu, ppu, t)\}$$

$M(pu, ppu, t) \geq 1$ a contradiction implies $pu = ppu$.

Hence $pu = ppu = pqu = qpu = qqu$

Hence pu is a common fixed point of p and q . Hence the theorem.

Example 1: Let $(X, M, ')$ is standard fuzzy metric space, where $X = [0,1]$, such as

$$M(x,y,t) = \begin{Bmatrix} 0, t = 0 \\ \dfrac{t}{t + |x - y|}, t > 0 \end{Bmatrix}$$

with$a'a \geq a, \forall a \in (0,1]$, Let$A, B, C, D$are four mapping from X to itself define as:

$$A(x) = \begin{Bmatrix} \frac{x}{3}, \ 0 \leq x < 1 \\ \frac{1}{2}, x = 1 \end{Bmatrix}, B(x) = \begin{Bmatrix} \frac{x}{2}, \ 0 \leq x < 1 \\ \frac{1}{4}, x = 1 \end{Bmatrix}$$

$$C(x) = \begin{Bmatrix} x, \ 0 \leq x < 1 \\ \frac{1}{2}, x = 1 \end{Bmatrix}, D(x) = \begin{Bmatrix} \frac{x}{2}, \ 0 \leq x < 1 \\ 1, x = 1 \end{Bmatrix}$$

i) Clearly, $A(X) \subset D(X), B(X) \subset C(X)$

ii) For sequence$x_n = \frac{1}{n}$, the pair (A, C) or (B, D) satisfies given property.

iii) For$k = \frac{1}{2}, t > 0$, and *(ii)* condition of the main theorem satisfied by mappingsA, B, C, D.

The pair (A, C) and(B, D) are weakly compatible at $x = 0$ which is coincident point ofA, B, C, D. Thus all conditions of theorem are satisfied then $x = 0$is unique common fixed point ofA, B, C and Din$X$.

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