



# DESIGN OF LOW PASS & HIGH PASS FILTER USING LATTICE WAVE DIGITAL FILTER

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**Abstract:** This paper presents design of low-pass and high-pass standard filter using Lattice Wave Digital Filter (LWDF). Standard filters like Butterworth, Chebyshev, Inverse Chebyshev, and Cauer Type I are reconstructed using LWDF coefficients and then the performance of every filter is compared. Results show that Cauer Type I Filter reconstructed using LWDF coefficients shows better filtering specifications than any other standard filter. In both Low-pass filter (LPF) and High-pass filter (HPF), Cauer Type I filter gives narrow transition width with constant stop-band attenuation which are favourable conditions for filtering process.

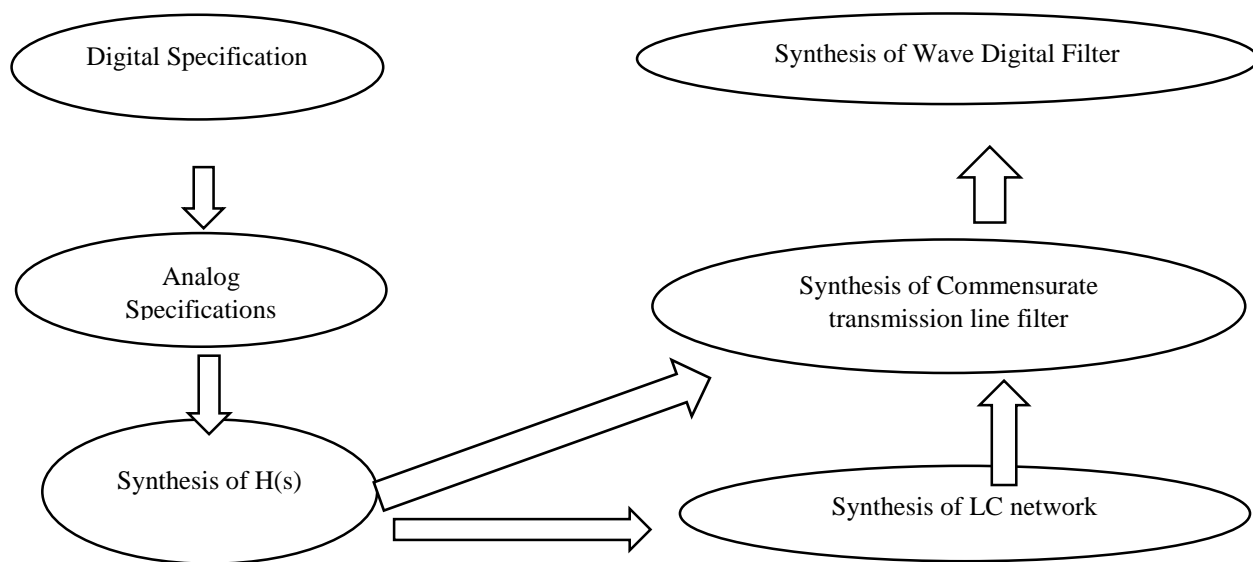
**Index Terms:** LWDF (Lattice wave digital filter), FIR (Finite Impulse Response), IIR (Infinite Impulse Response), LPF (Low-Pass Filter), HPF (High-Pass Filter)

## I. INTRODUCTION

Digital filtering [1] is one of the predominant signal processing operations whose aim is to pass certain frequency components of a signal by blocking the other frequency components. Types of digital filters are lowpass, high-pass, bandpass, and band-stop filters. These digital filters are broadly divided into two main categories: finite impulse response (FIR) filter and infinite impulse response (IIR) filter [2]. FIR filters are mostly applicable where linear phase response is particularly required. To implement the same order filter, FIR filters use more resources in comparison to its equivalent IIR filter [3]. IIR filters are used in applications in which cost is a major concern as they can be implemented using minimum hardware/software resources. IIR filter simply meets the desired specifications like sharp transition width, lower pass-band ripple, and higher stop-band attenuation with lower order as compared to the FIR filter.

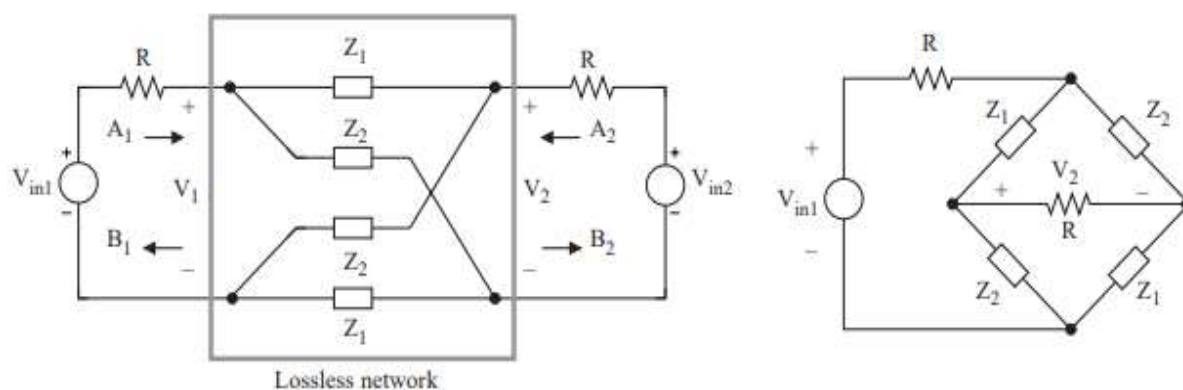
To avoid drift and aging of analog components digital filters are used. While moving towards optimization, the most important filter algorithm with guaranteed stability are non-recursive finite impulse response filters and Wave Digital Filters [4].

With quantized multiplier coefficients [5], frequency response of digital filter is different from that of desired digital filter with unquantized coefficients, which might be enough to make the practical digital filters unsuitable in most applications. It is thus important to develop digital filter structures that are less sensitive to coefficient quantization. The first perspective is based on the conversion of an inherently low sensitivity analog network (i.e., composed of inductors, capacitors, and resistors) to a digital filter structure by replacing each analog network component and their interconnections to a corresponding digital filter equivalent such that the overall structure “simulates” the analog prototype. The following digital filter structure is called a wave digital filter, with additional properties of its analog prototype.



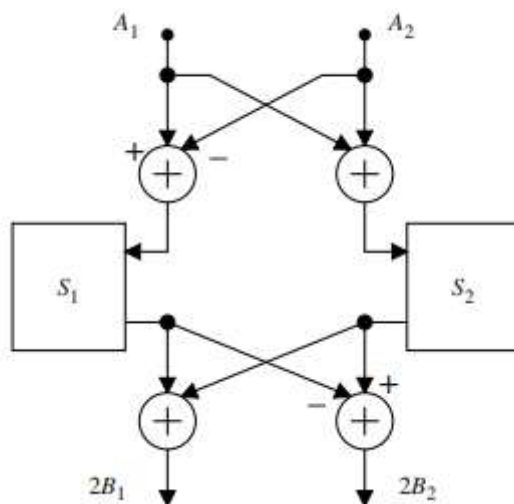
**Figure 1:** Flow chart of design process for wave digital filters using an analog reference filter [6]

There are some advantages of lattice structure over transversal structure [7]. Lattice structure is less sensitive to numerical round-off characteristics. It orthogonalizes the input signal stage-by-stage which induce fast convergence and efficient tracking capabilities. The lattice filter (predictor) can be interpreted as wave propagation in a stratified medium. An acoustical tube model of the human vocal tract can be represented by this, which is very useful in digital speech processing.



**Figure 2:** Analog Lattice filter

Lattice Wave Digital Filter (LWDFs) consists of two all-pass filters in parallel [8]. LWDFs were first proposed by Professor Alfred Fettweis in 1971. They are extremely stable and had a large dynamic range. LWDFs weren't sensitive to either limited word lengths or round off errors. Each all-pass section is made up of a two-port adapter (with two inputs A1 and A2 and two outputs B1 and B2) along with a one sample delay element.



**Figure 3:** Wave-flow diagram for a lattice filter.

The two inputs A1 and A2 correspond to the two input signals Vin1 and Vin2 in Figure 2 given as [9]-

$$S_1 = \frac{X_1 - R}{X_1 + R} \tag{1}$$

and

$$S_2 = \frac{X_2 - R}{X_2 + R} \tag{2}$$

Also,

$$B_1 = \frac{S_1 + S_2}{2} A_1 + \frac{S_2 - S_1}{2} A_2, \tag{3}$$

$$B_2 = \frac{S_2 - S_1}{2} A_1 + \frac{S_1 + S_2}{2} A_2 \tag{4}$$

Note that S1 and S2 are all-pass functions if X1 and X2 are reactance. Some of the characteristics of lattice wave digital filters includes better passband sensitivity than for ladder structures, a high degree of computational parallelism and can be pipelined, simple modular building blocks and simple to design.

## II. LITRATURE SURVEY

A paper by Abhay Sharma, Tarun Kumar Rawat and Anjali Agrawal, titled as “*Design and FPGA implementation of lattice wave digital notch filter with minimal transient duration*” [10], presented the design and field-programmable gate array (FPGA) implementation of the digital notch filter with the lattice wave digital filter (LWDF) structure. Variable notch bandwidth filter is designed for reducing the initial signal transient. To diminish signal transient, the notch filter has a wide bandwidth during the initial samples. The notch bandwidth reduces to accomplish the possible minimum width. This evaluates minimized transient duration notch filter with a adequately high-quality factor.

In 1971, Professor Alfred Fettweis presented a paper on “*Wave Digital Filters: Theory and Practice*” [11]. Some findings in this paper include analogy between LWDFs and reference analog filters, realization of LWDF, sensitivity to changes in multiplier coefficient, major one-port elements and sources and their realization in the WDF domain, realization of a Unit Element, a QUARL, and a Gyrator.

Capacitance	Inductance	Resistance	Short-circuit	Open-circuit	Resistive source	Voltage source
$V = IR / \psi$	$V = \psi RI$	$V = RI$	$V = 0$	$I = 0$	$V = E + RI$	$V = E$
$B = z^{-1} A$	$B = -z^{-1} A$	$B = 0$	$B = -A$	$B = A$	$B = E$	$B = 2E - A$
$b(t_n) = a(t_{n-T})$	$b(t_n) = -a(t_{n-T})$	$b = 0$	$b = -a$	$b = a$	$b = e$	$b = 2e - a$

(a)

Unit element	QUARL	Gyrator
$K = \frac{1}{\sqrt{1-\psi^2}} \begin{pmatrix} 1 & \psi R \\ \psi/R & 1 \end{pmatrix}$	$K = \frac{e^{\rho \Delta}}{\sqrt{1-\psi^2}} \begin{pmatrix} 1 & \psi R \\ \psi/R & 1 \end{pmatrix}$	$K = \begin{pmatrix} 0 & -R \\ 1/R & 0 \end{pmatrix}$
$B_1 = e^{-\rho T/2} A_2, B_2 = e^{-\rho T/2} A_1$	$B_1 = e^{-\rho T_{12}} A_2, B_2 = e^{-\rho T_{21}} A_1$	$B_1 = -A_2, B_2 = A_1$
$b_1(t_{1m}) = a_2(t_{1m} - T/2)$ $b_2(t_{2m}) = a_1(t_{2m} - T/2)$	$b_1(t_{1m}) = a_2(t_{1m} - T_{12})$ $b_2(t_{2m}) = a_1(t_{2m} - T_{21})$	$b_1 = -a_2$ $b_2 = a_1$

(b)

Figure 4: (a) Major One-Port Elements and Sources and Their Realization in the WDF Domain. (b) Realization of a Unit Element, a QUARL, and a Gyrator.

In 1985, Lajos Gazsi presented a paper “*Explicit Formulas for Lattice Wave Digital Filters*” [12]. Some findings of this paper include firstly explicit formulas for designing LWDF of the most common filter types, for Butterworth, Chebyshev, inverse Chebyshev, and Cauer parameter (elliptic) filter responses. Secondly, using these formulas a direct top-down design method is obtained.

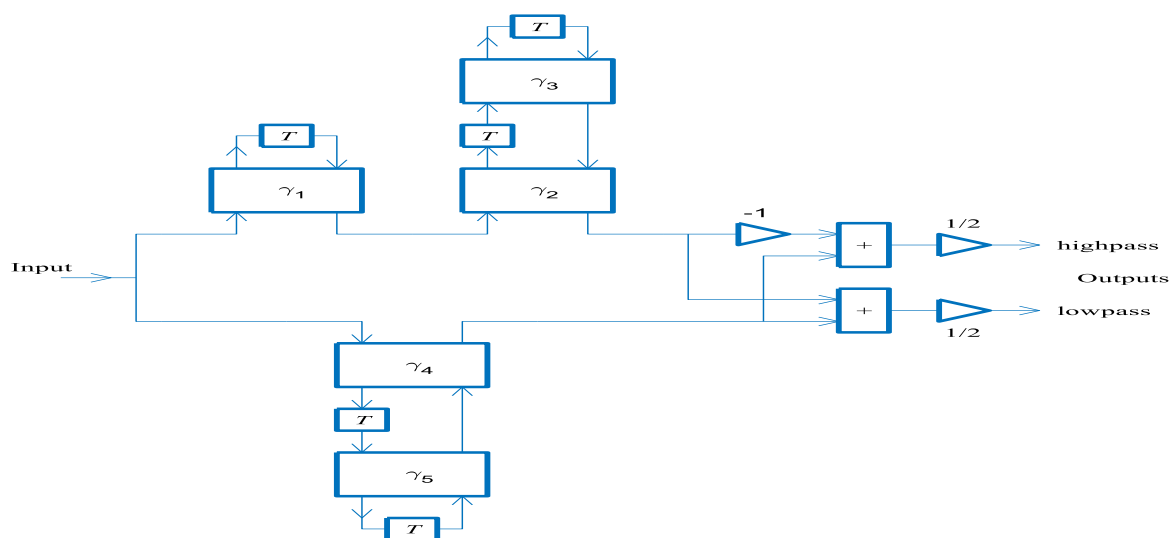
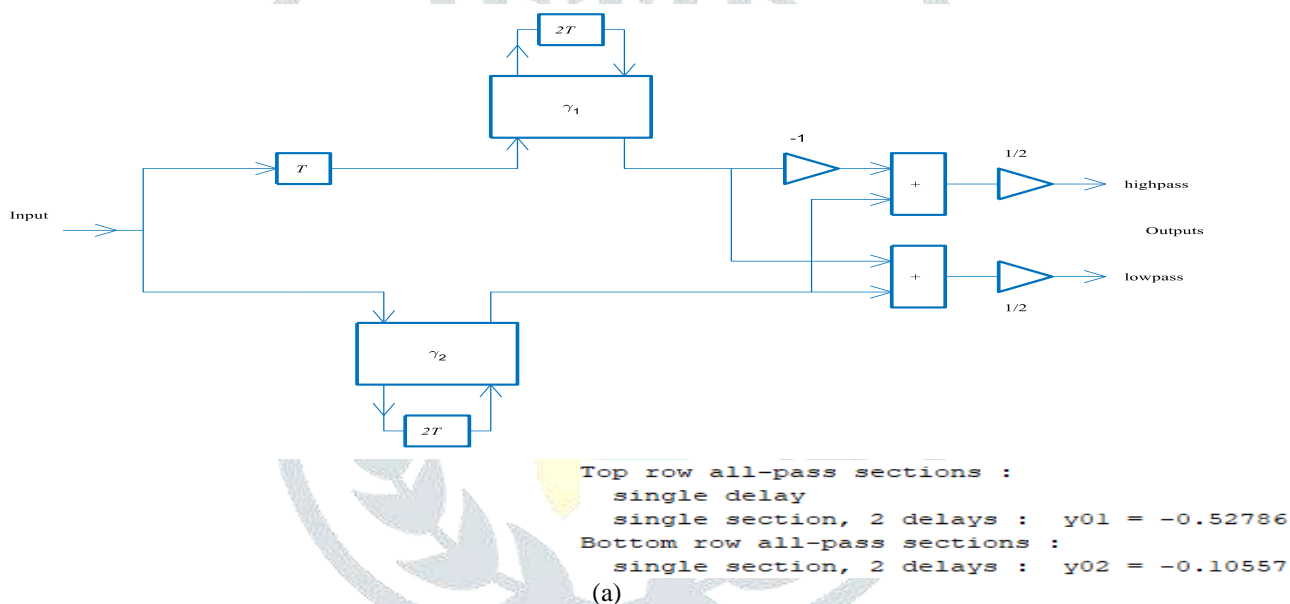
Zeintl and Brachtendorf, in 2018, presented a paper on “*Linear Phase Design of Lattice Wave Digital Filters*” [13] whose findings were mainly focused on a novel design method and its implementation for LWDF using approximation techniques having very low group delay variations. Design requires significantly less realization effort. The design procedure directly shows the effects of quantization of the coefficients of the transfer function. The results show the effectiveness of this design method compared to a reference Butterworth implementation.

Some ongoing application work on LWDF done by Dr. Tarun Kumar Rawat are: -

1. “*Efficient Implementation of LWDD using Unfolding Technique*” [14].
2. “*Optimal design of minimum multiplier lattice wave digital lowpass filter using metaheuristic techniques*” [15].
3. “*Lattice Wave Digital Filter based IIR System Identification with reduced coefficients*” [16].
4. “*FPGA implementation of Hilbert transformer based on lattice wave digital filters*” [17].
5. “*Minimum Multiplier Implementation of a Comb Filter using Lattice Wave Digital Filter*” [18].

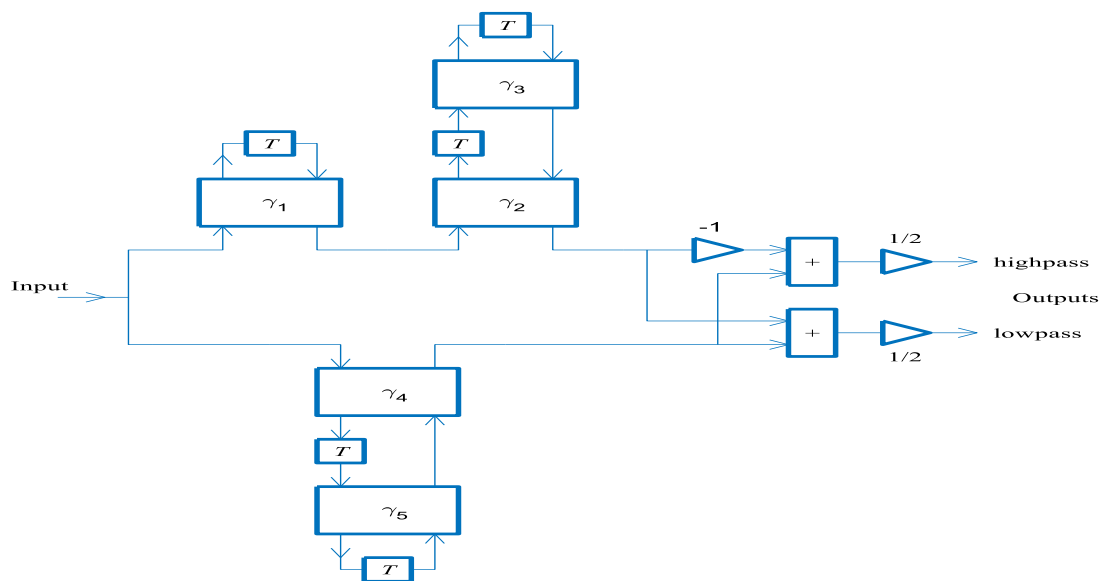
### III. FILTER DESIGNING AND COMPARISON USING LWDF

Now we will design low-pass and high-pass standard filters using LWDF and compare all the specifications. The design specifications include all the LWDF coefficients, its structure, transition width and stopband attenuation. For comparison purpose, some specifications like order of the filter, cut-off frequency, sampling frequency, pass-band ripple and stop-band ripple are kept constant. We will design LPF and HPF with order = 5, cut-off frequency  $\omega_c = 1$ (normalised), pass-band ripple = 0.0988 dB and stop-band attenuation = 40 dB and with DSP toolbox we design LPF and HPF filters using LWDF coefficients. Then we compare filter specification to conclude our study.



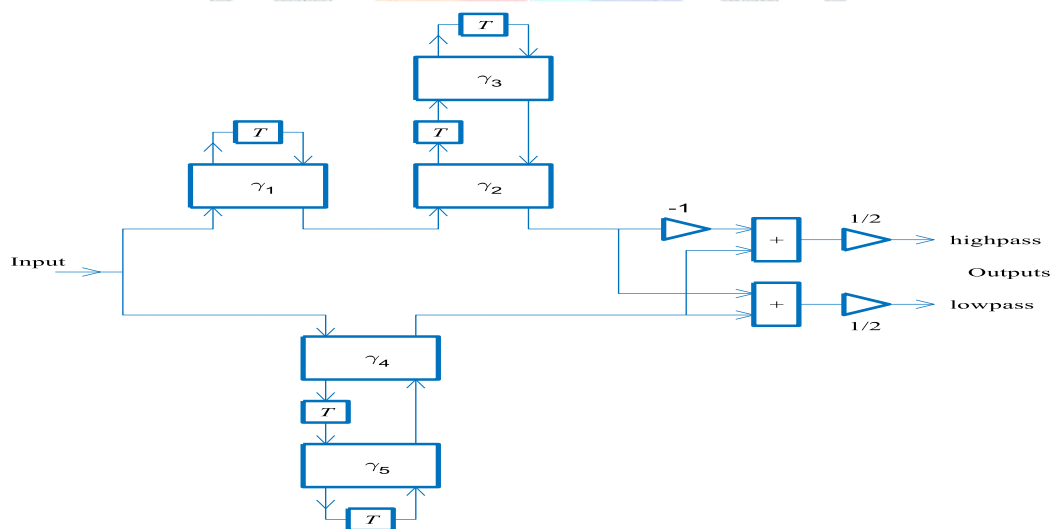
Top row all-pass sections :  
 1st degree section :  $y_{01} = 0.35514$   
 2nd degree section :  $y_{02} = -0.73531$   
 $y_{03} = 0.03729$   
 Bottom row all-pass sections :  
 2nd degree section :  $y_{04} = -0.31995$   
 $y_{05} = 0.33828$

(b)



Top row all-pass sections :  
 1st degree section :  $y_{01} = -0.12011$   
 2nd degree section :  $y_{02} = -0.60351$   
 $y_{03} = -0.01862$   
 Bottom row all-pass sections :  
 2nd degree section :  $y_{04} = -0.15837$   
 $y_{05} = -0.14326$

(c)



Top row all-pass sections :  
 1st degree section :  $y_{01} = 0.23633$   
 2nd degree section :  $y_{02} = -0.81814$   
 $y_{03} = 0.01727$   
 Bottom row all-pass sections :  
 2nd degree section :  $y_{04} = -0.37666$   
 $y_{05} = 0.19308$

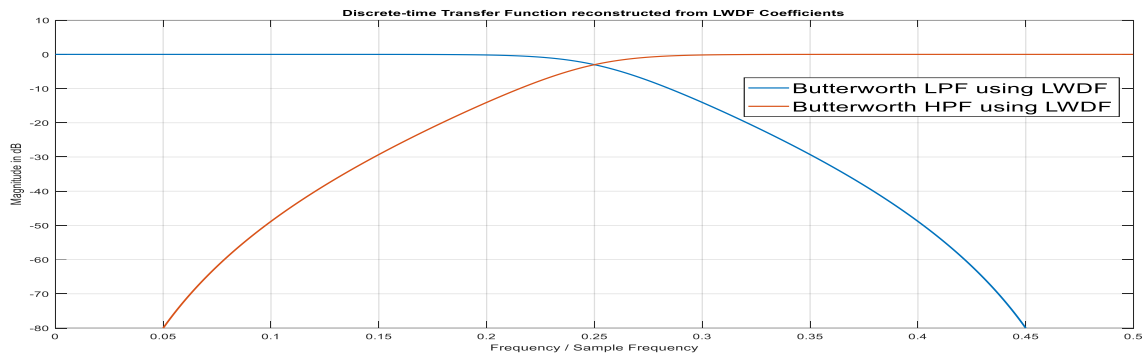
(d)

Figure 5: LWDF structures with coefficients

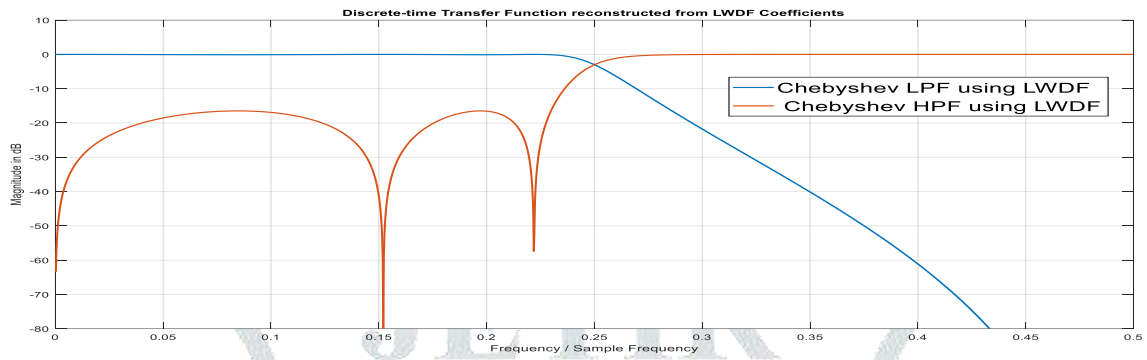
- a) Butterworth filter
- b) Chebyshev filter
- c) Inverse Chebyshev filter
- d) Cauer filter Type I

Figure 5 gives the LWDF structure and the corresponding coefficient values. Now we will see the discrete-time transfer function characteristic of these LPF and HPF.

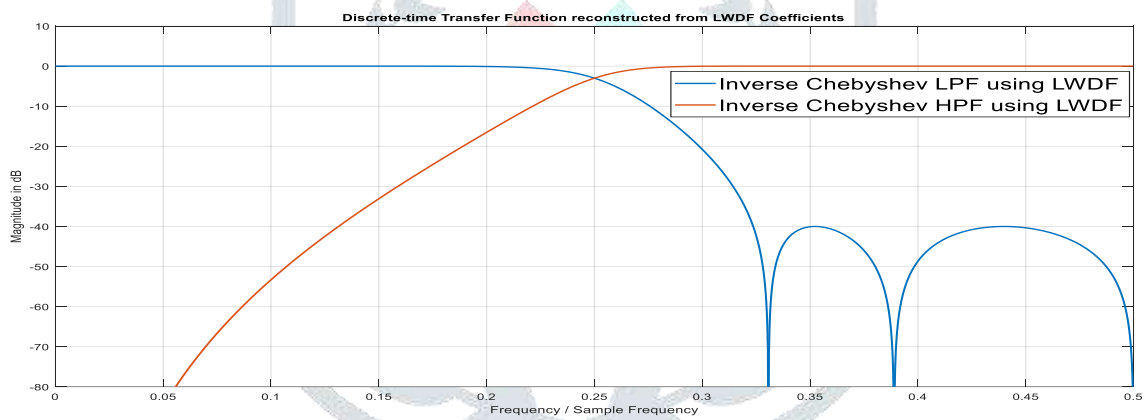




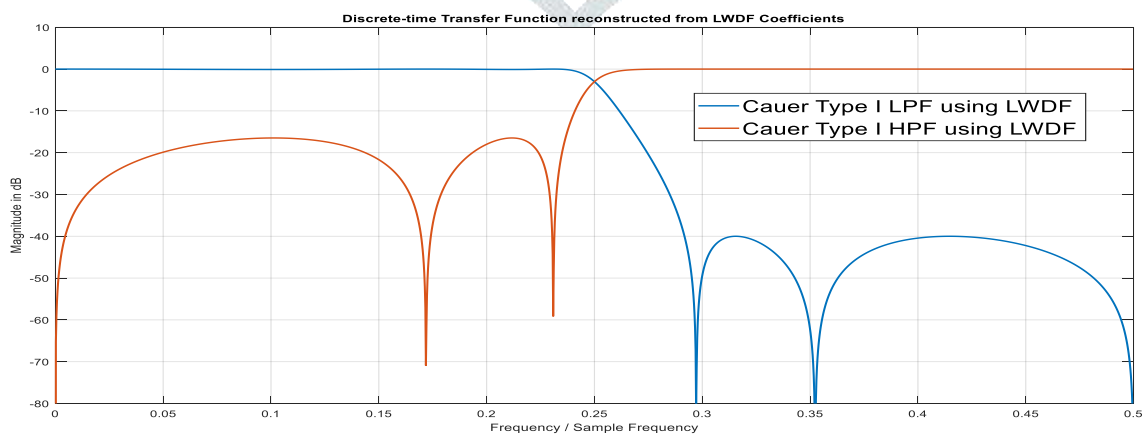
(a)



(b)



(c)



(d)

**Figure 6:** Transfer function reconstructed from LWDF coefficients of LPF and HPF of order 5

- a) Butterworth filter
- b) Chebyshev filter
- c) Inverse Chebyshev filter

## d) Cauer filter Type I

Firstly, we compare the characteristics of LPF using LWDF then we move towards the HPF specifications. At last, we conclude our findings. For comparison of different filters, Table 1 shows comparing parameters of LPF reconstructed using LWDF coefficients.

**Table 1:** Comparison of LPF and HPF specifications for different filters of order 5

Specifications	Standard Filters using LWDF coefficients							
	Butterworth		Chebyshev		Inverse Chebyshev		Cauer Type I	
	LPF	HPF	LPF	HPF	LPF	HPF	LPF	HPF
Transition width (normalized)	0.45	0.05	0.433	0.22	0.33	0.056	0.29	0.23
Stop-band Attenuation(dB)	-	-	-	16.48	40	-	40	16.48
Type	Bireciprocal or Half-band LPF	Bireciprocal or Half-band HPF	LPF	HPF	LPF	HPF	LPF	HPF

Table 1 data for Butterworth LPF show a large transition width. This is also in the case of Chebyshev LPF. While Inverse Chebyshev LPF shows smaller transition width and constant stop-band attenuation. Cauer Type I LPF have the least transition width and constant stop-band attenuation.

Just like that Table 1 data for Butterworth HPF show a large transition width. This is also in the case of Inverse Chebyshev HPF. While Chebyshev HPF shows smaller transition width and constant stop-band attenuation. Cauer Type I HPF also have a smaller transition width and constant stop-band attenuation.

#### IV. CONCLUSION

Evaluating the table and figure data, we conclude that Cauer Type I Filter reconstructed using LWDF coefficients shows better filtering specifications than any other standard filter. In both LPF and HPF, Cauer Type I filter gives narrow transition width with constant stop-band attenuation which are favourable conditions for filtering process.

#### ACKNOWLEDGEMENT

I am extremely thankful to University Grants Commission (UGC) for financial support in the form of Junior Research Fellowship (JRF).

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