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LRS BIANCHI TYPE-*I* BULK VISCOUS DARK ENERGY COSMOLOGICAL MODEL

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Abstract : In this paper, we have investigated LRS Bianchi type-I universe in the presence of pressureless dark matter and bulk viscous fluid of dark energy within the framework of general relativity. To find the solution of Einstein's field equations, we assumed that the deceleration parameter as linear function of Hubble parameter. Under this specification, we obtain the non-singular solution of LRS Bianchi type-I model depending upon the particular choice of the value of parameters. The cosmological implications of the model including the evolution of effective EoS parameter of bulk viscous dark energy, energy densities and bulk viscous coefficient ζ are investigated. It is shown that depending on the parameters of bulk viscous coefficient ζ, the bulk viscous dark energy model can behave as a quintessence or phantom dark energy. We have also studied the statefinder parameter of $r-s$ plane to characterize different phases of the evolution of the universe and our result is completely agree with the ΛCDM prediction in late time. The physical and geometric properties of the cosmological model are also well discussed. Therefore, the bulk viscosity of dark energy plays very important role in the accelerated expansion history of the universe.

Key words: LRS Bianchi type-I. Bulk viscous fluid. Dark energy. deceleration parameter

I. INTRODUCTION

The appearance of new cosmological models is connected with the discovery of the accelerated expansion of the universe. Cosmic acceleration can be introduced via dark energy or via modification of gravity (Nojiri and Odintsov [1]). A general review of dark energy cosmology was given in Bamba et al. [2]. Dark energy (DE) should have strong negative pressure and can be characterized by an equation of state parameter (EoS). Cosmological models that treat dark energy and dark matter as imperfect fluids with unusual equation of state are considered in Nojiri and Odintsov [3, 4], where viscous fluids are just one particular case. Bulk viscous cosmology is also an alternative to gravity modifying theories (Nojiri and Odintsov [1]) in that it alters the right hand side of Einstein's field equations instead of the left hand side. Bulk viscosity characterizes deviations from local equilibrium which modifies the energy-momentum tensor. It is necessary to take into account viscosity effects when considering turbulence (Brevik et al. [5]) or other realistic situations.

Dissipative dark energy models in which the negative pressure, which is responsible for the current acceleration, is an effective bulk viscous pressure have been proposed in order to avoid the occurrence of the big rip (Barrow [6]; McInnes [7]). Influence of bulk viscosity in the cosmic fluid plays an important role in the big rip phenomenon (Brevik et al. [8]). In this scenario which is based on the Eckart theorem (Eckart [9]), we consider the DE fluid with viscous. Evolution of the universe involves a sequence of dissipative process. In the case of isotropic and homogeneous model, the dissipative process is modeled as a bulk viscosity (Ren and Meng [10]; Hu and Meng [11]; Meng and Duo [12]). Brevik et al. [13] discussed the general account about viscous cosmology for early and late time universe. Norman and Brevik [14] analyze characteristic properties of two different viscous cosmological models for the future universe. Norman and Brevik [15] derived a general formalism for bulk viscous and estimated the bulk viscosity in the cosmic fluid.

Cosmological models where the modification of gravity is described in terms of a viscous fluid are explored in Myrzakul et al. [16], Myrzakulov and Sebastiani [17]. The possibility of a viscosity dominated late epoch of the universe with accelerated expansion was already mentioned by Padmanabhan and Chitre [18]. Recently, Velten et al.[19] have investigated phantom DE as an effect of bulk viscosity. It is worth noting that Brevik and Gorbunova [20] show that fluid which lies in the quintessence region can reduce its thermodynamical pressure and cross the barrier $\omega_{de}=-1$, and behave like a phantom fluid with the inclusion of a sufficiently large bulk viscosity. As the theoretical point of view, the bulk viscosity can originate due to the deviation from local thermodynamic equilibrium. It manifests as an effective pressure to bring back the system to its thermal equilibrium, which was broken when the cosmological fluid expands (or contracts) too fast. The bulk viscosity pressure thus generated, ceases as soon as the fluid reaches the equilibrium condition and it seems to play a more important role in constructions of cosmological model.

The bulk viscous driven inflation leads to a negative pressure term, which in process results in repulsive gravity and ultimately became a cause for the rapid expansion of the universe (Tripathy et al.[21]; Maartens [22]; Lima et al.[23]). The contribution of bulk viscosity to the cosmic pressure plays the role of accelerating the universe. In an expanding system, relaxation processes associated with bulk viscosity effectively reduce the pressure as compared to the value prescribed by the equation of state. For a large bulk viscosity, the effective pressure becomes negative and could mimic a dark energy behavior.

Brevik et al.[13] have investigated viscous cosmology in the early universe for both homogeneous and inhomogeneous EoS and examined the bulk viscosity effects on the various inflationary observables. Since viscosity appears to be an important dissipative phenomena in Friedman-Robertson-Walker cosmology, therefore it is expected that cosmological model with bulk viscosity fluid would produce some results in the two fluid situations. Moreover, viscosity cosmological model indicates a substantial contribution of bulk viscosity at the inflationary phase (Barrow [24]; Zimdahl [25]; Bafaluy and Pavon [26]).

Recently Planck collaboration revealed that this property of isotropic and homogeneity of the universe is well defined by the ΛCDM model in the FRW geometry. However, at low multipoles the ΛCDM cosmology shows a poor fit to the CMB temperature power spectrum (Ade et al. [27, 28]). This indicates that the isotropy and homogeneity were not the essential features of the early universe. Moreover, the recent Planck data results motivate us to construct and analyze the cosmological models with anisotropic geometry to get a deeper understanding on the evolution of the universe. In this regard, locally rotationally symmetric (LRS) Bianchi type-I space-time is of fundamental importance since it provides the stipulation framework. Several theoretical two fluids DE models either interacting or non-interacting have been discussed widely in the literature (Sheykhi and Setare [29]; Amirhashchi [30]; Amirhashchi et al. [31]; Tripathy et al. [32]; Kumar[33]). Santhi et al.[34] has studied bulk viscous string cosmological models in $f(R)$ gravity. The isotropic homogeneous spatially flat cosmological model with bulk viscous fluid discussed by Murphy [35]. In a similar approach of two fluids, DE cosmological models were constructed in different general scale factors (Mishra et al.[36]). So, all above informations give us motivation to investigate LRS Bianchi type-I space-time of bulk viscous fluid dark energy cosmological model in general relativity. In this paper, we study the behavior of an anisotropic LRS Bianchi type-I universe in the case of viscous dark energy and dark matter which are minimally coupled, that is, when there is no interaction between these two dark components. Moreover, we consider a bulk viscosity coefficient with a power law dependence on the energy density of dark energy $\zeta = \zeta_0(\rho_{de})^{\gamma_1}$ where ζ_0 and γ_1 are non negative constants.

This paper is organized as follows. In section 2, we present the metric and the field equations. In section 3, we describe the solutions of the field equations. We discuss some physical and geometrical properties of the model in section 4. Finally, concluding remarks are summarized in section 5.

2 Metric and field equations

We consider the spatially homogeneous and anisotropic LRS Bianchi type-I space-time in the form

$$ds^2 = dt^2 - A^2dx^2 - B^2(dy^2 + dz^2), \tag{1}$$

where A, B and are metric functions of cosmic time t alone. We define the following physical and geometric parameters to be used in formulating the law and further in solving Einstein's field equations for the metric (1). The spatial volume V, Hubble parameter H, expansion scalar θ, average anisotropy parameter A_m, the shear scalar σ^2 and deceleration parameter q for LRS Bianchi type-I space-time are defined as

$$V = a^3 = AB^2, \tag{2}$$

$$\theta = 3H = u^i_{;i} = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}, \tag{3}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = 2\frac{\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)^2}{\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)^2}, \tag{4}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right) = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2, \tag{5}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1, \tag{6}$$

where $u^i = (1,0,0,0)$ is the four velocity vector and assumed to satisfy $u^iu_i = 1$ and $\Delta H_i = H_i - H$ $(i = x, y, z)$. The directional Hubble parameters are defined as $H_1 = \frac{\dot{A}}{A}$ and $H_2 = H_3 = \frac{\dot{B}}{B}$ so that the mean Hubble parameter becomes $H = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)$.

We consider the universe filled with pressureless dark matter and viscous dark energy fluid. In this case the Einstein's field equations in gravitational units $(8\pi G = c = 1)$ are given by

$$R_i^j - \frac{1}{2}R\delta_i^j = -\left(T^j_{(m)i} + T^j_{(de)i}\right), \tag{7}$$

where $T^j_{(m)i}$ and $T^j_{(de)i}$ are the energy momentum tensors of dark matter and viscous DE fluid, respectively. These are given by

$$T^j_{(m)i} = (\rho_m, 0, 0, 0), \tag{8}$$

and

$$T^j_{(de)i} = (\rho_{de}, -p_{de}, -p_{de}, -p_{de}),$$
$$= diag(1, -\omega_{de}, -\omega_{de}, -\omega_{de})\rho_{de},$$

where ρ_m is the energy density of dark matter, ρ_{de} and p_{de} are, respectively, the energy density and pressure of viscous DE component while $\omega_{de} = \frac{p_{de}}{\rho_{de}}$ is the corresponding EoS parameter. The only change in the formalism because of bulk viscosity is that the thermodynamical pressure with the effective pressure p_{eff} and effective of EoS parameter ω_{eff}, defined as

$$p_{eff} = p_{de} + \Pi; \qquad \omega_{eff} = \frac{p_{eff}}{\rho_{de}}, \tag{10}$$

where $\Pi = -3\zeta H$ is the bulk viscosity pressure, ζ is the coefficient of bulk viscosity. The form of the above equation was originally proposed by Eckart [9] in the context of relativistic dissipative process occurring in thermodynamic systems went out of local thermal equilibrium. Hu and Meng [11], Kremer and Devecchi [37], Cataldo and Cruz [38], Fabris et al. [39] have used Eckart approach to explain the current acceleration of the universe with bulk viscous fluid. This motivates us to use Eckart formalism on viscous term, especially when one tries to look at recent acceleration of the universe. Here ω_{eff} is referred to as the effective equation of state parameter of viscous dark energy. Based on Landau and Lifshitz [40] in an irreversible process the positive sign of the entropy changes, ζ has to be positive. In a co-moving coordinate system $(u^i = \delta_0^i)$, Einstein's field equations (7) with (8) and (9) for locally rotational symmetric Bianchi type-I metric (1) subsequently lead to the following system of differential equations.

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = -\omega_{de}\rho_{de}, \tag{11}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}=-\omega_{de}\rho_{de}, \quad (12)$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}=\rho_m+\rho_{de}, \quad (13)$$

where the overhead dot denotes ordinary differentiation with respect to cosmic time t. From equations (11) and (12), we get

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}-\frac{\dot{B}^2}{B^2}=0. \quad (14)$$

Taking the second integral of equation (14), we obtain the following relation

$$\frac{A}{B}=b_2\exp\left[b_1\int\frac{dt}{AB^2}\right]=b_2\exp\left[b_1\int\frac{dt}{a^3}\right], \quad (15)$$

where b_1 and b_2 are constants of integration. From equations (2) and (15), the values of metric potentials are

$$A=b_3a\exp\left[\frac{2b_1}{3}\int\frac{dt}{a^3}\right], \quad (16)$$

$$B=b_4a\exp\left[-\frac{b_1}{3}\int\frac{dt}{a^3}\right], \quad (17)$$

where $b_3=b_2^{\frac{2}{3}}$ and $b_4=b_2^{-\frac{1}{3}}$ are constants of integrations such that $b_3b_4^2=1$. The law of energy-conservation equation ($T^{ij}_{;j}=0$) from (7) yields

$$\dot{\rho}_m+3\rho_mH+\dot{\rho}_{de}+3(1+\omega_{de})\rho_{de}H=0. \quad (18)$$

3 Solution of the field equations

The field equations (11)-(13) have three differential equations with five unknowns namely A, B, ω_{de}, ρ_m and ρ_{de}. To get determine solutions, we need extra conditions. One can classify models of the universe on the basis of the time dependence of Hubble parameter and deceleration parameter. When the Hubble parameter is constant, the deceleration term q is also constant and equal to -1, as in the de Sitter and steady state universes. In many universes, the deceleration term changes with time. Following Tiwari et al.[41, 42], we assume that the deceleration parameter q is a linear function of the Hubble parameter H as

$$q=p_1+p_2H, \quad (19)$$

where p_1 and p_2 are constants. If $\ddot{a}$ is positive, and in this case the deceleration parameter q will be negative, the universe is accelerating. Also recent observations (Tiwari et al. [42]; Perlmutter et al. [43]; Riess et al.[44-46]; Astier et al. [47]; Spergel et al. [48]; Eisentein et al.[49]) have suggested that the present universe is accelerating and that the value of the deceleration parameter q lies between 0 to -1. Therefore it is physically viable conditions which have been extensively used in the literature.

For mathematical simplicity, we take $p_1=-1$ and then, from equation (19), we obtain

$$a=e^{\frac{1}{p_2}\sqrt{2p_2t+k_1}}, \quad (20)$$

where k_1 is constant. This shows that at $t=-\frac{k_1}{2p_2}$, the scale factor a tends to a constant.

In terms of redshift z, the scale factor a and cosmic time t become

$$z=-1+\frac{1}{a}=-1+e^{-\frac{1}{p_2}\sqrt{2p_2t+k_1}}, \quad t=p_2\left(\ln(z+1)\right)^2-\frac{k_1}{p_2}. \quad (21)$$

Now from equations (16) and (17), we obtain the metric potentials as

$$A=b_3e^{\frac{1}{p_2}\sqrt{2p_2t+k_1}}\exp\left(\frac{-2b_1}{27}\left(3\sqrt{2p_2t+k_1}+p_2\right)e^{\frac{-3}{p_2}\sqrt{2p_2t+k_1}}\right), \quad (22)$$

$$B=b_4e^{\frac{1}{p_2}\sqrt{2p_2t+k_1}}\exp\left(\frac{b_1}{27}\left(3\sqrt{2p_2t+k_1}+p_2\right)e^{\frac{-3}{p_2}\sqrt{2p_2t+k_1}}\right). \quad (23)$$

Therefore, the metric (1) can be written as

$$\begin{aligned} ds^2 = \; & dt^2-b_3^2e^{\frac{2}{p_2}\sqrt{2p_2t+k_1}}\exp\left(\frac{-4b_1}{27}\left(3\sqrt{2p_2t+k_1}+p_2\right)e^{\frac{-3}{p_2}\sqrt{2p_2t+k_1}}\right)dx^2 \\ & -b_4^2e^{\frac{2}{p_2}\sqrt{2p_2t+k_1}}\exp\left(\frac{2b_1}{27}\left(3\sqrt{2p_2t+k_1}+p_2\right)e^{\frac{-3}{p_2}\sqrt{2p_2t+k_1}}\right)(dy^2+dz^2). \end{aligned} \quad (24)$$

From the metric (24), we obtain the non singular solution of LRS Bianchi type-I model depending upon the particular choice of the value of problem parameters.

4 Physical and geometrical properties of the model

The spatial volume V, Hubble parameter H, expansion scalar θ, shear scalar σ^2, anisotropy parameter A_m and deceleration parameter q are given by

$$V=e^{\frac{3}{p_2}\sqrt{2p_2t+k_1}}, \quad (25)$$

$$H=\frac{1}{\sqrt{2p_2t+k_1}}, \quad (26)$$

$$\theta=\frac{3}{\sqrt{2p_2t+k_1}}, \quad (27)$$

$$\sigma^2=\frac{b_1^2}{3e^{\frac{6}{p_2}\sqrt{2p_2t+k_1}}}, \quad (28)$$

$$A_m=\frac{2b_1^2}{9}\frac{(2p_2t+k_1)}{e^{\frac{6}{p_2}\sqrt{2p_2t+k_1}}}, \quad (29)$$

$$q=-1+\frac{p_2}{\sqrt{2p_2t+k_1}}. \quad (30)$$

The directional Hubble parameters are obtained as

$$H_1=\frac{\dot{A}}{A}=\frac{1}{\sqrt{2p_2t+k_1}}+\frac{2b_1}{3e^{\frac{3}{p_2}\sqrt{2p_2t+k_1}}}, \quad (31)$$

$$H_2=\frac{\dot{B}}{B}=\frac{1}{\sqrt{2p_2t+k_1}}-\frac{b_1}{3e^{\frac{3}{p_2}\sqrt{2p_2t+k_1}}}=H_3. \quad (32)$$

Equations (25) and (27) indicate that the spatial volume is constant at point $t = -\frac{k_1}{2p_2}$ and the expansion scalar is infinite at this point. But, it becomes decreasing as cosmic time increases. The volume increases with increasing cosmic time and the universe is expanding with increasing cosmic time.

The physical quantities Hubble parameter H, directional Hubble parameters, expansion scalar θ and shear scalar σ^2 diverge at $t = -\frac{k_1}{2p_2}$. As $t \to \infty$, the parameters H, θ and σ^2 tend to zero. We find that $\lim_{t\to\infty} \frac{\sigma^2}{\theta^2}$ converges to 0, which indicates that the model eventually approaches isotropy for late cosmic times t. Since anisotropy parameter is a measure of deviation from isotropic expansion, $t \to \infty$ gives the isotropic behavior of the model. The variation of deceleration parameter q versus redshift z is plotted in figure 1. From the figure it is seen that the deceleration parameter will be stabilised around -1 in the far future of the evolution of the universe and this is in confirmation with the previously obtained limit of q. By using Eckart theory and taking the velocity and acceleration dependence for the bulk viscous coefficient, we have $q \sim -0.64$ (Sasidharan and Mathew [50]). So even though the present model is predicting a never-ending accelerating phase, the universe is not reaching the exact de Sitter phase and this is in marked deviation from the corresponding models using the Eckart formalism (Sasidharan and Mathew [50]), in which the model evolves asymptotically to the de Sitter phase. It can be concluded that q is increasing with the increasing of redshift z and at late time, the value of q approaches -1 which shows the fastest rate of expansion of universe at late time. Present observational data obtained by Ade et al. [51] indicates that the universe is accelerating and the value of deceleration parameter lies somewhere in the range $-1 \le q < 0$. So in this case we can construct a present accelerating model of the universe which is in agreement with recent observations of cosmological data.

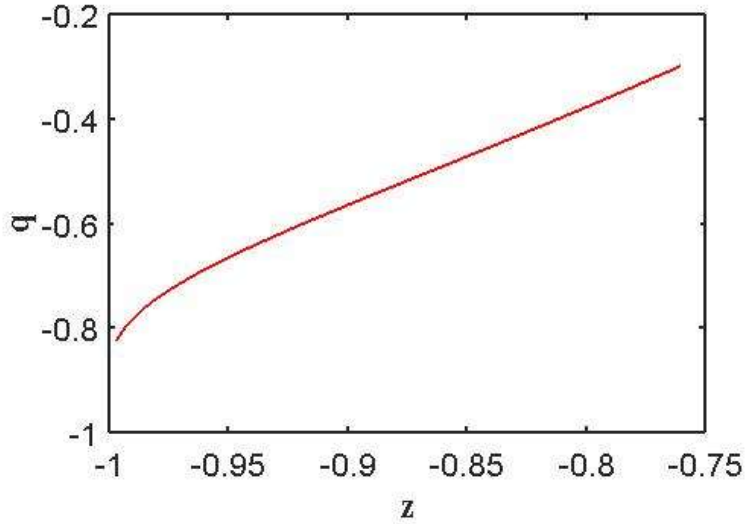

Figure 1: **P**lot of redshift z versus deceleration parameter q for $p_2 = 0.5$ and $k_1 = 0.5$.

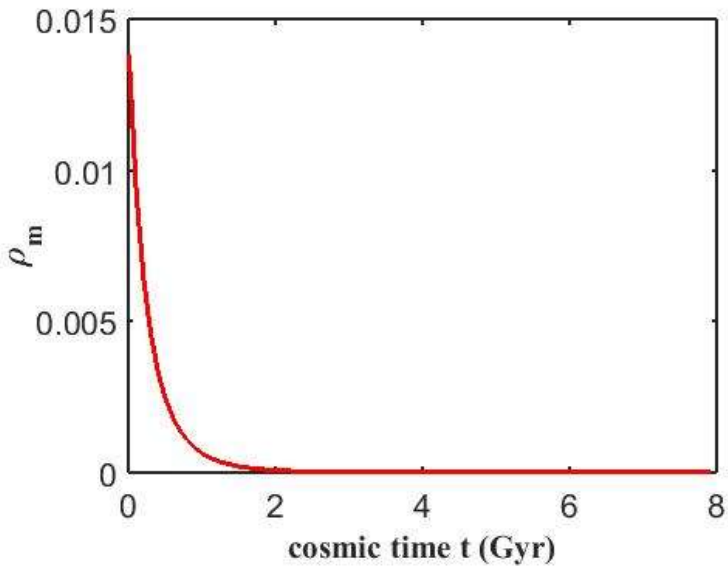

Figure 2: **P**lot of energy density of dark matter ρ_m versus cosmic time t for $\rho_{m_0} = 1$, $p_2 = 0.5$ and $k_1 = 0.5$.

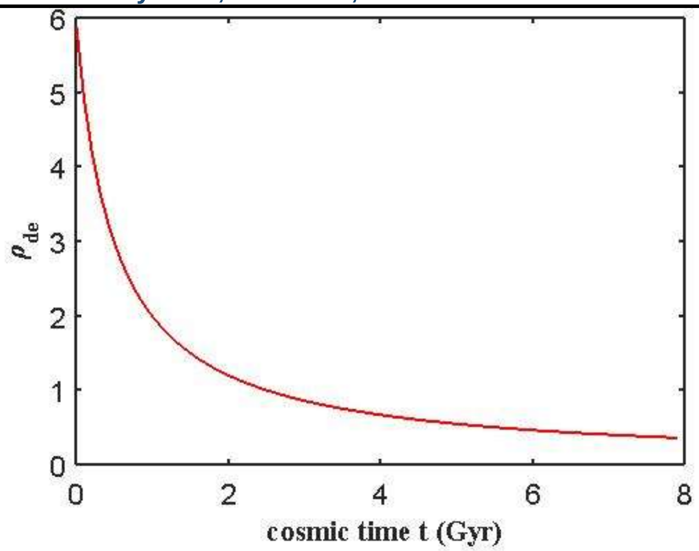

Figure 3: **P**lot of bulk viscous dark energy energy density ρ_{de} versus cosmic time t for $\rho_{m_0} = 1$, $b_1 = 0.5$, $p_2 = 0.5$ and $k_1 = 0.5$.

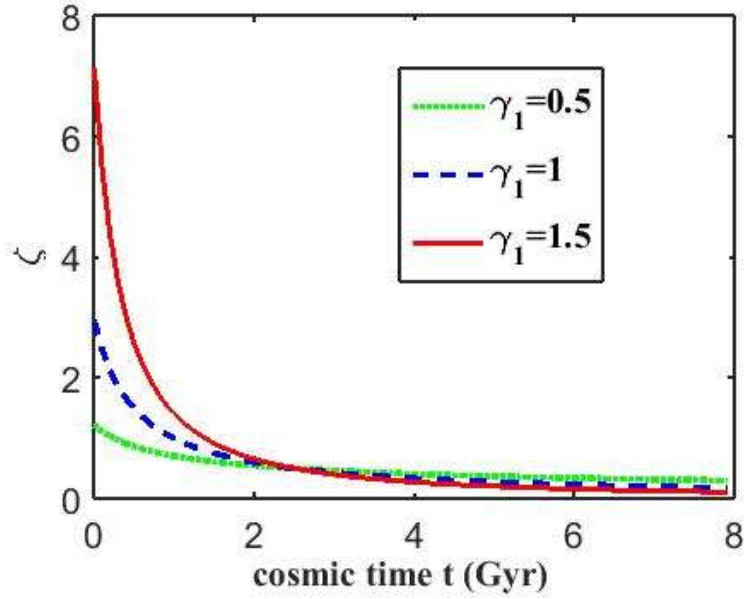

Figure 4: **P**lot of bulk viscous coefficient ζ versus cosmic time t for $b_1 = 0.5$, $\zeta_0 = 0.5$, $\rho_{m_0} = 1$, $p_2 = 0.5$ and $k_1 = 0.5$.

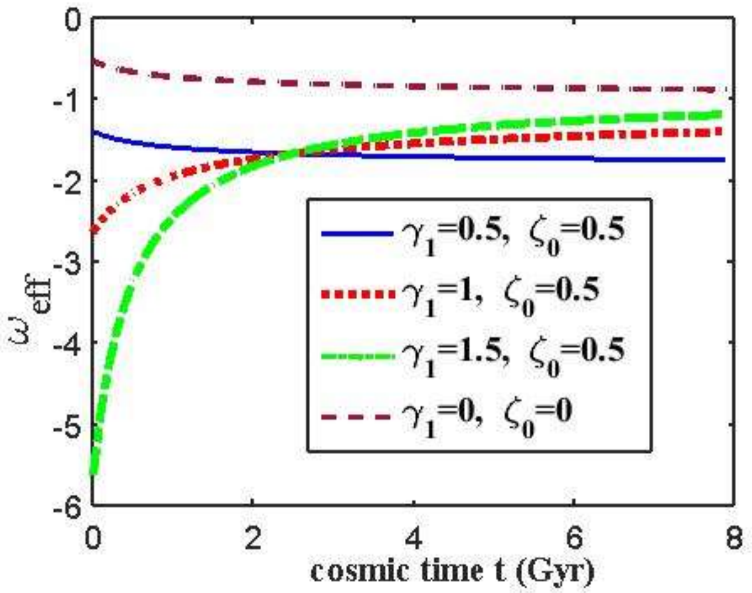

Figure 5: **P**lot of effective EoS ω_{eff} versus cosmic time t for $b_1 = 0.5$, $\rho_{m_0} = 1$, $p_2 = 0.5$ and $k_1 = 0.5$.

We assumed that the components of energy density of dark matter and viscous dark energy are interact minimally. Hence, the energy momentum tensors of the two sources may be conserved separately.

The energy conservation equation $T^{j}_{(m)i;j} = 0$, of the dark matter fluid from equation (18) leads to

$$\dot{\rho}_m + 3\rho_m H = 0, \tag{33}$$

Integrating of equation (33), using (21) leads to

$$\rho_m = \rho_{m_0} e^{\frac{-3}{p_2}\sqrt{2p_2 t + k_1}}, \tag{34}$$

where ρ_{m_0} is a positive constant of integration. By using equations (31), (32) and (34) in equation (13), we obtain

$$\rho_{de} = \frac{3}{2p_2t+k_1} - \frac{1}{3}b_1^2 e^{\frac{-6}{p_2}\sqrt{2p_2t+k_1}} - \rho_{m_0} e^{\frac{-3}{p_2}\sqrt{2p_2t+k_1}}. \quad (35)$$

The behavior of energy density of matter, ρ_m, and energy density of bulk viscous of dark energy, ρ_{de}, have been graphed in figure 2 and 3. It is evident that both physical parameters decrease with cosmic time and finally ρ_m drops to zero for late time and ρ_{de} is positive decreasing values with the passage of cosmic time t. These behavior of ρ_m and ρ_{de} match with observed universe. As a specific case, we consider the bulk viscous coefficient ζ as function of ρ_{de} (Amirhashchi [52]) has the following form

$$\zeta = \zeta_0(\rho_{de})^{\gamma_1} = \zeta_0\left(\frac{3}{2p_2t+k_1} - \frac{1}{3}b_1^2 e^{\frac{-6}{p_2}\sqrt{2p_2t+k_1}} - \rho_{m_0} e^{\frac{-3}{p_2}\sqrt{2p_2t+k_1}}\right)^{\gamma_1}, \quad (36)$$

where ζ_0 and γ_1 are non-negative constant parameters. From equation (36), we observe that the coefficient of bulk viscosity $\zeta \to \infty$ as $t \to -\frac{k_1}{2p_2}$, and as cosmic time t increases, ζ decreases and tending to a constant. Thus ζ is a decreasing function of cosmic time t. Figure 4 depicts the behavior of bulk viscosity coefficient ζ versus cosmic time for different values of γ_1. It is noted that as the value of γ_1 increases, the bulk viscosity coefficient approaches to zero for late time. Moreover, the influence of bulk viscosity more dense in early than late time.

From equation (11), we get

$$p_{de} = -\left(\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{b_1^2}{3a^6}\right),$$
$$= -\left(\frac{3}{2p_2t+k_1} - \frac{2p_2}{(2p_2t+k_1)^{\frac{3}{2}}} + \frac{b_1^2}{3} e^{\frac{-6}{p_2}\sqrt{2p_2t+k_1}}\right). \quad (37)$$

From equations (35) and (??), we get the equation of state parameter of non-viscous dark energy, ω_{de} as

$$\omega_{de} = \frac{p_{de}}{\rho_{de}} = -\frac{\left(\frac{3}{2p_2t+k_1} - \frac{2p_2}{(2p_2t+k_1)^{\frac{3}{2}}} + \frac{b_1^2}{3} e^{-\frac{6}{p_2}\sqrt{2p_2t+k_1}}\right)}{\frac{3}{2p_2t+k_1} - \frac{b_1^2}{3} e^{-\frac{6}{p_2}\sqrt{2p_2t+k_1}} - \rho_{m_0} e^{\frac{-3}{p_2}\sqrt{2p_2t+k_1}}}. \quad (38)$$

From equation (10), the equation of state parameter of viscous dark energy, ω_{eff}, is obtained as

$$\omega_{eff} = \omega_{de} + \frac{\Pi}{\rho_{de}} = \omega_{de} - \frac{3H\zeta}{\rho_{de}}. \quad (39)$$

Using equations (35), (36), (38) in equation (39), the effective EoS parameter of viscous dark energy is obtained as

$$\omega_{eff} = -\frac{\left(\frac{3}{2p_2t+k_1} - \frac{2p_2}{(2p_2t+k_1)^{\frac{3}{2}}} + \frac{b_1^2}{3} e^{-\frac{6}{p_2}\sqrt{2p_2t+k_1}}\right)}{\frac{3}{2p_2t+k_1} - \frac{b_1^2}{3} e^{-\frac{6}{p_2}\sqrt{2p_2t+k_1}} - \rho_{m_0} e^{\frac{-3}{p_2}\sqrt{2p_2t+k_1}}}$$
$$- \frac{3\zeta_0}{\sqrt{2p_2t+k_1}}\left[\frac{3}{2p_2t+k_1} - \frac{1}{3}b_1^2 e^{\frac{-6}{p_2}\sqrt{2p_2t+k_1}} - \rho_{m_0} e^{\frac{-3}{p_2}\sqrt{2p_2t+k_1}}\right]^{\gamma_1 - 1}. \quad (40)$$

By using equations (26), (36) and (39), we get effective pressure as

$$p_{eff} = -\left(\frac{3}{2p_2t+k_1} - -\frac{2p_2}{(2p_2t+k_1)^{\frac{3}{2}}} + \frac{b_1^2}{3} e^{\frac{-6}{p_2}\sqrt{2p_2t+k_1}}\right)$$
$$- \frac{3\zeta_0}{\sqrt{2p_2t+k_1}}\left(\frac{3}{2p_2t+k_1} - \frac{1}{3}b_1^2 e^{\frac{-6}{p_2}\sqrt{2p_2t+k_1}} - \rho_{m_0} e^{\frac{-3}{p_2}\sqrt{2p_2t+k_1}}\right)^{\gamma_1}. \quad (41)$$

Figure 5 depicts the behavior of effective equation of state parameter ω_{eff}) of bulk viscous dark energy versus cosmic time t for values for $\gamma_1 = 0, 0.5, 1, 1.5$ and $\zeta_0 = 0, 0.5$. It can be observed from equation (??) that for $\zeta_0 = 0$, the effective equation of bulk viscous dark energy reduced to non-viscous dark energy fluid. Figure 5 shows that ω_{eff} of non-viscous DE ($\zeta_0 = 0$) is only varying in the quintessence region whereas for ($\zeta_0 \neq 0$) the variation of bulk viscous DE starts from the relatively high phantom region goes to low phantom regions. The EoS parameter of non-viscous dark energy, ω_{de}, is decreasing rapidly till present and it approaches -1 asymptotically in the future. So, in that case the model represents ΛCDM model for the future evolution of the universe.

Density parameters

The viscous dark energy density parameter Ω_{de}, dark matter density parameter Ω_m, and total density parameter Ω are given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \left(\frac{2p_2t+k_1}{3}\right)\rho_{m_0} e^{\frac{-3}{p_2}\sqrt{2p_2t+k_1}}, \quad (42)$$

$$\Omega_{de} = \frac{\rho_{de}}{3H^2} = 1 - \left(\frac{2p_2t+k_1}{3}\right)\left[\frac{b_1^2}{3} e^{\frac{-6}{p_2}\sqrt{2p_2t+k_1}} + \rho_{m_0} e^{\frac{-3}{p_2}\sqrt{2p_2t+k_1}}\right], \quad (43)$$

$$\Omega = \Omega_m + \Omega_{de} = \frac{\rho_m}{3H^2} + \frac{\rho_{de}}{3H^2} = 1 - \left(\frac{2p_2t+k_1}{3}\right)\frac{b_1^2}{3} e^{\frac{-6}{p_2}\sqrt{2p_2t+k_1}}. \quad (44)$$

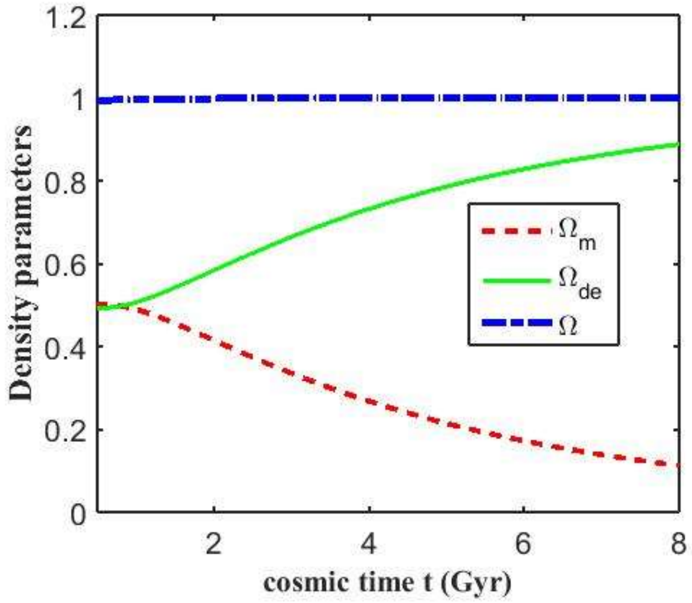

Figure 6: **P**lot of energy density parameters ($\Omega_m, \Omega_{de}, \Omega$) versus cosmic time t for $b_1 = 0.5$, $p_2 = 0.5$ and $k_1 = 0.5$.

From equation (42), the parameter of energy density of dark matter Ω_m is taken to be 0 for late time. However, from equation (43) the density parameter of viscous DE ultimately approaches 1. It is evident that for chosen values of free parameters, the overall density parameter of derived model is about 1 at late time which matches with observational value of the flatness of universe at present epoch.

State finder parameters

Since many number of dark energy models have been proposed to explain the accelerated expansion of the universe, it is very important to find a way to discriminate between the various contenders in a model-independent manner. Sahni et al. [53] have published a geometric diagnostic technique for contrasting various models of dark energy. For all models predicting the Hubble parameter, scale factor, deceleration parameter etc., to distinguish between the models, it is better to use quantities involving higher derivatives of H or the scale factor. The statefinder parameter pair (r, s) introduced by them depends on the third order derivative of the scale factor. A characteristic property of the statefinder parameter pair is that $(r, s) = (1,0)$, is a fixed point for the ΛCDM model. Evolutionary trajectories of these parameters and their difference from the fixed ΛCDM point distinguishes the models from each other and also from the standard ΛCDM model. The statefinder parameters are defined and given in our model as

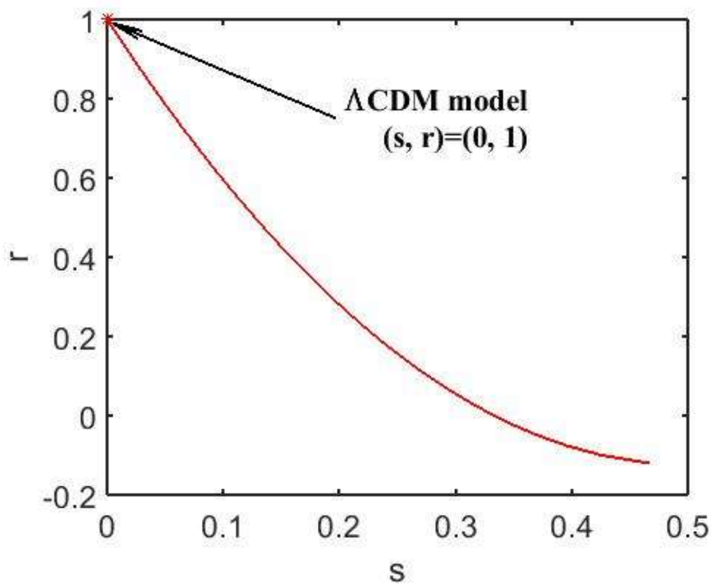

Figure 7: **P**lot of r versus s for $p_2 = 0.5$ and $k_1 = 0.5$.

$$r = \frac{\dddot{a}}{aH^3} = 1 - \frac{3p_2}{\sqrt{2p_2t+k_1}} + \frac{2p_2^2}{2p_2t+k_1}, \tag{45}$$

$$s = \frac{r-1}{3\left(q-\frac{1}{2}\right)} = \frac{-\frac{p_2}{\sqrt{2p_2t+k_1}}+\frac{2p_2^2}{3(2p_2t+k_1)}}{\frac{-3}{2}+\frac{p_2}{\sqrt{2p_2t+k_1}}}. \tag{46}$$

The evolution of the statefinder parameters in the $r - s$ plane is shown in figure 7. It is pointed out that the $r - s$ plane for the model corresponds to ΛCDM limit at late times. Its trajectories pointed out that the $r - s$ plane for model possesses the regions of the quintessence and phantom model.

5 Conclusions

In this paper, we have studied locally rotational symmetric Bianchi type-I universe filled by a pressureless dark matter and viscous dark energy in general relativity. To get the exact solutions of the Einstein's field equations, we assumed that the deceleration parameter (q) is the linear function of the Hubble parameter (H) i.e., $q = p_1 + p_2 H$, which yields scale factor a as $a = e^{\frac{1}{p_2}\sqrt{2p_2 t + k_1}}$ (Tiwari et al. 2015, 2016). We considered a case when viscous dark energy and pressureless dark matter do not interact with each other and the coefficient of bulk viscosity is taken to be proportional to some power function of the energy density. We observe that the volume of the model increases with increasing cosmic time. Hubble parameter, expansion scalar, shear scalar and directional Hubble parameters become converge to zero at late times showing a uniform spatial expansion of the universe.

Some of the physical behavior of the cosmological parameters is studied through their graphical representation. The followings are the interesting observations in the model: It is realistic because of the fact that the energy densities of matter and viscous dark energy are always non-negative and decrease with increasing cosmic time (see figures 2, 3). The behavior of deceleration parameter with respect to redshift shows that the model exhibits accelerating universe (figure 1). The deceleration parameter is found to be negative $q \simeq -1$ at present in the derived model which is supported by recent observations. It is observed that the bulk viscous coefficient decreases with cosmic time for the chosen three values of parameters γ_1 (figure 4).

Also, it is interesting to mention that as bulk viscous coefficient converges to zero, viscous dark energy reduced to dark energy density. We observe that in the absence of bulk viscosity $(\zeta_0 = 0)$, DE EoS parameter does not cross the phantom divide line and it varies in the quintessence region. However, when viscous dark energy is considered, its effective EoS parameter could lay in the phantom region which is shown in figure 5 for parameters $\gamma_1 = 0.5, 1, 1.5$, $\zeta_0 = 0,\ 0.5$, $b_1 = 0.5$, $p_2 = 0.5$ and $k_1 = 0.5$. As it is shown from the figure 5, for $\gamma_1 \geq 1$ it ultimately tends to the cosmological constant $(\omega_{\text{eff}} = -1)$ for late times. This special behavior of the effective EoS parameter is because of our choice of bulk viscosity which is a decreasing function of cosmic time in an expanding universe. Moreover from figure 6, the overall density parameter shows that it is constant and $\Omega \cong 1$ which can show satisfactory behavior of flat universe, which is in agreement with the observational data.

It is observed that the effective energy density parameter of dark energy dominates the energy density parameter of dark matter at late times. By illustrating the evolutionary trajectories in $r - s$ plane, the model corresponds to ΛCDM limit (figure 7) in late time. The above discussions show that the bulk viscosity of dark energy plays very important role in the expansion history of the universe and it gives a good agreement with the recent scenario of modern cosmology.

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