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SUPRA SOFT REGULAR OPEN(CLOSED) SETS ON FUZZY TOPOLOGICAL SPACES

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Abstract:

The Main Purpose of this paper is to introduce the concept of fuzzy supra soft topological spaces called fuzzy supra soft regular open (closed)sets. Which is a generalization to the notion of fuzzy soft topological spaces and supra soft topological spaces. We discussed certain properties and relations of fuzzy supra soft regular open sets and fuzzy supra soft regular closed sets in fuzzy supra soft topological spaces are established in the work. Finally, we establish some interesting properties of this notion.

Keywords: Fuzzy supra soft regular open sets, Fuzzy supra soft regular closed sets, Fuzzy supra soft topological spaces.

Introduction

The most appropriate theory for dealing with un-certainties is the theory of fuzzy sets, introduced by Zadeh [16] in 1965. This theory brought a paradigmatic change in mathematics. But, there exists a difficulty, how to set the membership function in each particular case. In 1968, Chang [5] introduced fuzzy topological space, and in 2011, subsequently, Cagman et al. [4] and Shabir et al. [13] introduced soft topological spaces and they defined basic notions of soft topological spaces. In 1999, Molodtsov [8] initiated the concept of soft theory as a new mathematical tool for dealing with uncertainties that are free from the above difficulties. Soft set theory has rich potential for applications in several directions, few of which had been shown by Molodtsov in his pioneer work. Applications of Soft Set Theory in other disciplines and real-life problems are now catching momentum. Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, theory of probability, theory of measurement, and so on. Maji et.al introduced Fuzzy soft sets [7]. In 2011, Tanay et al. [15] introduced the notion of fuzzy soft topological spaces. In 1937, M. Stone [14] introduced the notion of regular open sets in topological spaces. In 1981, K.K. Azad [3] introduced the concept of a fuzzy regular open set in fuzzy topological spaces. In 1987, M.E. Abd El-Monsef et al. [1] introduced fuzzy supra topological space and defined the basic notion of fuzzy supra topological spaces. In 2019, A.M. Abd El-Latif [2] introduced fuzzy supra soft topological spaces. This paper aims to introduce and study the notion of fuzzy supra soft topological spaces, which is a generalization to the notion of fuzzy soft topological spaces [9]and supra soft topological spaces [6]. It is hoped that our findings will help enhance this study on fuzzy supra soft topological spaces for the researchers.

2. Preliminaries

We start our work with the following definitions, which will be needed in this paper.

Definition2.1[16]

A fuzzy set A in a non-empty set X is characterized by a membership function μ_A: X → [0, 1] = I whose value μ_A (x) represents the "degree of membership" of x in A for $x \in$ X. Here, I^X denotes the family of all fuzzy sets on X.

Definition 2.2[8]

Let X be an initial universe and E be a set of parameters. Let P(X) denote the power set of X and A be a non-empty subset of E. A pair (F, A), denoted by F_A, is called a soft set over X, where F is a mapping given by F : A → P(X). For particular e ∈ A, F(e) may be considered the set of e-approximate elements of the soft set (F, A) and if e ∉A, then F(e) = φ i.e F_A = {(e, F(e)) : e ∈ A ⊆ E, F : A → P(X)}. The family of all these soft sets over X is denoted by $SS(X)_A$.

Definition 2.3[7]

Let A ⊆ E. A pair (f, A), denoted by f_A, is called a fuzzy soft set over X, where f is a mapping given by f : A → I^X defined by $f_A(e) = \mu^e_{f_A}$ where $\mu^e_{f_A}$=0 if e∉A and $\mu^e_{f_A} \neq 0$ if e ∈A, where $\overline{0}$ is the membership function of null fuzzy set over X, which takes value 0 for all $x \in X$ i.e, $\bar{0}(e) = 0\ \forall\ x \in X$. The family of all these fuzzy soft sets over X denoted by F $SS(X)_A$.

Definition 2.4[12]

The complement of a fuzzy soft set (f, A), denoted by $(f, A)^c$, is defined $(f, A)^c =$

(f^c, A) $f^c{}_A: A \to I^X$ is a mapping given by $\mu^e{}_{f^c{}_A} = \bar{1}\mu^e_{f_A}\ \forall\ e \in A$, where $\overline{1}(e) = 1\ \forall\ x \in X$. clearly $(f^c{}_A)^c =$ f_A.

Definition 2.5[7]

A fuzzy soft set f_A over X is said to be a null fuzzy soft set, denoted by $\tilde{0}_A$, if for all $e \in A$, $f_A(e) = \bar{0}$.

Definition 2.6[7]

A fuzzy soft set f_A over X is said to be an absolute fuzzy soft set, denoted by $\tilde{1}_A$, if for all $e \in A$, $f_A(e) = \bar{1}$, where $\bar{1}$ is the membership function of absolute fuzzy set over X, Which takes value 1 for all $x \in X$. Clearly, we have $(\tilde{1}_A)^c = \check{0}_A$ and $(\tilde{0}_A)^c = \tilde{1}_A$.

Definition 2.7[7]

Let f_A, $g_B \in$ F $SS(X)E$. Then, f_A is fuzzy soft subset of g_B, denoted by $f_A \sqsubseteq g_B$, if $A \subseteq B$ and $\mu^e_{f_A} \subseteq \mu^e_{g_B}\ \forall\ e \in A$, i.e. $\mu^e_{f_A}(x) = \mu^e_{g_B}(x)\ \forall\ x \in X$ and $\forall\ e \in A$.

Definition 2.8 [7]

The union of two fuzzy soft sets f_A and g_B over the common universe X is also a fuzzy soft set h_C, where $C = A \cup B$ and for all $e \in C$, $h_C(e) = \mu^e_{h_c} = \mu^e_{f_A} \vee \mu^e_{g_B}\ \forall e \in C$. Here, we write $h_C = f_A \sqcup g_B$.

Definition 2.9[7]

The intersection of two fuzzy soft sets f_A and g_B over the common universe X is also a fuzzy soft set h_C, where $C = A \cap B$ and for all $e \in C$, $h_C(e) = \mu^e{}_{h_C} = \mu^e_{f_A} \wedge \mu^e_{g_B}\ \forall e \in C$. Here, we write $h_C = f_A \sqcap g_B$.

Definition 2.10[15]

Let τ be a collection of fuzzy soft sets over a universe X with a fixed set of parameters E, then τ is called a fuzzy soft topology on X if

(1) $\tilde{1}E, \tilde{0}E \in \tau$, where $\widetilde{0}E(e) = 0$ and $\tilde{1}E(e) = 1, \forall e \in E$,

(2) The union of any members of τ, belongs to τ.

(3) The intersection of any two members of T, belongs to τ.

The triplet (X, τ, E) is called a fuzzy soft topological space over X. Also, each member of τ is called a fuzzy open soft in (X, τ, E). We denote the set of all fuzzy open soft sets by F OS

(X,τ, E), or FOS(X).

Definition 2.11[15]

Let (X, τ, E) be a fuzzy soft topological space. A fuzzy soft set f_A over X is said to be fuzzy closed soft set in, if its relative complement $f^c{}_A$ is fuzzy open soft set. We denote the set of all fuzzy closed soft sets by FCS(X, τ, E), or F CS(X).

Definition 2.12[11]

Let (X, T, E) be a fuzzy soft topological space and $f_A \in$ F SS(X)E. The fuzzy soft closure of f_A, denoted by F cl(f_A) is the intersection of all fuzzy closed soft super sets of f_A. i.e., Fcl(f_A) = $\sqcap\{h_D : h_D$ is fuzzy closed soft set and $f_A \sqsubseteq h_D\}$.

The fuzzy soft interior of g_B, denoted by F int(g_B) is the fuzzy soft union of all fuzzy open soft subsets of g_B. i.e. Fint(g_B) = $\sqcup\{h_D : h_D$ is fuzzy open soft set and $h_D \sqsubseteq g_B\}$.

Definition 2.13 [9]

The fuzzy soft set $f_A \in$ F SS(X)E is called fuzzy soft point if for the element e $\in$ E, $\mu^e{}_{fA}(x) \neq \bar{0}$ and $\mu^{e'}{}_{fA}$ (x) = $\bar{0}$ for each $e' \in E - \{e\}$, and this fuzzy soft point is denoted by $f^e{}_A$.

Definition 2.14 [9]

The fuzzy soft point $f^e{}_A$ is said to be belonging to the fuzzy soft set g_B, denoted by $f^e{}_A \breve{\in} g_B$, if for the element e $\in$ A $\cap$ B, $\mu^e F_A$ (x) $\leq \mu^e_{g_B}$ (x).

Definition 2.15[12]

Let T be a collection of fuzzy soft sets over a universe X with a fixed set of parameters E, then T is called a fuzzy supra soft topology on X if

- $\tilde{1}E, \tilde{0}E \in T$, where $\tilde{0}E(e) = 0$ and $\tilde{1}E(e) = 1$, $\forall e \in E$,
- The union of any members of T, belongs to T.

The triplet (X, T, E) is called a fuzzy supra soft topological space (fssts for short) over X. Also, each member of T is called a fuzzy supra open soft in (X, T, E). A fuzzy soft set f_A over X is said to be fuzzy supra closed soft set in X, if its relative complement f^{c}_{A} is a fuzzy supra open soft set. We denote the set of all fuzzy supra open (closed) soft sets by F SOS(X) (F SCS(X)).

Definition 2.16[2]

A fuzzy soft set g_B in a fssts (X, T, E) is called fuzzy supra soft neigh- borhood (briefly: fss neighborhood) of the fuzzy soft point $f^{e}_{A}\breve{\in}X_E$ if there exists a fuzzy supra open soft set h_C such that $f^{e}_{A}\breve{\in}h_C \subseteq g_B$. The fuzzy supra soft neighborhood sys- tem of a fuzzy soft point f^{e}_{A}, denoted by $N\tau(f^{e}_{A})$,is the family of all its fuzzy supra soft neighborhoods.

Definition 2.17[2]

Let (X, T, E) be a fssts and Y ⊆ X. Let y_E be a fuzzy soft set over (Y,E) defined by y_E: $E\rightarrow I^{Y}$ such that

$$y_E(e)=\mu e_{y_E} \text{ where } \mu e_{Y_E}(x) = \begin{cases} 1, & x \in Y, \\ 0, & x \in Y. \end{cases}$$

Then, the fssts

$Ty_E = \{y_E \sqcap g_B : g_B \in T\}$ is called the fuzzy supra soft subspace topology for y_E and (Y, Ty_E, E) is called fuzzy supra soft subspace of (X,T,E).

Theorem.1[2]

Let (X, T, E) be a supra soft topological space and f_A, $g_B \in SS(X)E$. Then

- $Fint^{fs}(\tilde{1}E) = \tilde{1}E$ and $F\, int^{fs}(\tilde{0}E) = \tilde{0}E$
- $F\, cl^{fs}(\tilde{1}E) = 1E$ and $F\, cl^{fs}(\tilde{0}E) = \tilde{0}E$.
- f_A is fuzzy supra open soft if and only if $F\, int^{fs}(f_A) = f_A$.
- $Fint^{fs}(F\, int^{fs}(f_A)) = F\, int^{s}(f_A)$ and $F\, int^{fs}(f_A) \subseteq f_A$.
- f_A is fuzzy supra closed soft set if and only if $Fcl^{fs}(f_A) = f_A$
- $Fcl^{fs}(Fcl^{fs}(f_A)) = F\, cl^{fs}(f_A)$ and $f_A \subseteq F\, cl^{fs}(f_A)$.
- If $f_A \subseteq g_B$,then $Fint^{fs}(f_A) \subseteq Fint^{fs}(g_B)$ and $Fcl^{fs}(f_A) \subseteq (g_B)$
- $Fcl^{fs}(f_A) = [Fint^{s}(f_A^{c})]^{c}$.

Theorem.2[2]

Let (X, T, E) be a fssts on X and $f_A \in SS(X)E$. Then:

- $F\, int^{fs}(f^{c}_{A}) = \tilde{1}_E - [Fcl^{fs}(f_A)]$.
- $F\, cl^{fs}(f^{c}_{A}) = \tilde{1}_E - [F\, int^{fs}(f_A)]$.

3. Fuzzy supra soft Regular open (closed) sets

This section aims to introduce the notion of fuzzy supra soft topological spaces. We also introduce some basic definitions of fuzzy supra soft topological spaces and theorems of concept.

Definition 3.1:

Let (F, A) be a fuzzy soft set in a fuzzy supra soft topological space (X, E, T*), then

(F, A) is called

- a fuzzy supra soft regular open set if int*(cl* (F, A)) = (F, A)
- a fuzzy supra soft regular closed set if cl*(int*(F, A)) = (F, A).

Theorem3.2:

Let (F, A) be a fuzzy soft set in a fuzzy supra soft topological space (X, E, T*), then

(F, A) is a fuzzy supra soft regular open set if and only if $(F, A)^c$ is a fuzzy supra soft regular closed set.

Proof:

Let (F, A) be a fuzzy supra soft set then,

$$int^*(cl^*(F, A)) = (F, A)$$

$$[int^* (cl^*(F, A)]^c = (F, A)^c$$

$$cl^*[(cl^* (F, A))^c]= (F,A)^c$$

$$cl^*(int^*[(F, A)^c)]) = (F,A)^c$$

$$cl^* (int^*(F, A)^c) = (F,A)^C$$

(F^C, A) is a fuzzy supra soft regular closed set.

Conversely, Let $(F, A)^c$ be a fuzzy supra soft regular closed set.

Consider,

$$cl^*(int^* ((F, A)^c)) = (F, A)^c$$

$$[cl^*(int^* ((F , A)^c))]^c = [(F, A)^c]^c$$

$$int^* [(int^* ((F, A)^c))^c] = (F, A)$$

$$int^* (cl^* [((F, A)^c)^c]) = (F , A)$$

$$int^* (cl^* (F, A)) = (F , A).$$

Since (F, A) is a fuzzy supra soft regular open set.

Theorem 3.3:

Every fuzzy supra soft regular open set is a fuzzy supra soft open set.

Proof:

Let (F, A) be a fuzzy supra soft regular open set in (X, E, T*). Then int*(cl* (F, A)) = (F, A). By definition of fuzzy supra soft interior, int* (F, A) is the largest fuzzy supra soft open set contained in (F, A). int*(cl*(F, A)) is the largest fuzzy supra soft open set contained in cl*(F, A). (F, A) is a fuzzy supra soft open set.

Theorem 3.4:

Every fuzzy supra soft regular closed set is a fuzzy supra soft closed set.

Proof:

Let (F, A) be a fuzzy supra soft regular closed set (X, E, T*). cl*(int*(F, A)) = (F, A). By the definition of fuzzy supra soft closure, cl*(F, A) is the smallest fuzzy supra soft closed set containing (F, A). cl*(int*(F, A)) is the smallest fuzzy supra soft closed set containing int*(F, A). (F, A) is a fuzzy supra soft closed set.

Theorem 3.5:

The fuzzy supra soft closure of a fuzzy supra soft open set is a fuzzy supra soft regular closed set.

The fuzzy supra soft interior of a fuzzy supra soft closed set is a fuzzy supra soft regular open set.

Proof:

Let (F, A) be a fuzzy supra soft open set in a fuzzy supra soft topological space

(X, E, T*).

By the definition of fuzzy supra soft interior,

$$\text{int}^* (F, A) \leq (F, A)$$

$$\text{Since} \quad \text{int}^*(\text{cl} (F, A)) \leq \text{cl}^*(F, A)$$

$$\text{cl}^*(\text{int}^*(\text{cl}^*(F, A))) \leq \text{cl}^*(\text{cl}^*(F, A))$$

$$\text{cl}^*(\text{int}^*(\text{cl}^*(F, A))) \leq \text{cl}^*(F, A) \;\ldots\ldots\ldots\ldots\ldots\ldots\; (1)$$

$$\text{since } (F, A) \leq \text{cl}^*(F, A)$$

$$\text{int}^* (F, A) \leq \text{int}^*(\text{cl}^*(F ,A))$$

$$(F, A) \leq int^*(cl^* (F, A))$$

$$cl^*(F, A) \leq cl^*(int^*(cl^*(F, A))) \text{ ………..(2)}$$

From (1) & (2)

$$cl^*(int^* (cl^*(F, A))) = cl^* (F, A)$$

Since fuzzy supra soft closure of a fuzzy supra soft open set is a fuzzy supra soft regular closed set.

Let (F, A) be a fuzzy supra soft closed set in a fuzzy supra soft topological space (X, E, T*). By the definition of fuzzy supra soft closure, $(F, A) \leq cl^* (F,A)$.

$$int^*(int^*(F, A) \leq int^*(cl^*(int^*(F, A)))$$

$$int^*(F, A) \leq int^*(cl^*(int^*(F, A))) \text{ ……(3)}$$

Since $int^*(F, A) \leq (F, A)$

$$cl^*(int^*(F, A)) \leq cl^* (F, A)$$

$$cl^*(int^*(F, A)) \leq (F, A)$$

$$int^*(cl^*(int^*(F, A))) \leq int^* (F, A) \text{………(4)}$$

From (3) & (4)

$$int^* (cl^* (int^*(F, A))) = int^*(F, A)$$

Since fuzzy supra soft interior of a fuzzy supra soft closed set is a fuzzy supra soft regular open set.

Definition 3.6:

Let (X, E, T*) be a fuzzy supra soft topological space and (F, A) be a fuzzy soft set in X, then the fuzzy supra soft regular closure and fuzzy supra soft regular interior of (F, A) is denoted and defined respectively as

$$rcl^*(F, A) = \wedge\{(G, B) | (G, B) \text{ is a fuzzy supra soft regular closed set in X and } (F, A) \leq (G, B)\}$$

$$rint^*(F,A) = \vee\{(G,B) | (G,B) \text{ is a fuzzy supra soft regular open set in X and } (G, B) \leq (F, A)\}$$

Remark3.7:

Obviously, rint* (F, A) is a fuzzy supra soft regular open set, and rcl* (F, A) is a fuzzy supra soft regular closed set.

Theorem 3.8:

For any fuzzy soft set (F, A) in a fuzzy supra soft topological space (X, E, T*),

- $[\text{rint}^*(F, A)]^c = r\, cl^*(F, A^c)$
- $[r\, cl^*(F, A)]^c = r\, int^*(F, A^c)$

Proof:

Consider

$$r\, int^*(F, A) = \{(G, B) \mid (G, B) \text{ is a fuzzy supra soft regular open set in } X \text{ and } (G, B) \le (F, A)\}$$

$$[\text{rint}^*(F, A)]^c = 1 - \vee\{(G, B) \mid (G, B) \text{ is a fuzzy supra soft regular open set in X and } (g, B) \le (F, A)\}$$

$$= \wedge\{(g, B)^c \mid (g, B)^c \text{ is a fuzzy supra soft regular closed set in X and } (g, B)^c \ge (f, A)^c\}$$

$$= \text{rcl}^*(F, A^c)$$

$$r\, cl^*(F,A) = \wedge\{(G, B) \mid (G, B) \text{ is a fuzzy supra soft regular set in X and } (F, A) \le (G, B)\}$$

$$[r\, cl^*(F, A)]^c = 1 - \wedge\{(G, B) \mid (G, B) \text{ is a fuzzy supra soft regular closed set in X and } (F, A) \le (G, B)\}$$

$$= \vee\{(G, B)^c \mid (G, B)^c \text{ is a fuzzy supra soft regular open set in X and } (F, A)^c \ge (G, B)^c\}$$

$= r\ int^*(F, A^c)$

Theorem 3.9:

For any two fuzzy soft sets (F, A) and (G, B) in fuzzy supra soft topological space (X, E, T*), Then if $(F, A) \leq (G, B)$ implies

- $rint^*(F, A) \leq r\ int^*(G, B)$
- $r\ cl^*(F, A) \leq r\ cl^*(G, B)$

Proof:

$rint^*(F, A) = \vee\{(h, c) \mid (h, c)$ is a fuzzy supra soft regular open set in X and $(h, c) \leq (F, A)\}$

since rint*(F, A) is the largest fuzzy supra soft regular open set contained in (F,A).

$r\ int^*(F, A) \leq (F, A)$

$(F, A) \leq (G, B)$ implies that

$rint^*(F, A) \leq (F, A) \leq (G, B)$

implies $rint^*(F, A) \leq (G, B)$

since r int* (F, A) is the fuzzy supra soft regular open set contained in (G, B).

But rint* (G, B) is the largest fuzzy supra soft regular open set contained in (G, B).

Since $rint^*(F, A) \leq r\ int^*(G, B)$.

$(F, A) \leq (G, B)$ iff $(F, A)^c \geq (G, B)^C$

Consider $(G, B)^c \leq (F, A)^c$

$rint^*(G, B)^c \leq r\ int^*(F, A^c)$

$(rcl^*(G, B))^c \leq (rcl^*(F, A))^c$

Since $rcl^*(F, A) \leq rcl^*(G, B)$.

Conclusion:

This paper introduces and studies the notion of fuzzy supra soft topological spaces. We introduce some new concepts in fuzzy supra soft topological spaces. Certain properties and relations of fuzzy supra soft regular open sets and fuzzy supra soft regular closed sets in fuzzy supra soft topological spaces are established in the work. In the end, we must say that, this paper is just a beginning of a new structure and we have studied a few ideas only, it will be necessary to carry out more theoretical research to establish a general framework for the practical application. It is hoped that our findings will help enhancing this study on fuzzy supra soft topological spaces for the researchers.

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