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Graph theory In Linear programming

[#1]**M.Durgadevi** (MCA,M.Sc)Assistant Professor, Department of M.Sc Mathematics,
CH.S.D.ST.THERESA'S COLLEGE FOR WOMEN(A), ELURU.

Email: m.devi.mca.06@gmail.com

[#2]**G. Bhavana Priya**, pursuing M.Sc. Mathematics, **CH.S.D.ST.THERESA'S COLLEGE FOR WOMEN(A), ELURU.**

Email: bhavana321priya@gmail.com

[#3]**S.Naga durga, Msc,** Assistant Professor, Department of M.Sc Mathematics,
CH.S.D.ST.THERESA'S COLLEGE FOR WOMEN(A), ELURU.

Email: sndurga210@gmail.com

Abstract:

In mathematics, graph proposition is the study of graphs, which are fine structures used to model brace wise relations between objects. A graph in this environment is made up of vertices (also called bumps or points) which are connected by edges (also called links or lines). Graphs can be used to model numerous types of relations and processes in physical, natural, social and information systems. Numerous practical problems can be represented by graphs. Emphasizing their operation to real- world systems, the term network is occasionally defined to mean a graph in which attributes are associated with the vertices and edges, and the subject that expresses and understands the real- world systems as a network is called network wisdom. Linear programming is an important subject. Linear programming is used for carrying the most optimal result for a problem with given constraints. In direct programming, we formulate our real- life problem into a fine model. It involves an objective function, direct inequalities with subject to constraints. When you have a problem that involves a variety of resource constraints, direct programming can induce the stylish possible result. Whether it's maximizing effects like profit or space, or minimizing factors like cost and waste, using this tool is a quick and effective way to structure the problem, and find a result.

Key words: Graph theory, linear programming, Real world systems, Networks, Maximizing and Minimizing.

Introduction:

In order to have a graph we need to define the rudiments of two sets vertices and edges. The vertices are the fundamental units that a graph must have, in order for it to live. It's customary to put on graphs the condition that they must have at least one vertex, but there's no real theoretical reason why this is the case. Vertices are fine abstractions corresponding to objects associated with one another by some kind of criterion. Edges rather are voluntary, in the sense that graphs with no edges can still be defined. The idea behind edges is that they indicate, if they are present, the actuality of a relationship between two objects, that we imagine the edges to connect. In distinction with vertices, edges cannot live in sequestration. This derives from the consideration that graphs themselves bear vertices in order to live, and those edges live in relation to a graph.

Graphs whose edges connect further than two vertices also live and are called hyperactive graphs. This tutorial doesn't concentrate on them, but we've to mention their actuality because of their literal and contemporary significance for the development of knowledge graphs.

Linear programming is a simple fashion where we depict complex connections through direct functions and also find the optimum points. The important word in the former judgment is depicted. The real connections might be much more complex but we can simplify them to direct connections. Operations of programming are everyplace around you. You use direct programming at particular and professional fronts. You're using direct programming when you're driving from home to work and want to take the shortest route. Or when you have a design delivery you make strategies to make your platoon work efficiently for on- time delivery.

Using direct programming requires defining variables, chancing constraints and chancing the objective function, or what needs to be maximized. In some cases, direct programming is rather used for minimization, or the lowest possible objective function value. Linear programming requires the creation of inequalities and also graphing those to break problems. While some direct programming can be done manually, relatively frequently the variables and computations come too complex and bear the use of computational software.

Graph theory Used in Linear programming:

- An **arc** is a directed line (a pair of ordered vertices).
- An **edge** is line joining a pair of nodes.
    - **Incident** edges are edges which share a vertex. An edge and vertex are **incident** if the edge connects the vertex to another.
- A **loop** is an edge or arc that joins a vertex to itself.
- A **vertex**, sometimes called a **node**, is a point or circle. It is the fundamental unit from which graphs are made.
    - **Adjacent** vertices are vertices which are connected by an edge.
    - The **degree** of a vertex is simply the number of edges that connect to that vertex. Loops count twice.
    - A **predecessor** is the node (vertex) before a given vertex on a path.
    - A **successor** is the node (vertex) following a given vertex on a path.
- **A walk** is a series of vertices and edges.
    - A **circuit** is a closed walk with every edge distinct.
    - A **closed walk** is a walk from a vertex back to itself; a series of vertices and edges which begins and ends at the same place.
    - A **cycle** is a closed walk with no repeated vertices (except that the first and last vertices are the same).
    - A **path** is a walk where no repeated vertices. A **u-v** path is a path beginning at u and ending at v.
    - A **u-v walk** would be a walk beginning at u and ending at v.

In Linear programming the graphs are used in different concepts like graphical method and Critical path method and different networks. Graphs are places major role in linear programming. The Linear programming techniques and graph theory having interconnection between them. The terminology used in Graph theory which also present in linear programming.

Applications Of Graph Theory in Linear programming:

Graphical method:

PROBLEM:

solve the following LPP by using graphical method max $z=6x_1+8x_2$; subject to the constraints $5x_1+10x_2\leq60$ and $4x_1+4x_2\leq 40$ where x_1, $x_2\geq0$.

Solution:

1) the given problem is LPP

convert all the inequalities into equations

$5x_1+10x_2=60$ ………… (1)

$4x_1+4x_2= 40$ ………… (2)

2) in equation-(1) put $x_1=0$

$5x_1+10x_2=60$

$x_2=6$

The corresponding coordinates is (0,6)

In equation-(1) put $x_2=0$

$5x_1+10x_2=60$

$x_1=12$

The corresponding coordinates is (12,0)

In equation-(2) put $x_1=0$

$4x_1+4x_2= 40$

$x_2=10$

The corresponding coordinate is (0,10)

In equation-(2) put $x_2=0$

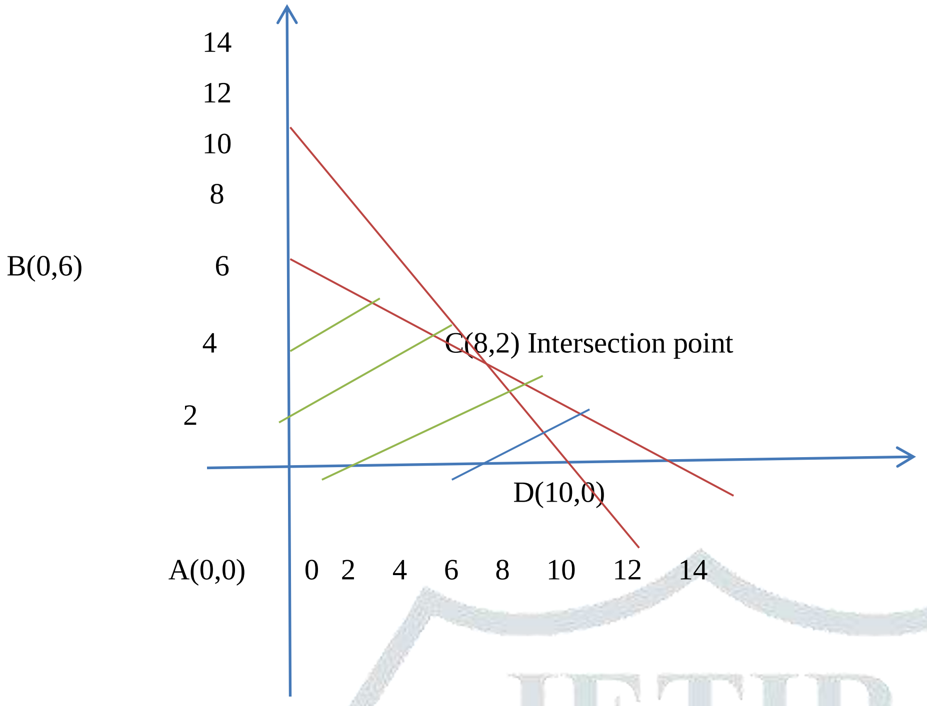
$x_1=10$

The corresponding coordinate id (10,0)

3) plot the points (0,6), (12,0), (0,10), (10,0) in the x_1x_2-plane

equation-(1) ………… (0,6), (12,0)

equation-(2) ………… (0,10), (10,0)



Hence A, B, C, D is a closed polygon were

A= (0,0), B= (0,6), C= (8,2), D= (10,0)

C (8,2) is the intersection of two lines are

$5x_1+10x_2=60$& $4x_1+4x_2=40$

Then we have $x_1=8$ & $x_2=2$ where c is (8,2)

The given objective function is,

Max $z=6x_1+8x_2$ for A= (0,0)

Max z =6(0) +8(0)

Max z=0

For B= (0,6)

Max z=6(0) +8(6)

Z(B)=48

For c= (8,2)

Z(c)=6(8) +8(2)

Z(c)=64

For D (10,0)

Z(D)=6(10) +8(0)

z(D)=60

since the given problem is of maximization type the optimization solution is $x_1^*=8$, $x_2^*=2$

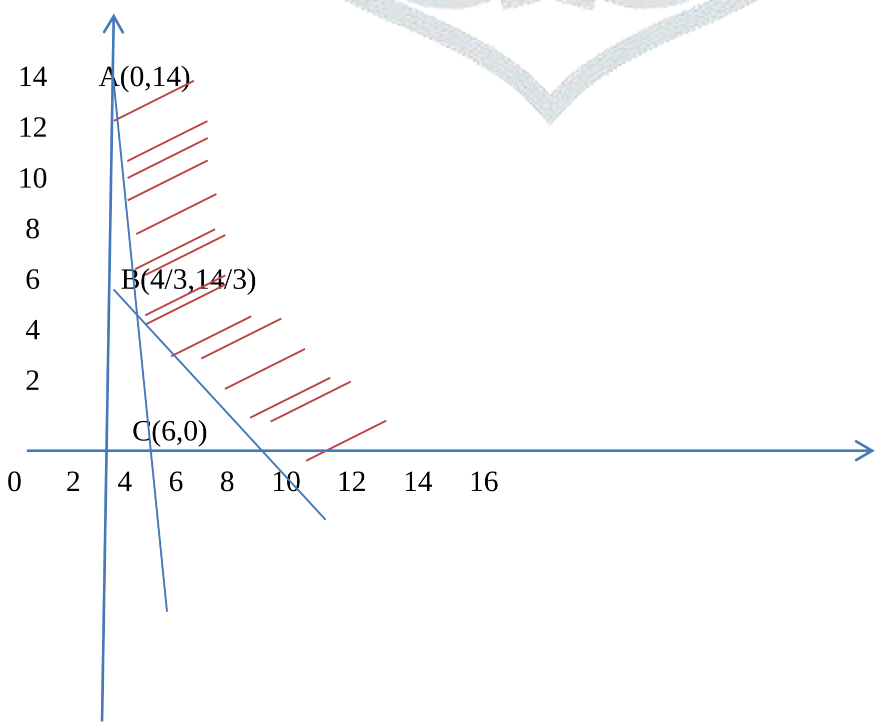
therefore, max z=64.

PROBLEM:

solve the following LPP by using graphical method, min $z=2x_1+3x_2$ subject to the constraints $x_1+x_2=6$ and $7x_1+x_2\geq14$ where $x_1, x_2\geq0$.

Solution:

1) the given LPP is
   convert all the inequalities into equations
   $x_1+x_2=6$………… (1)
   $7x_1+x_2=14$………… (2)
2) the equation-(1)
   put $x_1=0$ then $x_2=6$
   the corresponding coordinate is (0,6) in equation-(1); put $x_2=0$ then $x_1=6$
   the corresponding coordinate is (6,0) in equation-(1); put $x_1=0$ then $x_2=14$
   the corresponding coordinate is (0,14) in equation-(2); put $x_2=0$ then $x_1=2$
   the corresponding coordinate is (2,0)
3) put the points (0,6), (6,0), (0,14), (2,0) in x_1x_2-plane



Hence A, B, C are closed polygon where A= (0,14), B= $\left(\frac{4}{3},\frac{14}{3}\right)$, C= (6,10) is the intersection of two lines are

$x_1+x_2=6, 7x_1+x_2=14$

Then we have $x_1=\frac{4}{3}$, $x_2=\frac{14}{3}$ where c $\left(\frac{4}{3},\frac{14}{3}\right)$

4) The given objective function is,

Min $z=2x_1+3x_2$ for A (0,14)

Min z=2(0) +3(14) =42=z(A)

Min $z=2x_1+3x_2$ for B= $\left(\frac{4}{3},\frac{14}{3}\right)$

Min $z=2(\frac{4}{3})+3(\frac{14}{3})$

=16.7=z(B)

Min $z=2x_1+3x_2$ for C (6,0)

=2(6) +3(0) =12=z(C)

Since the given problem is of minimization type the optimization solution is $x_1^*=6$, $x_2^*=0$.

Therefore, min z=12.

GRAPHICAL METHOD FOR (2*n) AND (m*2) GAMES

The optimal strategies for a (2*n) or (m* 2) matrix game can be located easily by a simple graphical method. This method enables us to reduce the 2 x n or m x 2 matrix game to 2x2 games that could be easily gloved by the earlier methods.

If the graphical method is used for a particular problem, then the same reasoning can be used to solve any game with mixed strategies that has only two un dominated pure strategies for one of the players

Optimal strategies for both the players assign non-zero probabilities to the same number of pure strategies. It is clear that if one player has only two strategies, the other wilt also uses two strategies. Hence, graphical method can be used to find two strategies of the player. The method can be applied to 3x n or mx3 games also by carefully dewing three-dimensional diagram.

GRAPHICAL METHOD FOR (2*n) GAMES

Consider the (2 X n) game, assuming that the game does not have a saddle point.

Since the player A has two strategies, it follows that x=1-x, x. Thus, for each of the pure strategies available to the player B, the expected payoff for the player A. Would be as follows:

This shows that the player A's expected payoff varies linearly with x_1

According to the maxi min criterion for mixed as Pure Strategies strategy games; the player A should select the value of x, so as to maximize his minimum expected payoff. This may be done by plotting the following straight lines:

B's pure strategies	**A's expected pay off $e_j[x_1]$**
B_1	$V_{11} X_1+V_{21}(1-X_1) =(V_{11}-V_{21}) X_1+V_{21}$
B_2	$V_{12} X_1+V_{22}(1-X_1) =(V_{12}-V_{21}) X_1+V_{22}$
B_N	$V_{1n} X_1+V_{2n}(1-X_1) =(V_{1n}-V_{2n}) X_1+V_{2n}$

$E_1(x_1) = (v_{11}-V_{21}) X_1 + V_{21}$

$E_2(x_1) = (v_{12}-V_{22}) X_1 + V_{22}$

. $En(x_1) = (v_{1n}-V_{2n}) X_1 + V_{2n}$

as functions of x. The lowest boundary of these lines will give the minimum expected payoff as function of the highest point on this lowest boundary would then give the maxi min expected payoff and the optimum value of $x_1[=x_1*]$.

Now determine only two strategies for player B corresponding to those two lines which pass through the maxi min point P. This way, it is possible to reduce the game to 2 x 2 which can be easily solved either by using formulae given or by arithmetic method.

EXAMPLE: Outlines of Graphical Method:

To determine maxi min value p_1 we take different values of $x_1$the horizontal line and values of $E(x_1)$ on vertical axis. Since $0<=x_1<=1$, the straight-line $E_j(x_1)$ must pass through the points {0, Ej (0)}and {1, Ej (1)}, where E_j (0) =v2j and E_j (1) =v1j. Thus, the lines $E_j(x_1) =(v_{1j}-v_{2j}) x_1+ v_{2j}$

for j=1,2, …., n can be drawn as follows:

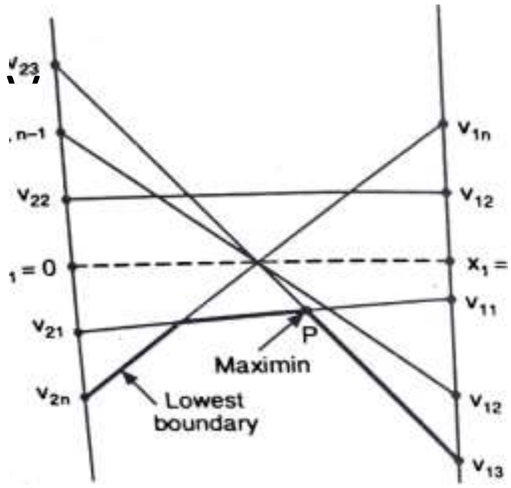
Step1. Construct two vertical axes, axis 1 at the point $x_1 = 0$ and axis 2 at the point $x_1 = 1$.

Step2. Represent the payoffs v_{2j}, j=1,2, ..., n on axis 1 and payoff $v_{1j,}$ j =1, 2......, on axis 2.

Step3. Join the point representing v on Axis 2 to the point representing v_{2j}on axis 1. The resulting straight line is the expected payoff line $E_j(x_1)$ =1, 2…. n.

Step4. Mark the lowest boundary of the lines $E_j(x_1)$ so plotted, by thick line segments. The highest point on this lowest boundary gives the maxi min point P and identifies the two critical moves of player B.

If there are more than two lines passing through the maxi min point P, there are ties for the optimum mixed strategies for player B. Thus, any two such lines with opposite sign slopes will define an alternative optimum for B.



Graphical solution of 2xn games

Graphical Solution of m x 2 Games

The (m x 2) games are also treated in the like manner except that the mini max point P is the lowest point on the uppermost boundary instead of highest point on the lowest boundary.

From this discussion, it is concluded that any (2 x n) or (mx2) game is basically equivalent game to a (2 x 2) game.

Now each point of the discussion is explained by solving numerical examples for (2xn) and (m x 2) games.

Q. Explain the graphical method of solving (2x m) and (m x 2) games.

EXAMPLE: Solve the following (2 x 3) game graphically.

y_1	y_2	y_3
1	3	11
8	5	2

SOLUTION: This game does not have a saddle point. Thus, the player A's expected payoff corresponding to the player B's strategies are given (*Table).*

Three expected payoff lines are:

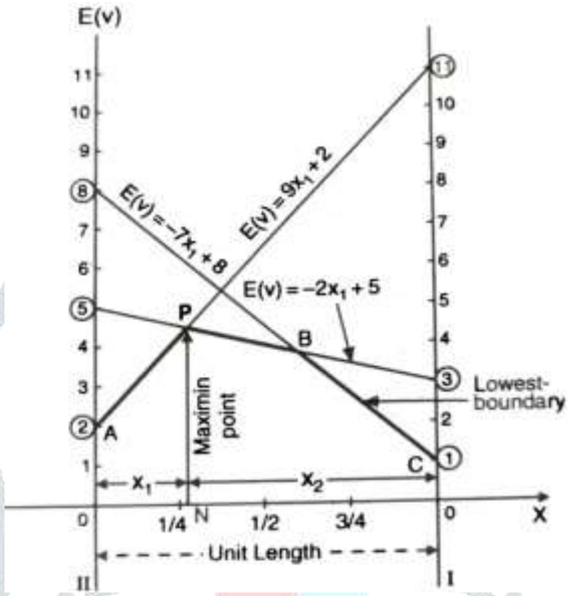
$E(x_1) = -7x_1 + 8$, $E(x_1) = -2x_1 + 5$ and $E(x_1) = 9x_1 + 2$

and can be plotted on a graph as follows

B's Pure strategies	A's Expected Payoff $E(X_1)$
1	$E(x_1) = 1x_1+8(1-x_1) = -7x_1+8$
2	$E(x_1) = 3x_1+5(1-x_1) = -2x_1+5$
3	$E(x_1) = 11x_1+2(1-x_1) = 9x_1+2$

First, draw two parallel lines one unit apart and mark a scale on each. These two lines will represent two strategies available to player A. Then draw lines to represent each of player B's strategies.

For example, to represent player B's 1st strategy, join mark I on scale 1 to mark8 on scale II: to represent player g's second strategy, join mark 3on the scale I to mark 5 on scale 1, and so on. Since the expected payoff $E(x_1)$ is the function alone, these three expected payoff lines can be drawn by taking $x_1$as the x-axis and $E(x_1)$ as the y-axis.



GRAPHICAL REPRESENTATION FOR SOLVING (2XN) GAME.

Points A. P. B, C on the lowest boundary represent the lowest possible expected gain to player A for any value of $x_1$between 0 and 1. According to the maxi min criterion, player A chooses the best of these worst outcomes.

Clearly, the highest point P on the lowest boundary will give the largest expected gain PN to A. So best strategies for player 8 are those which pass through point P. Thus, the game is reduced to 2 x 2 (Table 19-31). Now, by solving the simultaneous s equations.

$3x_1 + 5x_2 = v$, $11x_1 + 2x_2 = v$, $x_1 + x_2 = 1$ (For player A)

$3y_2 + 11y_3 = v$, $5y_2 + 2y = v$, $y_2 + y = 1$ (For player B)

the solution of the game is obtained as follows:

(1) The player A chooses the optimal mixed strategy $(x_1, x_2) = (3/11, 8/11)$.

(2) The player B chooses the optimal mixed strategy $(y_1, y_2, y_3) = (0, 2/11, 9/11)$.

(3) The value of the game to player A is $v = 49/11$.

EXAMPLE: Solve the game graphically whose pay-off matrix for player A is given in Table 19:32:

Solution. The game does not have a saddle point. Let y_1 and $y_2 = (1-y_1)$ be

Mixed strategies of player B. The four straight lines thus obtained are

$E(y_1)$ =-2y_1 +4, $E(y_1)$ =-y_1+3,

$E(y_1)$ = y_1+2, $E(y_1)$ = -8y_1 +6,

A's Pure Strategies	B's Expected Payoff $E(y_1)$
1	$E(y_1) = 2y_1 + 4(1-y_1)$
2	$E(y_1) = 2y_1+ 3(1-y_1)$
3	$E(y_1) = 3y_1 + 2(1-y_1)$
4	$E(y_1) = -2y_1 + 6(1-y_1)$

and these are plotted in Fig. 19-3. In this case, the mini max point is determined as the lowest point P on the uppermost boundary. Lines intersecting at the mini max point P correspond to player A's pure strategies I and III. This indicates x= x=0. Thus, the reduced game is given in Table.

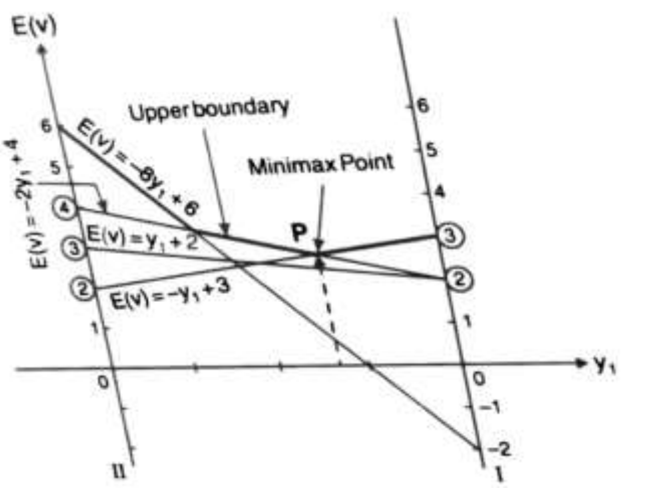
Now, solve this (2x2) game by solving the simultaneous equations:

to get the solution:

(1) The player A chooses the optimal mixed strategy, $(x_1, x_2, x_3, x_4) = (1/3, 0, 2/3, 0)$.

(2) The player B chooses the optimal mixed strategy, $(y_1, y_2) = (2/3, 1/3)$.

(3) The value of the game to player A is v=8/3.



Graphical representation for [mx2] game.

EXAMPLE: Two firms A and B make colour and black & white television sets. Firm A can make either 150 colour sets in a week or an equal number of black & white sets, and make a profit of Rs. 400 per colour set and Rs. 300 per black & white se. Firm B can, on the other hand, make either 300 colour sets, or 150 colour 150 black & white sets, or 300 black & white sets per week. It also has the same profit margin on the two ez as A. Each week there is a market of 150 colour sets and 300 black & white sets and the manufacturers would share market in the proportion in which they manufacture a particular type of set. Write the pay-off matrix of A per week. Obtain graphically A's and B's optimum strategies and value of the game.

Solution. For firm A, the strategies are:

A_1: make 150 colour sets.

A_2: make 150 black & white sets

For firm 8, the strategies are:

B_1: make 300 colour sets,

B_2: make 150 colour and 150 black & white sets,

B_3: make 300 black and white sets.

For the combination A, B_1, the profit to firm A would be:150/ (150 + 300) x 150 x 400 = Rs. 20,000

wherein (150/150 + 300) represents share of market for A, 150 is the total market for colour television sets and 400 is the profit per set. In a similar way, other profit figures may be obtained as shown in the following pay-off matrix:

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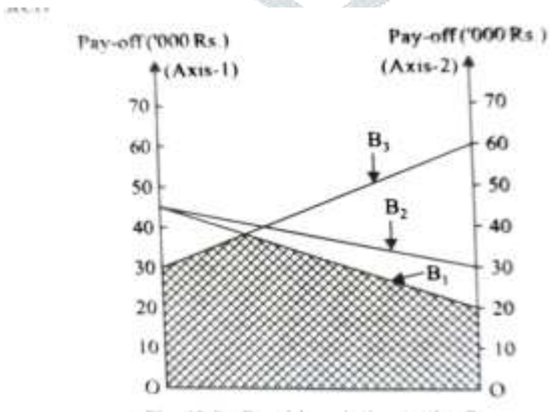
B's strategy

	B_1	B_2	B_3
A_1	20000	30000	60000
A_2	45000	45000	30000

Since no saddle point exists, we shall determine optimum mixed strategy. The data are plotted on graph as shown in the adjoining Fig.

Lines joining the pay-offs on axis I with the pay-offs on axis II represents each of B's strategies. Since firm A wishes to maximize his minimum expected pay-off, we consider the highest point P on the lower envelope of A's expected pay-off equation. This point P represents the maxi min expected value of the game for firm A. The lines B_1 and B3 passing through P, define the relevant moves B, and B, that alone from B needs to adopt. The solution to the original 2 x 3 game, therefore, reduces to that of the simple game with 2 x 2 pay-off matrix as follows:



GRAPHICAL SOLUTION TO THE GAME

A's strategy

	B_1	B_2
A_1	20,000	60,000
A_2	45,000	30,000

Correspondingly,

$P_1 = (a_{22}+a_{21})/((a_{11}+a_{22}) – (a_{12}+a_{21})) = (30,000-40,000)/ ((20,000+30,000) – (60,000+45,000)) =$ **3/11**

$q_1 = (a_{22}-a_{21})/((a_{11}+a_{22}) – (a_{12}+a_{21})) = (30,000-60,000)/ ((20,000+30,000) – (60,000+45,000)) =$ **6/11**

$v = (a_{11}a_{22}-a_{12}a_{21})/((a_{11}+a_{22}) – (a_{12}+a_{21})) = (20,000 \times 30,000-60,000 \times 45,000)/ ((20,000+30,000)$

$(60,000+45,000)) =$ **38,132.**

Critical Path Method (CPM):

The critical path method is used to determine the shortest possible time to complete the project

The CPM is sequence of activities of a project's starting activity and time the project's finish time and activity

This tool is totally based on mathematical calculation & is used for scheduling project activities

The CPM was developed by JAMES E. KELLY and MORGAN R. WALKER in the 1950's

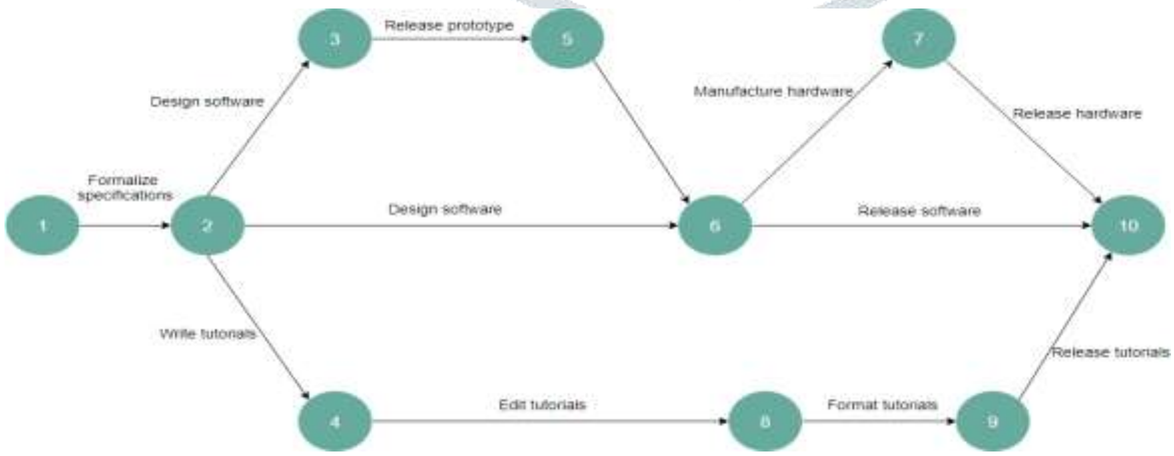
Initially, the CPM _method was used for managing plant maintenance project

CPM_ method is used for planning and scheduling activity

This method also shows the interrelationship between the time & cost of the project

CPM method helps used to assess the different possibilities of project planning

This method is one of the parts of the Arrow Diagram



CPM method is uses:

CPM method is used in the below fields

- Software Development
- Construction
- Aerospace& Defence
- Research Projects
- Product Development& many more fields

Steps in CPM:

These are the 5 steps in the CPM method as mentioned below.

STEP_1 List out the activity

STEP_2 Forward pass calculation

STEP_3 Backward passes calculations

STEP_4 Float calculations for each activity

STEP_5 Identifying critical path

Advantages of CPM method:

This tool is used for planning scheduling, monitoring, and controlling various types of projects

It helps us for proper communications between departments and functions

We can estimate the expected project completion date

This tool is very easy to use we can identify the critical path with help of this tool

It is very useful in monitoring costs

This tool is visually very effective so all people can easily understand

We can also reduce the project completion time lines by using this method

CPM method improves the decision-making ability of the team

By using this tool, we can able to determine which activity can we delayed without delaying the project.

Conclusion:

In the above Paper we explained how the graph theory plays an important role in linear programming. The Graph theory used in different ways in Operations research. In Game theory and Graphical Method in Linear programming and Critical Programming method etc. Likewise, we are using the graph theory widely in real world problems also.

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