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ON τ^*-GENERALIZED α CONTINUOUS MULTIFUNCTIONS IN TOPOLOGICAL SPACES

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ABSTRACT:

In this paper, we introduce the concept of τ^*-generalized α continuous multifunctions in topological spaces and study some of their properties where τ^* is defined by τ^*={G: $cl^*(G^c)$= G^c}

KEY WORDS: τ^*-gα open set, τ^*-gα closed set, τ^*-gα continuous

1. INTRODUCTION

In 1965, Njastad [9] introduced a weak form of open sets called α-sets and Mashhour et al. [8] introduced the concept of α- continuous mappings in 1983. In 1982, the author Noiri. T [13] defined a function from a topological space into a topological space to be strongly semi-continuous if the inverse image of each open set is an α-set. In 1986, Neubrunn [10] extended these functions to multifunctions and introduced the notion of upper (lower) α-continuous multifunctions. Various types of functions play a significant role in the theory of classical point set topology. A great number of papers dealing with such functions have appeared, and many of them have been extended to the setting of multifunctions. A multifunction is a set-valued function.

Multifunction in topological spaces have been extensively studied by general topologists. For any two sets X and Y, F: X→ Y is a multifunction, if for each $x \in$ X and F(x) is a non empty subset of Y. For a multifunction F: X→ Y, upper and lower inverse subset B of Y is denoted by F^+(B) and F^-(B) respectively that is F^+(B)={$x \in$ X: F(x)⊆B} and F^- (B)={x∈X:F(x) ∩B≠ ∅}. The graph G(F) of the multifunctions F: X → Y is strongly closed if for each(x, y) ∉ G(F), there exist open sets U and V containing x and y respectively such that (U×cl(V)∩G(F)= ∅. Throughout this paper, the space (X, τ*) and (Y, σ) always mean topological spaces and F: X→Y represents a multivalued function. The concept of multifunctions has advanced in a variety of ways and applications of this theory can be found, specially in functional analysis and fixed point theory.

2 .Preliminaries:

Definition: 2.1[9]

Let X be a topological space and A be a subset of X. The closure of A and interior of A are denoted by cl A and int(A), respectively.

A subset A is said to be α-open if A⊂int(cl(int(A))).

Definition: 2.2[15]

A subset A is called an α-neighbourhood of a point x in X if there exists U ∈ $\alpha(x)$ such that x ∈U⊂A.

Lemma: 2.3 [1]

The following are equivalent for a subset A of a topological space X:

(i) A∈ $\alpha(x)$
(ii) U⊂A⊂int(cl (U)) for some open set U.
(iii)U⊂A⊂scl (U) for some open set U.
(iv)A⊂scl (int(A)).

Lemma: 2.4 [12]

The following properties hold for a subset A of a topological spaces X:

(i) A is α-closed in X if and only if sint(cl(A))⊂A;
(ii) sint(cl(A))= cl (int(cl(A)));
(iii) αcl(A)= A∪ cl (int(cl(A))).

Definition: 2.5 [7]

A subset A of a topological space X is called an α-generalized closed (briefly αg-closed) if cl_{α}(A)⊆G whenever A⊆G and G is open in X.

Definition: 2.6 [2]

For the subset A of a topological space X, the generalized closure operator Cl^{*} is defined by the intersection of a g-closed sets containing A.

Definition: 2.7 [2]

For the subset A of a topological space X, the topology τ^{*} is defined by τ^{*}={G:$Cl^{*}(G^{C})$=G^{C}}.

Definition: 2.8

A subset A of a topological space X is called τ^{*}-generalized α closed set (briefly τ^{*}-gα closed). If cl^{*} [(cl_{α} (A) ⊆ G .Wherever A ⊆ G and G is τ^{*}open. The complement of τ^{*}-generalized α closed set is called the τ^{*} -generalized α open set (briefly τ^{*}-gα open).

Definition: 2.9 [6]

A function f: X→Y is said to be feebly continuous if for every open set V of Y, f^{-1}(v) is feebly open in X.

Definition: 2.10 [8]

A function f : X →Y is said to be α-continuous if for every open set V of Y, f^{-1}(v) is α-open in X.

Definition: 2.11[4]

A function f : X→ Y from a topological space X into a topological space Y is αg-continuous if the inverse image of a closed set Y is αg-closed in X.

Definition: 2.12 [3]

A function f : X→ Y from a topological space X into a topological space Y is τ^*-g continuous if the inverse image of a g-closed set in Y is τ^*-g closed in X.

Definition: 2.13

A function f : X→ Y from a topological space X into a topological space Y is τ^*-generalized α continuous function (briefly τ^*-gα continuous) if the inverse image of a gα-open set in Y is τ^*-g open in X.

Remark: 2.14

If X is α -compact and f : X→ Y is an α-continuous surjective, then Y is compact. [11]

If f, g: X→ Y are α continuous and Y is Hausdroff, then the set {x ∈ X/f(x) = g(x)} is α-closed in X. [12]

3. ON τ^*-GENERALIZED α CONTINUOUS MULTIFUNCTIONS :

Definition: 3.1

A multifunction F: (X, τ*)→ (Y, σ) is said to be,

(i) Upper τ^*-generalized α continuous at a point x of X if for any open set V of Y such that F(x)⊂ V, there exists U ∈ τ^*-gα(X) containing x such that F(U) ⊂V;
(ii) Lower τ^*-generalized α continuous at a point x ∈ X if for any open set V of Y such that F(x) ∩ V≠ ∅, there exists U ∈ τ^*-gα(X) containing x such that F(u)∩ V≠ ∅ for every u ∈U;
(iii) Upper (resp. lower) τ^* -generalized α continuous if it is upper (resp. lower) τ^* -generalized α continuous at every point of X.

Theorem: 3.2

The following are equivalent for a multifunction F: (X, τ^*)→ (Y, σ)

(i) F is upper τ*-gα continuous at a point x ∈ X;
(ii) x ∈ τ*-gscl (int F⁺(V)) for any open set V of Y containing F(x);
(iii) For and U ∈ τ*-gsO(X) containing x and any open set V of Y containing F(x), there exists a nonempty open set U_V of X such that U_V ⊂ U and F(U_V)⊂ V .

Proof :

(i)⇒ (ii) Let V be any open set such that F(x)⊂ V. Then there exists U ∈ τ*-gα(X) containing x such that F(U) C V ; hence x ∈ U ⊂ F^+(V). Since U is τ*-gα open,
By Lemma 2.3 (iv) we have $x \in U \subset \tau^*$-gscl (int(V)) ⊂ τ^*-gscl (int F^+V))) .
(ii)⇒ (iii) Let V be any open set of Y such that F(x) ⊂ V . Then $x \in \tau^*$-gscl (int F^+V))). Let U be any τ^*-generalized semi-open set containing x. Then U ∩ int (F^+V))≠ ∅ and U ∩ int(F^+(V)) ∈ τ^*-gs*O(X)*. Put U_V = int[U∩ int(F^+V))] , then U_V is a nonempty open set of Y, $U_V \subset U$ and $F(U_V) \subset V$.
(iii)⇒ (i) Let τ^*-gsO(X, x) be the family of all τ^*-generalized semi-open sets of X containing x . Let V be any open set of Y containing F*(x).* For each $U \in \tau^*$-gsO(X, x), there exists a nonempty open set U_V such that $U_V \subset U$ and $F(U_V) \subset$ V. Let W=∪{U_V/U∈ τ^*-gsO(X, x)}. Then W is open in X, $x \in \tau^*$-gscl (W) and *F(W)* ⊂ V. Put *S*=*W*∪{x}, then W⊂S⊂ τ^*-gscl (w). Then by lemma 2.3 (iii), $x \in \tau^*$-gs ∈ τ^*-gα(X) and *F(S)* ⊂ V . Hence *F* is upper τ^*-gα continuous at x.

Theorem: 3.3

The following are equivalent for a multifunction F : (X, τ*)→ (Y, σ)

(i) F is upper τ*-gα continuous.
(ii) F^+(V) ∈ τ*-gα (X) for any open set V of Y .
(iii) F^-(V) is τ*-gα closed in X for any closed set V of Y .
(iv) τ*-gsint(cl(F^-(B)) ⊂ F^-(cl (B)) for any set B of Y .
(v) τ*-gαcl (F^-(B))⊂ F^-(cl (B)) for any set B of Y .

(vi) For each point x of X and each neighbourhood V of F(x), $F^+(V)$ is an α-neighbourhood of x .
(vii) For each point $x \in X$ and each neighbourhood V of F(x) , there exists an α-neighbourhood U of x such that F(U)⊂ V .

Proof:

(i)⇒(ii) Let V be any open set of Y and let $x \in F^+(V)$. By Theorem 3.2, $x \in \tau^*$-gscl (int $F^+(V)$). Then $F^+(V) \subset \tau^*$-gscl (int (F^+ (V))). By Lemma 2.3 (i), $F^+(V) \in \tau^*$-gα(X) .

(ii)⇒(iii)In fact that $F^+(Y- B)= X - F^-(B)$ for any subset B of Y .

(iii)⇒(iv)Let B be any subset of Y. Then $F^-(cl(B))$ is τ^*-gα closed in Y. By Lemma 2.4 (i), we have τ^*-gsint(cl $F^-(B)$)⊂ τ^*-gsint(cl (F^- cl (B)))) ⊂ $F^-(cl$ (B)) .

(iv)⇒(v)Let B be any subset of Y. By Lemma 2.4 (iii), we have τ^*-gαcl(F^- (B))= $F^-(B) \cup \tau^*$-gsint(cl $F^-(B)$))⊂ $F^-(cl(B))$.

(v)⇒(iii)Let V be any closed set of Y. Then we have τ^*-gαcl($F^-(V)$)⊂ $F^-(cl$ (V))= $F^-(V)$. Hence F^- (V) is τ^*-gα closed in X.

(ii)⇒(vi) Let $x \in X$ and V be a neighbourhood of F(x) , there exists a open set G of Y such that F(x) ⊂ G⊂ V. So that $x \in F^+(G) \subset F^+(V)$. Since F^+ (G) ∈ τ^*-gα(X), $F^+(V)$ is an α-neighbourhood of x.

(vi)⇒(vii) Let $x \in X$ and V be a neighbourhood of F(x). Put U =$F^+(V)$, then U is an α-neighbourhood of x and F(U) ⊂ V.

(vii)⇒(i) Let $x \in X$ and V be a any open set of Y such that F(x) ⊂ V. There exists an α-neighbourhood U of x such that F(U) ⊂ V. Then A∈ τ^*-gα(X) such that $x \in$ A⊂ U, hence F(A) ⊂ V.

Theorem: 3.4

The following are equivalent for a multifunction F : (X, τ^*)→ (Y, σ)

(i) F is lower τ^*-gα continuous.
(ii) $F^-(V) \in \tau^*$-gα (X) for any open set V of Y .
(iii) $F^+(V)$ is τ^*-gα closed in X for any closed set V of Y .
(iv) τ^*-gsint(cl F^+(B)) ⊂ $F^+(cl(B))$ for any subset B of Y .
(v) τ^*-gα cl (F^+(B))⊂ $F^+(cl(B))$ for any subset B of Y .
(vi) F(τ^*-gαcl (A))⊂ cl (F(A)) for any subset A of X.
(vii) F(τ^*-gsint(cl(A)))⊂ cl (F(A)) for any subset A of X.
(viii)F(cl (int(cl (A))))⊂ cl (F(A)) for any subset A of X.

Proof:

The proof (i)⇒(ii), (ii)⇒(iii), (iii)⇒(iv), (iv)⇒(v) are similar to the above theorem.

(v)⇒(vi)Let A be any subset of X. Since A⊂ F^+ (F(A)), we have τ^*-gαcl(A) ⊂ τ^*-gαcl (F^+(F(A))) ⊂((F^+(cl (F(A))) and F(τ^*-gαcl(A))⊂ cl (F(A)).

(vi)⇒(vii) By lemma 2.4 (ii), F((τ^*-gsint(cl (A)))⊂ cl (F(A)) for any subset A of X.

(vii)⇒(viii) This is obvious.

(viii)⇒(i) Let $x \in X$ and V be any open set such that $F(x) \cap V \neq \emptyset$. Then $x \in F^-(V)$. Now $F^-(V) \in$ (τ^*-gα(X). By the hypothesis, F(cl (int(cl(F^+(Y-V))) ⊂ cl (F(F^+(Y-V)) ⊂ Y-V and hence cl (int(cl (F^+(Y-V)))⊂ F^+(Y-V)=X-F^- (V). Then we have $F^-(V)$ ⊂int(cl(int($F^-(V)$)) and hence $F^-(V) \in$ τ^*-gα(X). put U= $F^-(V)$, we have x ∈U ∈ τ^*-gα(X) and F(u)∩V≠∅ for every u ∈ U . Hence F is *lower* τ^*gα continuous.

Note : 3.5 [12]

A function f : X→Y is α-continuous if and only if it is pre continuous and semi- continuous.

Definition: 3.6 [15]

A subset A of a topological space X is said to be α-paracompact if every cover of A by open sets of X is refined by a cover of A which consists of open sets of X and is locally finite in X.

Definition: 3.7 [5]

A subset A of topological space X is said to be α- regular if for each point $x \in A$ and each open set U of X containing x, there exists an open set G of X such that $x \in G \subset cl(G) \subset U$.

Lemma: 3.8

If A is an α -regular α -paracompact subset of a topological space X and U is an open neighbourhood of A , then there exists an open set G of X such that $A \subset G \subset cl(G) \subset U$.[5]

A multifunction F: X → Y is said to be punctually α-paracompact (resp. punctually α-regular) if for each $x \in X$, $F(x)$ *is α* -paracompact (resp. α -regular). By $\alpha cl(F): X \to Y$, we shall denote a multifunction defined as follows: $[\alpha cl(F)(x)] = \alpha cl(F(x))$ for each point $x \in X$.

Lemma: 3.9

If $F:(X,\tau^*) \to (Y,\sigma)$ is punctually α-regular and punctually α -paracompact, the $[\tau^*\text{-}g\alpha cl(F)^+](V) = F^+(V)$ for every open set V of Y .

Proof:
Let V be any open set of Y and $x \in [\tau^*\text{-}g\alpha cl(F)^+](V)$. Then $\tau^*\text{-}g\alpha cl(F(x)) \subset V$ and hence $F(x) \subset V$. Then, $x \in F^+(V)$ and hence $[\tau^*\text{-}g\alpha cl(F)^+](V) \subset F^+(V)$. Conversely, let V be any open set of Y and $x \in F^+(V)$. Then $F(x) \subset V$. Since $F(x)$ is α -regular and α -paracompact, by lemma 3.8 there exists an open set G such that $F(x) \subset G \subset cl(G) \subset V$;hence $\tau^*\text{-}g\alpha cl(F(x)) \subset cl(G) \subset V$. So that $x \in [\tau^*\text{-}g\alpha cl(F)^+](V)$ and hence $F^+(V) \subset [\tau^*\text{-}g\alpha cl(F)^+](V)$.So, that $[\tau^*\text{-}g\alpha cl(F)^+](V) = F^+(V)$.

Theorem: 3.10

Let $F:(X,\tau^*) \to (Y,\sigma)$ be punctually α -regular and punctually α -paracompact. Then F is upper τ^*-gα continuous if and only if $\tau^*\text{-}g\alpha cl(F): (X,\tau^*) \to (Y,\sigma)$ is upper τ^*-gα continuous.

Proof:

Suppose that F is upper τ^*-gα continuous. Let $x \in X$ and V be any open set of Y such that $\tau^*\text{-}g\alpha cl(F)(x) \subset V$. By Lemma 3.9, we have $x \in [\tau^*\text{-}g\alpha cl(F)]^+(V) = F^+(V)$. Since F is upper τ^*-gα continuous, there exists $U \in \tau^*\text{-}g\alpha(X)$ containing x such that $F(U) \subset V$. Since $F(u)$ is α -paracompact and α -regular for each $u \in U$, by Lemma 3.8, there exists an open set H such that $F(u) \subset H \subset cl(H) \subset V$. Then, we have $\tau^*\text{-}g\alpha cl(F(u)) \subset cl(H) \subset V$ for each $u \in U$ and hence $\tau^*\text{-}g\alpha cl(F(U)) \subset V$. Hence $\tau^*\text{-}g\alpha cl(F)$ is upper τ^*-gα continuous.

Conversely assume that $\tau^*\text{-}g\alpha cl(F): (X,\tau^*) \to (Y,\sigma)$ is upper τ^*-gα continuous. Let $x \in X$ and V be any open set of Y such that $F(x) \subset V$. By Lemma 3.9, we have $x \in F^+(V) = [\tau^*\text{-}g\alpha cl(F)^+](V)$ and hence $[\tau^*\text{-}g\alpha cl(F)(x) \subset V$. Since $\tau^*\text{-}g\alpha cl(F)$ is upper τ^*-gα continuous, there exists $U \in \tau^*\text{-}g\alpha(X)$ containing x such that $\tau^*\text{-}g\alpha cl(F(U)) \subset V$; hence $F(U) \subset V$. Then F is upper τ^*-gα continuous.

Theorem: 3.11

A multifunction $F:(X,\tau^*) \to (Y,\sigma)$ is lower τ^*-gα continuous if and only if $\tau^*\text{-}g\alpha cl(F): (X,\tau^*) \to (Y,\sigma)$ is lower τ^*-gα continuous.

Theorem: 3.12

If $F: (X,\tau^*) \to (Y,\sigma)$ is upper (resp. lower) τ^*-gα continuous and F(X) is endowed with subspace topology, then $F: X \to F(X)$ is upper(resp. lower) τ^*-gα continuous.

Proof:

Since $F:(X,\tau^*) \to (Y,\sigma)$ is upper(resp. lower) τ^*-gα continuous for every open subset V of Y, $F^+(V \cap F(X)) = F^+(V) \cap F^+(F(X)) = F^+(V)$ is τ^*-gα open. Hence $F: X \to F(X)$ is upper (resp.lower) τ^*gα continuous.

Theorem: 3.13

Let F : $(X,\tau^*) \to (Y,\sigma)$ and G : $(Y, \sigma) \to (Z, \mu)$ be two multifunctions. Then G∘F is upper (resp. lower) τ^*-gα continuous, if G is semi continuous and F is τ^*-gα continuous.

Proof:

Let V be an open set in Z. Since G is semi continuous then $(G^+(V))$ is an open set in Y and since F is τ^*-gα continuous then $F^+(G^+(V)) = (G \circ F)^+(V)$ is an τ^*-gα open set in X. Thus G∘F is upper (resp. lower) τ^*-gα continuous.

Theorem: 3.14

Let F: $(X,\tau^*) \to (Y,\sigma)$ be a multifunction and A be an open subset of X. If F *is* upper (resp. lower) τ^*-gα continuous, then $F/_A$: A →Y is upper (resp. lower) τ^*-gα continuous multifunction.

Proof:

Let V be any open subset of Y. Since F is upper (resp. lower) τ^*-gα continuous, then $F^+(V)$ is τ^*-gα open in X. Since $A \cap F^+(V) = F^+|_A(V)$, then $F^+|_A(V)$ is τ^*-gα open. Hence $F|_A$ is upper (resp. lower) τ^*-gα continuous multifunction.

Theorem: 3.15

Let F:$(X, \tau^*) \to (Y, \sigma)$ be a multifunction and $\{U\alpha : \alpha \in \Delta\}$ be an open cover of X. If the restriction function $F|_{U_\alpha}$ is upper τ^*-gα continuous for each $\alpha \in \Delta$, then F is upper τ^*-gα continuous.

Proof:

Let V be any open subset of Y. Since $F/_{U\alpha}$ is upper τ^*-gα continuous for each $\alpha \in \Delta$, then $F^+/_{U\alpha}(V) = U_\alpha \cap F^+(V)$ is τ^*-gα open set.

Then $\cup_{\alpha\in\Delta}(U_\alpha \cap F^+(V)) = \cup_{\alpha\in\Delta}(U_\alpha) \cap F^+(V) = X \cap F^+(V) = F^+(V)$ is τ^*-gα open set. Hence, F is upper τ^*-gα continuous.

4. SOME PROPERTIES

Lemma:4.1 [15]

Let A and B be subsets of a topological space X.

(i)*If* $A \in sO(X) \cup pO(X)$ *and* $B \in \alpha(X)$, *then* $A \cap B \in \alpha(A)$.

(ii)If $A \subset B \subset X$, $A \in \alpha(B)$ and $B \in \alpha(X)$, then $A \in \alpha(X)$.

Theorem: 4.2

If a multifunction, F :$(X, \tau^*) \to (Y,\sigma)$ is upper(resp. lower) τ^*-gα continuous $X_0 \in \tau^*$-gpO(X)∪ τ^*-gsO(X), then the restriction F / X_0: $X_0 \to Y$ is upper(resp. lower) τ^*-gα continuous.

Proof:

We prove that, F is upper τ^*-gα continuous. Let $x \in X_0$ and V be any open set of Y such that $(F/X_0)(x) \subset V$. Since F is upper τ^*-gα continous and $(F/X_0)(x)$=F(x) there exists U∈ τ^*-gα(X) containing x such that F(U) ⊂V. Set $X_0 = F \cap X_0$, then by lemma 4.1, $x \in X_0 \in \tau^*$-gα(X_0) and $(F/X_0)\, X_0 \subset V$. Hence (F/X_0) is upper τ^*-gα continous .

Theorem: 4.3

A multifunction *F: (X,* τ^**)*→ (Y,σ) *is upper*(resp. lower) τ^*-gα continuous if for each x∈X there exists $X_0 \in \tau^*$-gα(X)containing x such that the restriction $F/X_0 : X_0 \to Y$ is upper (resp.lower)τ^*-gα continuous.

Proof:

We prove that, F is upper τ^*-gα continuous. Let x∈X and V be any open set of Y such that F(x)⊂V. There exists $X_0 \in \tau^*$-gα(X) containing x such that F/X_0 is upper τ^*-gα continuous.

Then there exists $\cup_0 \in \tau^*$-gα(X_0) containing x such that $(F/X_0) \subset X_0$. By lemma 4.1 $\cup_0 \in \tau^*$-gα(X) and F(u)= (F/X_0)(u) for every $u \in U_0$. Hence, *F: (X,* τ^**)*→ (Y,σ) *is upper* τ^*-gα continuous.

Theorem: 4.4

If *F : (X,* τ^**)*→ (Y,σ) *is* upper τ^*-gα continuous multifunction into a Hausdroff space Y and F(x) is compact for each x∈X, then the graph G(F) is τ^*-gα closed in X×Y

Proof:

Let (x, y)∈ X×Y-G(F). Then y ∈Y-F (x). For each a ∈F(x), there exist open sets V(a) and W(a) containing a and y, respectively, such that V(a)∩W(a)=∅. The family {V((a))⁄a∈F(x)} is an open cover of F(x) and there exist a finite number of points in F(x), say, a_2, $a_1 \ldots a_n$ such that F(x)⊂∪{V(x_i)/1≤i≤n}. Set V=∪{V(x_i)/1≤i≤n} and W=n{W(a_i))/1≤i≤n}. Since F(x) ⊂ V and F is upper τ^*-gα continuous, there exists U∈ τ^*-gα(X) such that x∈U and F(U) ⊂V. So, we obtain F(U)∩W=∅ and hence (U×W)∩G(F)=∅. Since U×W is τ^*-gα-open X ×Y and (x, y)∈ U×W, (x,y)∉ τ^*-gαcl ($G(F)$) and G(F) is τ^*-gα closed in X×Y.

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