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Rational Type Fixed Point Theorem in Complex Valued Metric Spaces.

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Abstract:

In this paper, we have established a common fixed point theorem for rational type maps in complex valued metric spaces. We have proved the result for three maps by using weakly compatible maps. Our result generalizes the result of Pathak [24], Deepak *et.al* [7] Sendersija *et.al* [2] and many others.

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Keywords: Contractive map, common fixed point, rational type mapping, complex valued metric space, partial ordered set.

1.0 Introduction:

Azam *et. al* [1] in 2011 introduced the idea of complex valued metric space, and a common fixed points of a pair of mappings satisfying a contractive condition. Later on Bhatt [ ], F. Riuzkard *et.al* [9] [10] [11] and many others established fixed point theorems in complex valued metric spaces.

In this paper, we have established a common fixed point theorem for rational type maps in complex valued metric spaces. We have proved the result for three maps by using weakly compatible maps.

2.0 Preliminaries:

Here we will discuss some basic notions and established results which will be needed in the sequel. The following definitions is recently introduced by Azam *et.al* [1].

Let C be the set of Complex numbers and $z_1, z_2 \in \mathbb{C}$. Define a partial order $\precsim$ on C as follows:

$z_1 \precsim z_2$ if and only if $Re(z_1) \leq Re(z_2)$ and $Im(z_1) \leq Im(z_2)$.

Consequently, we can infer that $z_1 \precsim z_2$ if one of the following conditions is satisfied:

$Re(z_1) = Re(z_2)$ and $Im(z_1) < Im(z_2)$.

$Re(z_1) < Re(z_2)$ and $Im(z_1) = Im(z_2)$.

$Re(z_1) < Re(z_2)$ and $Im(z_1) < Im(z_2)$.

$Re(z_1) = Re(z_2)$ and $Im(z_1) = Im(z_2)$.

In particular, we write $z_1 \precsim z_2$ if $z_1 \neq z_2$ and one of (i), (ii), and (iii) is satisfied and we write $z_1 \prec z_2$ if only (iii) is satisfied.

We Note that $0 \precsim z_1 \precnsim z_2 \Rightarrow |z_1| < |z_2|$, and $z_1 \precsim z_2$, $z_2 \prec z_3 \Rightarrow z_1 \prec z_3$.

The following definition is recently introduced by Azam et al. [1].

Definition (2.1):[1]: "Let X be a nonempty set whereas C be the set of Complex numbers. Suppose that the mapping $d : X \times X \to C$, satisfies the following conditions:

(d1): $0 \precsim d(x, y)$, for all $x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$;

(d2): $d(x, y) = d(y, x)$ for all $x, y \in X$;

(d3): $d(x, y) \precsim d(x, z) + d(z, y)$, for all $x, y, z \in X$.

Then d is called a Complex valued metric on X, and (X, d) is called a Complex valued metric space.

Example(2.2):[1]:"Let X = C . Define the mapping d: X x X $\to C$ by

$d(z_1, z_2) = e^{ik} \, | z_1 - z_2|$, where $k \in R$.

Then (X,d) is a complex valued metric space."

Definition(2.3):[1]:Let (X,d) be a Complex valued metric space and $\{x_n\}_{n\geq 1}$ be a sequence in X and $x \in X$. We say that

(i) the sequence $\{x_n\}_{n\geq 1}$ converges to x if for every $c \in C$, with $0 \prec C$ there is $n_0 \in N$ such that for all $n > n_0$, $d(n_0, x) \prec c$. We denote this by $\lim_{n\to\infty}\{x_n\} = x$, or $\{x_n\} \to x$, as $n \to \infty$.

(ii) the sequence $\{x_n\}_{n\geq 1}$ is Cauchy sequence if for every $c \in \mathbb{C}$ with $0 \prec c$ there is $n_0 \in N$ such that for all $n > n_0$, $d(x_n, x_{n+m}) \prec c$,

(iii) the metric space (X, d) is a Complete Complex valued metric space if every Cauchy sequence is Convergent.

Definition(2.4): [1]: "A point $x \in X$ is said to be a fixed point of $T: X \to X$ if $Tx = x$.

Definition (2.5):[1]: "A point $x \in X$ is said to be a common fixed point of T and S if $Tx = x,\ and\ Sx = x$.

Definition (3.2.16):[17]:"Let (X,d) be a complex valued metric space. A mapping T = X→X is called a contraction on X if there exists a constant k such that

$d(Tx,Ty) \preccurlyeq kd(x,y)$ for all x,y in X. when $k \in (0,1)$".

3.0 Main Results:

Theorem (3.1) Let E, F and T be three self maps of a complete complex valued metric space such that:

(i) T is continuous

(ii) $\lfloor E,T\}\ and\ \{F,T\} are\ weakly\ Compatible.$

(iii) $E(X) \subseteq T(X)F(X) \subseteq T(X) \subseteq T(X)$

(iv) $\lambda\mathbb{C}_d(Ex,Fy) \le q\max\{\mathbb{C}_d(Tx,Ty),\mathbb{C}_d(Tx,Ex),\mathbb{C}_d(Ty,Fy),\mathbb{C}_d(Tx,Fy),$

$$\mathbb{C}_d(Ty,Ex)\}$$

$$+q_2\frac{\mathbb{C}_d(Tx,Ex)\mathbb{C}_d(Ty,Fy)}{1+\mathbb{C}_d(T_x,T_y)}$$

$$+q_3\frac{\mathbb{C}_d(Tx,Ex)\mathbb{C}_d(Ty,Fy)}{1+\mathbb{C}_d(Tx,Ty)+\mathbb{C}_d(Tx,Fy)+\mathbb{C}_d(Tx,Ex)}$$

$$+q_4\frac{\mathbb{C}_d(Ty,Ex)\mathbb{C}_d(Tx,Fy)}{1+\mathbb{C}_d(Tx,Ty)+\mathbb{C}_d(Tx,Ex)+\mathbb{C}_d(Ty\ Fy)}$$

$$+q_5\frac{\mathbb{C}_d(Tx,Ex)\mathbb{C}_d(Ty,Fy)}{1+\mathbb{C}_d(Ty,Xy)+\mathbb{C}_d(Tx,Ex)+\mathbb{C}_d(Ty\ Fy)}$$

for all $x,y \in X$ where $\lambda, q_{1,}q_{2,}q_{3,}q_{4,}q_5 \in C_+$, and o $< q_1+q_2+q_3+q_4+\ q_5 < \lambda$

Then E, F and T have a unique common fixed point in X.

Proof:

Let x_0 be any arbitrary point of X. Since E (X) $\subseteq T(X)$ we can choose a point in X_1 that $Tx_1 = Ex_{0.}$ Also F(x) $\subseteq T(x)$ We can choose a point $x_2$such that $Tx_2 = Fx_{1.}$

$$In\ general, Tx_{2p+1} = Ex_{2p}\ and\ Tx_{2p+2} = Fx_{5p+1}\ for\ p = 0,1,2,\ldots$$

Now,

$\lambda\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2}0)$

$$= \mathbb{C}_d(Ex_{2p}, Fx_{2p+1})$$

$$\leq q_1 \max \left\{ \begin{array}{c} \mathbb{C}_d(Tx_{2p}, T_{x2p+1}), \mathbb{C}_d(Tx_{2p}, Ex_{2p}), \mathbb{C}_d(Tx_{2p+1}, Fx_{2p+1}), \\ \mathbb{C}_d(Tx_{2p}, Fx_{2p+1}), \mathbb{C}_d(Tx_{2p+1}, Ex_{2p}\}. \end{array} \right\}$$

$$+q_2 \frac{\mathbb{C}_d(Tx_{2p}, Ex_{2p}) + \mathbb{C}_d(Tx_{2p+1}, Fx_{2p+1})}{1 + \mathbb{C}_d(Tx_{2p}, Tx_{2p+1})}$$

$$+q_3 \frac{\mathbb{C}_d(Tx_{2p}, Ex_{2p})\mathbb{C}_d(Tx_{2p+1}, Fx_{2p+1})}{1 + \mathbb{C}_d(Tx_{2p}, Tx_{2p+1}) + \mathbb{C}_d(Tx_{2p}, Fx_{2p+1}) + \mathbb{C}_d(Tx_{2p+1})}$$

$$+q_4 \frac{\mathbb{C}_d(Tx_{2p+1}, Ex_{2p})\mathbb{C}_d(Tx_{2p}, Fx_{2p+1})}{1 + \mathbb{C}_d(Tx_{2p}, Tx_{2p}) + \mathbb{C}_d(Tx_{2p}, Ex_{2p}) + \mathbb{C}_d(Tx_{2p+1}, Fx_{2p+1})}$$

$$+q_5 \frac{\mathbb{C}_d(Tx_{2p}, Ex_{2p}) + \mathbb{C}_d(Tx_{2p+1}, Fx_{2p+1})}{1 + \mathbb{C}_d(Tx_{2p}, Tx_{2p+1}) + \mathbb{C}_d(Tx_{2p}, Ex_{2p}) + \mathbb{C}_d(Tx_{2p+1}, Fx_{2p+1})}$$

Or, $\lambda\mathbb{C}_d(T_{x2p+1}, Tx_{2p+2})$

$$\preccurlyeq q_1 \max. \{\mathbb{C}_d(Tx_{2p}, Tx_{2p+1}), \mathbb{C}_d(Tx_{2p}, px_{2p+1}),$$

$$\mathbb{C}_d(Tx_{2p+1}, Tx_{2p+2,}), \mathbb{C}_d(Tx_{2p}, Tx_{2p+2}), \mathbb{C}_d(Tx_{2p+1}, Tx_{2p+1})\}$$

$$+q_2 \frac{\mathbb{C}_d(Tx_{2p}, Tx_{2p+1}), \mathbb{C}_d(Tx_{2p+1}, Tx_{2p+2})}{1 + \mathbb{C}_d(Tx_{2p}, Tx_{2p+1})}$$

$$+q_3 \frac{\mathbb{C}_d(Tx_{2p}, Tx_{2p+1})\mathbb{C}_d(Tx_{2p+1}, Fx_{2p+2})}{1 + \mathbb{C}_d(Tx_{2p}, Tx_{2p+2}) + \mathbb{C}_d(Tx_{2p+1}, Tx_{2p+1}) + \mathbb{C}_d(Tx_{2p}, Tx_{2p+1})}$$

$$+q_4 \frac{\mathbb{C}_d(Tx_{2p+1}, Tx_{2p+1})\mathbb{C}_d(Tx_{2p}, Tx_{2p+2})}{1 + \mathbb{C}_d(Tx_{2p}, Tx_{2p+1}) + \mathbb{C}_d(Tx_{2p}, Tx_{2p+1}) + \mathbb{C}_d(tx_{2p+1}, Tx_{2p+2})}$$

$$+q_5 \frac{\mathbb{C}_d(Tx_{2p}, Tx_{2p+1})\mathbb{C}_d(Tx_{2p+1}, Tx_{2p+2})}{1 + \mathbb{C}_d(Tx_{2p}, Tx_{2p+1}) + \mathbb{C}_d(Tx_{2p}, Tx_{2p+1}) + \mathbb{C}_d(Tx_{2p+1}, Tx_{2p+2})}$$

Since

$$\mathbb{C}_d(Tx_{2p}, Tx_{2p+1}) \leq 1 + \mathbb{C}_d(Tx_{2p}, Tx_{2p+1})$$

$$\mathbb{C}_d(Tx_{2p}, Tx_{2p+1}) \leq 1 + \mathbb{C}_d(Tx_{2p}, Tx_{2p+1}) + \mathbb{C}_d(Tx_{2p}, Tx_{2p+2})$$

$$\mathbb{C}_d(Tx_{2p}, Tx_{2p+1}) \leq 1 + \mathbb{C}_d(Tx_{2p}, Tx_{2p+1}) + \mathbb{C}_d(Tx_{2p}, Tx_{2p+1}) + \mathbb{C}_d(Tx_{2p+1}, Tx_{2p+2})$$

Therefore,

$$\lambda\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2}) \preccurlyeq q_1\{\mathbb{C}_d(Tx_{2p},Tx_{2p+1}) + \mathbb{C}_d(Tx_{2p+1},Tx_{2p+2})\}$$
$$+q_2\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2}) + q_3\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2})$$
$$+q_4.0 + q_5\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2})$$

Or, $\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2}) \preccurlyeq \frac{q_1}{\lambda-(q_1+q_2+q_3+q_5)}\mathbb{C}_d(Tx_{2p},Tx_{2p+1})$

Again,

$$\lambda\mathbb{C}_d(Tx_{2p+2},Tx_{2p+3})$$
$$\preccurlyeq \mathbb{C}_d(F_{x2p+1},Ex_{2p+2})$$
$$= \mathbb{C}_d(Ex_{2p+2},Fx_{2p+1})$$
$$\preccurlyeq q_1 \max\{\mathbb{C}_d(Tx_{2p+2},Tx_{2p+1}),\mathbb{C}_d(Tx_{2p+2},Ex_{2p+2}),\mathbb{C}_d(Tx_{2p+1},Fx_{2p+1}),$$
$$\mathbb{C}_d(Tx_{2p+1},Ex_{2p+2}),\mathbb{C}_d(Tx_{2p+2,}Fx_{2p+1})\}$$
$$+q_2\frac{\mathbb{C}_d(Tx_{2p+2},Ex_{2p+2})\,\mathbb{C}_d(Tx_{2p+1},Fx_{2p+1})}{1+\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2})}$$
$$+q_3\frac{\mathbb{C}_d(Tx_{2p+2},Ex_{2p+2})\,\mathbb{C}_d(Tx_{2p+1},Fx_{2p+1})}{1+\mathbb{C}_d(Tx_{2p+2},Tx_{2p+1})+\mathbb{C}_d(Tx_{2p+2},Fx_{2p+1})+\mathbb{C}_d(Tx_{2p+1},Ex_{2p+2})}$$
$$+q_4\frac{\mathbb{C}_d(Tx_{2p+1},Ex_{2p+2})\mathbb{C}_d(Tx_{2p+2},Fx_{2p+1})}{1+\mathbb{C}_d(Tx_{2p+2},Tx_{2p+1})+\mathbb{C}_d(Tx_{2p+2},Ex_{2p+2})+\mathbb{C}_d(Tx_{2p+1},Fx_{2p+1})}$$
$$q_5\frac{\mathbb{C}_d(Tx_{2p+2},Ex_{2p+2})\mathbb{C}_d(Tx_{2p+1},Fx_{2p+1})}{1+\mathbb{C}_d(Tx_{2p+2},Tx_{2p+1})+\mathbb{C}_d(Tx_{2p+2},Ex_{2p+2})+\mathbb{C}_d(Tx_{2p+1},Fx_{2p+1})}$$

Or, $\lambda\mathbb{C}_d(Tx_{2p+2},Tx_{2p+3}) \preccurlyeq q_1\max\{\mathbb{C}_d(Tx_{2p+2},Tx_{2p+1}),$

$$\mathbb{C}_d(Tx_{2p+2},Tx_{2p+3}),$$
$$\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2}) + \mathbb{C}_d(Tx_{2p+1},Tx_{2p+3}),$$
$$\mathbb{C}_d(Tx_{2p+2},Tx_{2p+2})\}$$
$$+q_2\frac{\mathbb{C}_d(Tx_{2p+2},Tx_{2p+3})\mathbb{C}_d(Tx_{2p+1,}Tx_{2p+2})}{1+\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2})}$$
$$+q_3\frac{\mathbb{C}_d(Tx_{2p+2},Tx_{2p+3})\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2})}{1+\mathbb{C}_d(Tx_{2p+2},Tx_{2p+1})+\mathbb{C}_d(Tx_{2p+2},Tx_{2p+2})+\mathbb{C}_d(Tx_{2p+1},Tx_{2p+3})}$$
$$+q_4\frac{\mathbb{C}_d(Tx_{2p+1},Tx_{2p+3})\mathbb{C}_d(Tx_{2p+2},Tx_{2p+2})}{1+\mathbb{C}_d(Tx_{2p+2},Tx_{2p+1})+\mathbb{C}_d(Tx_{2p+2},Ex_{2p+3})+\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2)}}$$

$$+q_5\frac{\mathbb{C}_d(Tx_{2p+2},Tx_{2p+3})\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2})}{1+\mathbb{C}_d(Tx_{2p+2},Tx_{2p+1})+\mathbb{C}_d(Tx_{2p+2},Tx_{2p+3})+\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2})}$$

Since,

$$\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2}) \le 1+\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2})$$

$$\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2}) \le 1+\mathbb{C}_d(Tx_{2p+2},Tx_{2p+1})+\mathbb{C}_d(Tx_{2p+1},Tx_{2p+3})$$

$$\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2}) \le 1+\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2})+\mathbb{C}_d(Tx_{2p+2},Tx_{2p+3})$$
$$+\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2})$$

So,

$$\lambda\mathbb{C}_d(Tx_{2p+2},Tx_{2p+3}) \le q_1[\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2})+\mathbb{C}_d(Tx_{2p+2},Tx_{2p+3})]$$
$$+q_2\mathbb{C}_d(Tx_{2p+2},Tx_{2p+3})+q_3\mathbb{C}_d(Tx_{2p+2},Tx_{2p+3})$$
$$+q_4.0+q_5\mathbb{C}_d(Tx_{2p+2},Tx_{2p+3})$$

Or,

$$\lambda\mathbb{C}_d(Tx_{2p+2},Tx_{2p+3}) \preccurlyeq \frac{q_1}{\lambda-(q_1+q_2+q_3+q_5)}\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2})$$

$Let\ h=\dfrac{q_1}{\lambda-(q_1+q_2+q_3+q_5)}<1, we\ have$

$$\begin{aligned}\mathbb{C}_d(Tx_{2p+1},Tx_{2p+2}) &\preccurlyeq h\mathbb{C}_d(Tx_{2p},Tx_{2p+1})\\ &\preccurlyeq h^2\mathbb{C}_d(Tx_{2p-1},Tx_{2p})\\ &\vdots\\ &\preccurlyeq h^{2p}\mathbb{C}_d(Tx_0,Tx_1)\end{aligned}$$

Thus

$$|\mathbb{C}_d(Tx_{2p},Tx_{2p+1})| \le h^{2p}||\mathbb{C}_d(x_0+x_1)| \to 0\ as\ p\to\infty$$

Hence $\{Tx_{2p}\}$ is convergent. Let z be the limit point of this sequence in X, and is subsequences $Ex_{2p}\ and\ Fx_{2p+1}$ of sequence$\{Tx_{2p}\}$ also converges to points to z.

Since $E(X)\subseteq T(X)$ to there is $u\in X\ such\ that\ z=Tu$ we now show that $Fu=Tu=z$

$$\begin{aligned}\mathbb{C}_d(Fu\ z) &\preccurlyeq \mathbb{C}_d(Fu,Ex_{2p})+\mathbb{C}_d(Ex_{2p},z)\\ &\preccurlyeq \mathbb{C}_d(Ex_{2p},z)\\ &\preccurlyeq q_1\max\{\mathbb{C}_d(Tu,Tx_{2p}),\mathbb{C}_d(Tu,Fu),\mathbb{C}_d(Tx_{2p},Ex_{2p}),\\ &\qquad \mathbb{C}_d(Tu,Ex_{2p}),\mathbb{C}_d(Tx_{2p},Fu)\}\end{aligned}$$

$$+q_2 \frac{\mathbb{C}_d(Tx_{2p}, Ex_{2p})\mathbb{C}_d(Tu, Fu)}{1 + \mathbb{C}_d(Tx_{2p}, Tu)}$$

$$+q_3 \frac{\mathbb{C}_d(Tx_{2p}, Ex_{2p})\mathbb{C}_d(Tu, Fu)}{1 + \mathbb{C}_d(Tx_{2p}, Tu) + \mathbb{C}_d(Tx_{2p}, Fu) + \mathbb{C}_d(Tu, Ex_{2p})}$$

$$+q_4 \frac{\mathbb{C}_d(Tu, Ex_{2p})\mathbb{C}_d(Tx_{2p}, Fu)}{1 + \mathbb{C}_d(Tx_{2p}, Tu) + \mathbb{C}_d(Tx_{2p}, Ex_{2p}) + \mathbb{C}_d(Tu, Fu)}$$

$$+q_5 \frac{\mathbb{C}_d(Tu, Ex_{2p})\mathbb{C}_d(Tu, Fu)}{1 + \mathbb{C}_d(Tx_{2p}, Tu) + \mathbb{C}_d(Tx_{2p}, Ex_{2p}) + \mathbb{C}_d(Tu, Fu)}$$

Taking limit as n approaches to infinity we get

$$\mathbb{C}_d(Fu, z) \preccurlyeq \mathbb{C}_d(z, Fu)$$

$$or\ (1 - q_1)\mathbb{C}_d(Fu, z) \preccurlyeq 0 \leftrightarrow so\ that\ Fu = z$$

Since $z = Fu \in F(X) \subseteq T(X)$, so there exists $v\ is\ X$ such that $z = T\vartheta$.We can show that $E\vartheta = T\vartheta = z$ using the above argument. Therefore, we get $Fu = Eu = Tu = \mathfrak{z}$.

Now we claim that z is the unique common fixed point of E, F & T, Since E and T are weakly compatible so $ETu = TEu$ This gives $E\mathfrak{z} = T\mathfrak{z}$

Thus $\mathbb{C}_d(E\mathfrak{z}, z) = \mathbb{C}_d(E\mathfrak{z}, Fu)$

$\preccurlyeq q_1 \max\{\mathbb{C}_d(Tz, Tu), \mathbb{C}_d(Tz, Ez), \mathbb{C}_d(Tu, Fu), \mathbb{C}_d(Tz, Fu), \mathbb{C}_d(Tu, Ez)\}$

$$+q_2 \frac{\mathbb{C}_d(Tz, Ez)\mathbb{C}_d(Tu, Fu)}{1 + \mathbb{C}_d(Tz, Tu)} + q_3 \frac{\mathbb{C}_d(Tz, Ez)\mathbb{C}_d(Tu, Fu)}{1 + \mathbb{C}_d(Tz, Tu) + \mathbb{C}_d(Tz, Fu) + \mathbb{C}_d(Tu, Ez)}$$

$$+q_4 \frac{\mathbb{C}_d(Tu, Ez)\mathbb{C}_d(Tz, Fu)}{1 + \mathbb{C}_d(Tz, Tu)} + q_5 \frac{\mathbb{C}_d(Tz, Ez)\mathbb{C}_d(Tu, Fu)}{1 + \mathbb{C}_d(Tz, Tu) + \mathbb{C}_d(Tz, Ez) + \mathbb{C}_d(Tu, Fu)}$$

Or $\mathbb{C}_d(Ez, z) \preccurlyeq (q_{1+}q_4)\mathbb{C}_d(Ez, z)$

$$\mathbb{C}_d(Ez, z) \preccurlyeq (q_1 + q_4)\mathbb{C}_d(E\mathfrak{z}, \mathfrak{z})$$

Or, $1 - (q_1 + q_4)d(Ez, z)$ which implies $d(Ez, z) = 0$ and so $Ez = z = Tz$ By similar argument $Fz = Tz = z$ Thus we prove that z is a common fixed point of E, F and T. Let z and w be common fixed point of E, F and T.

Then,

$$d(\mathfrak{z}, w) = d(E\mathfrak{z}, Fw)$$

$$\preccurlyeq q_1 \max\{d(Tz, Tw), d(Tz, Ez), d(Tz, Fw), d(Tw, Ez)\}$$

$$+q_2 \frac{d(Tz, Ez)d(Tw, Fw)}{1 + d(Tz, Tw)} q_3 \frac{d(Tz, Ez)d(Tw, Fw)}{1 + d(Tw, Tw) + d(Tz, Fw) + d(Tw, Ez)}$$

$$+q_4\frac{d(Tw,Ez)d(Tz,Fw)}{1+d(Tz,Tw)+d(Tz,Ez)+d(Tw,Fw)}$$

$$+q_5\frac{d(Tz,Ez)d(Tw,Fw)}{1+d(Tz,Tw)+d(Tw,Ez)+d(Tw,Fw)}$$

$$d(z,w)\precsim q_1d(z,w)+q_4d(z,w)$$

$$or, 1-(q_1+q_4)\precsim d(z,w), a\ contradiction.$$

Thus, $d(z,w)\precsim 0\ and\ so\ z=w$

Therefore z is unique common fixed point of E,F and T.

Corollary :(3.2) Let S and T be two self-maps of a complete complex value metric space (X,$\mathbb{C}_d$) such that

$$\lambda\mathbb{C}_d(Sx,Ty)\precsim\ A\max\{\mathbb{C}_d(x,y),\mathbb{C}_d(x,Sx),\mathbb{C}_d(y,Ty),\mathbb{C}_d(x,Ty),\mathbb{C}_d(y,Sx)\}$$

$$+\mathrm{B}\frac{\mathbb{C}_d(x,Sx)\mathbb{C}_d(y,Ty)}{1+\mathbb{C}_d(x,y)}+\mathrm{C}\frac{\mathbb{C}_d(x,Sx)d(y,Ty)}{1+\mathbb{C}_d(x,y)+\mathbb{C}_d(x,Ty)+\mathbb{C}_d(y,Sx)}$$

$$+\mathrm{D}\frac{d(y,Sx)d(x,Ty)}{1+d(x,y)+d(x,Sx)+d(y,Ty,)}+\mathrm{E}\frac{d(x,Sx)d(y,Ty)}{1+d(x,y)+d(x,Sx)+d(y,Ty)}$$

for all $x,y\in X$ where λ, A, B, C, D, E $\in\not\subset^+$ and o<A+B+C+D+E< λ.

Then S and T have a unique common fixed point.

Proof: We put S=E, T=F and T=I(Identity map) and the result follows.

Corollary (3.3) Let (X, $\mathbb{C}_d$) be a complete complex valued metric space and T : X →X be self-mapping satisfying the following condition.

$$\lambda\mathbb{C}_d(T^nx,T^ny)\leq A\max\{\mathbb{C}_d(x,y),\mathbb{C}_d(x,T^nx\ ),\mathbb{C}_d(y,T^ny),\mathbb{C}_d(x,T^ny),\mathbb{C}_d(y,T^nx)\}$$

$$+B\frac{\mathbb{C}_d(x,T^nx)\mathbb{C}_d(y,T^ny)}{1+\mathbb{C}_d(x,y)}+C\frac{\mathbb{C}_d(x,T^nx)\mathbb{C}_d(y,T^ny)}{1+\mathbb{C}_d(x,y)+\mathbb{C}_d(x,T^ny)+\mathbb{C}_d(y,T^nx)}$$

$$+D\frac{\mathbb{C}_d(y,T^nx)\mathbb{C}_d(x,T^ny)}{1+\mathbb{C}_d(x,y)+\mathbb{C}_d(x,T^nx)+\mathbb{C}_d\ (y,T^ny)}$$

$$+E\frac{\mathbb{C}_d(x,T^nx)\mathbb{C}_d(y,T^ny)}{1+\mathbb{C}_d(x,y)+\mathbb{C}_d(x,T^nx)+\mathbb{C}_d(y,T^ny)}$$

for all $x,y\in X$, where $\lambda,A,B,C,D\in\not\subset\ and\ 0<A+B+C+D+$

$E<\lambda$. Then T has a unique fixed point.

Corollary (3.4) [1]: Let (X, $\mathbb{C}_d$) be a complete complex valued metric space and let the mapping $S,T:X\rightarrow X$ satisfy.

$$\mathbb{C}_d\left(Sx,Ty\right)\leq\alpha\mathbb{C}_d(x,y)+\beta\frac{\mathbb{C}_d(x,Sx)\mathbb{C}_d(y,Ty)}{1+\mathbb{C}_d(x,y)}$$

for all $x, y \in X$, where α, β are non-negative reals with $\alpha + \beta < 1$. Then S and T have a unique common fixed point.

Proof: The required proof can be obtained by assuming the following in the theorem 3.1

1. E=S, F=T and T=I(identity map)
2. $A \max\{\mathbb{C}_d(x,y), \mathbb{C}_d(x,Sx), \mathbb{C}_d(y,Ty), \mathbb{C}_d(x,Ty), \mathbb{C}_d(y,Sx)\}$

$$= \alpha \mathbb{C}_d(x,y)$$

3. $\beta = B. and\ C = D = E = 0. and\ \lambda = 1$

Corollary (3.3.) Theorem 2.1 of [24]: Let (X, $\mathbb{C}_d$) be a complete Complex valued metric space and mapping $S, T: X \to X$ satisfying

$$\mathbb{C}_d(Sx,Ty) \le h\, max\{\mathbb{C}_d(x,y), \mathbb{C}_d(x,Sx), \mathbb{C}_d(y,Ty), \mathbb{C}_d(x,Ty), \mathbb{C}_d(y,Sx)\}$$

for all $x, y \in X$ $where$ $0 < h < {}^{1}/_{2}$.Then S and T have unique

common fixed point in X.

Proof. Putting B = C = D = E = 0 in Theorem (3.1) and putting h = A and $\lambda = 1$. we get the proof.

Corollary (3.3.6): Theorem 3.1. of [7]

Let (x, Pz) be a complex valued metric space and $S, T: X \to X$ be self mappings satisfying the following conditions:

$$P_e(Sx,Ty) \le \alpha P_e(x,y) + \beta \frac{P_e(x,Sx)P_e(y,Ty)}{1 + P_e(x,y)}$$

$$+\gamma \frac{P_e(x,Sx)P_e(y,Ty)}{1 + Pe(x.y) + P_e(x,Ty) + Pe(y,Sx)}.$$

For all x, y $\in X$, where α, β, γ are non-negative reals with $\alpha + \beta + \gamma < 1$. Then S, T have a unique common fixed point.

Proof: Putting in the theorem (3.1)

1. E =S, F=T and T=I(identity map)
2. A max $\{\mathbb{C}_d(x,y), \mathbb{C}_d(x,Sx), \mathbb{C}_d(y,Ty), \mathbb{C}_d(d(y,Sx), \mathbb{C}_d(x,Ty)\}$

$$= \alpha P_e(x,y)$$

3. $\beta = B, : \gamma = C : D = E = O.$
4. $\lambda = 1$ we will get the result.

References:

1 A. Azam, B. Fisher, and M. Khan, Common fixed point theorems in complex valued metric spaces. *Numerical Functional Analysis and Optimization*, vol. 32, no. 3, pp. 243–253, 2011.

2 AntimaSendersiya AkleshPariya Nirmala Gupta and V.H.Badshah, Common Fixed Point Theorems in Complex Valued metric spaces. Adv. Fixed Point Theory 7 (2017) No. 4, 572 – 579.

3 Anuradha Gupta and Manu Rohilla, Common Fixed Point results in Complex valued metric spaces via simulation functions . arXiv.org > math > arXiv:1905.03653 (2019)

4 *Anil Kumar Dubey, Shweta Bibay, R. P. Dubey, M. D. Pandey*, Some Fixed Point Theorems in C-complete Complex Valued Metric Spaces Communications in Mathematics and Applications Vol. 9, No. 4, pp. 581–591, 2018

5. Deepak Kumar, Amal Chacko, Some Results on Common Fixed Points for Rational Type Contraction Mappings in Complex Valued Metric Spaces International Journal of Maps in Mathematics, Vol. I Issue 1, (2018)

6. F. Rouzkard, Common fixed point theorems for two pairs of self-mappings in complex valued metric spaces *Eurasian Math. J., 2019, Volume 10, Number 2, Pages 75–83*

7 M. Kumar, P. Kumar, S. Kumar and S. Araci, Weakly compatible maps in complex valued metric spaces and an application to solve Urysohn integral equation, Filomat 30(10) (2016), 2695 – 2709

8. R.K.Verma H.K.Pathak, Common fixed point theorems for a pair of mappings in complex valued metric spaces. Journal of Mathematics and Computer Science 6(2013) , 18 – 26.

9.	*R. K. Verma, H. K. Pathak*	Common Fixed Point Theorems in Complex Valued Metric Space and Application The Thai Journal of Mathematics Vol 17, No 1 (2019)
10.	S. Bhatt S. Chankiyal and R.C. Dimri	Common Fixed points of mappings satisfying rational uniqulity in complex valued metric space International Journal of Pure and applied Mathematics, 73(2), 2011, 159-164.
11.	Sunarsini , Aufa Biahdillah and Sentot Didik Surjanto	Application of Banach Contraction Principle in Complex Valued Rectangular b-Metric Space 2020 J. Phys.: Conf. Ser. 1490 012003
12.	Sanjay Kumar Tiwari & Sandip Kumar Mishra,	A Common Fixed Point Theorem in Complex Valued Metric Spaces. International Journal of Research and Analytical Reviews, olume 9, Issue 2, April 2022.
13	Yihao Sheng , Jianping Ren &LinanZhong	Common fixed point theorems for maps satisfying φϕ– contractions in complex valued metric spaces European Journal of Mathematics and Computer Science Vol. 5 No. 2, 2018.

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