JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

"KUSHARE TRANSFORM" FOR SOLVING THE PROBLEMS ON GROWTH AND DECAY"

D. P. Patil[1], P. S. Nikam[2], S. D. Shirsath[3], A. T. Aher [4]

[1]Professor, Department of the Mathematics, K.T.H.M. College, Nashik.

[2,3,4] M.Sc. students, Department of Mathematics, K.T.H.M. College, Nashik.

ABSTRACT:

In this paper we apply **KUSHARE** transform for solving problems on population growth and decay. KUSHARE transform is introduced by **Sachin Kushare** in September 2021**.**

Keywords: Integral Transform, Growth and Decay problems, Differential Equations, KUSHARE Transform.

1. INTRODUCTION:

Nowadays many researchers are interested in introducing various integral transforms. Recently in September 2021, Kushare and Patil [1] introduced Kushare transform to facilitate the process of solving ordinary and partial differential equations in the time domain. In January 2022, Rohidas Sanap and D. P. Patil [2] use this transform for Newton's law of cooling. Soham transform is introduced by D. P. Patil and S. S. Khakale [3]. In October 2021, D. P. Patil [4] used Sawi transform in Bessel functions. Patil [5] used Sawi transform of Error function for evaluating improper integrals. Patil [6] further used Laplace and Shehu transform in chemical sciences. Sawi transform and convolution theorem is used for solving wave equation by Patil [7]. Patil [8] used Mahgoub transform for solving parabolic boundary value problems. Double Laplace and Double Sumudu transform are used to obtain solution of wave equation by Patil [9]. Dr. Patil [10] also obtained dualities between double integral transforms.

Laplace, Elzaki, and Mahgoub transforms for solving system of first order first degree differential equations by Kushare [11]. Patil [12] used Aboodh and Mahgoub transform in boundary value problems of system of ordinary differential equations.

In this paper we use Kushare transform to solve the problems on population growth and decay. Paper is organized as follows. Some useful formulas of Kushare transform are stated in section 2. Third section is devoted to application in population growth and decay problems. Lastly conclusion is stated.

2. DEFINATION OF KUSHARE TRANSFORM

A new integral transform said to be KUSHARE change characterized for capacity of outstanding request. KUSHARE transform defined on the set A

$$A = \{f(t)/\exists\ M, \tau_1, \tau_2 > 0, |f(t)| < Me^{\frac{|t|}{\tau_j}}, \text{ if } t \in (-1)^j \times [0, \infty)\}$$

For a given function in the set A, the constant M must be finite number, τ_1, τ_2 may be finite or infinite. KUSHARE TRANSFORM denoted by function f. The purpose of this study is to show the applicability of this interesting transform and operator s(v) defined by the integral equations.

$$K[f(t)] = S(v) = v\int_0^\infty f(t)e^{-tv^\alpha}\,dt, \qquad t \geq 0, \qquad \tau_1 \leq v \leq \tau_2$$

If α=1 then this transform is called "Mahgoub Transform".

If α= 2 then this transform is called "Pourreza Transform".

If α= -1 then this transform is called "Elzaki Transform".

2.1 KUSHARE TRNSFORM OF THE SOME FUNCTIONS

Sr. No.	Functions	Kushare Transform
1	1	$\frac{1}{v^{\alpha-1}}$
2	t^n	$\frac{\Gamma(n+1)}{v^{\alpha(n+1)}-1}$
3	e^{at}	$\frac{v}{v^{\alpha}-a}$
4	$\sin(at)$	$\frac{av}{v^{2\alpha}+a^2}$
5	$\cos(at)$	$\frac{v^{\alpha+1}}{v^{2\alpha}+a^2}$

2.2 INVERSE KUSHARE TRANSFORM

Sr. No.	Inverse Kushare Transform	Functions
1	$\frac{1}{v^{\alpha-1}}$	1
2	$\frac{\Gamma(n+1)}{v^{\alpha(n+1)}-1}$	t^n
3	$\frac{v}{v^{\alpha}-a}$	e^{at}
4	$\frac{av}{v^{2\alpha}+a^2}$	$\sin(at)$
5	$\frac{v^{\alpha+1}}{v^{2\alpha}+a^2}$	$\cos(at)$

2.3 KUSHARE TRANSFORM of the derivative of the function $f(t)$ is

$$K[f'(t)] = v^{\alpha}s(v) - vf(0)$$

3. Applications of KUSHARE Transform in Population Growth and Decay Problems.

3.1 KUSHARE Transform for Growth Problems:

In this section, we discuss KUSHARE Transform for population growth problem as follows.

The population growth (growth of plant, or a cell, or an organ, or a species) is governed by the first order linear ordinary differential equation

$$\frac{dN}{dt} = PN \qquad \dots\dots(1)$$

With initial condition as

$$N(t_0) = N_0 \qquad \dots\dots(2)$$

Where P is a positive real number, N is the amount of population at time t and N_0 is the initial population at time t_0.

Applying KUSHARE transform on the both sides of (1)

$$K\left\{\frac{dN}{dt}\right\} = PK\{N(t)\}$$

Now applying the property, KUSHARE Transform of derivative of function, on above equation

$$v^{\alpha}\, s(v) - v\, N(0) = P\, s(v)$$

Since $t_0 = 0,\ N = N_0$,

$$(v^{\alpha} - P)\, s(v) = v\, N_0$$

$$\Rightarrow s(v) = \frac{vN_0}{v^{\alpha} - P}$$

Apply inverse KUSHARE transform on above equation

$$K^{-1}[s(v)] = K^{-1}\left[\frac{vN_0}{v^{\alpha} - P}\right]$$

$$\Rightarrow N(t) = N_0 e^{Pt}$$

which is required amount of the population at time t.

3.2 KUSHARE Transform for Decay Problems:

In this section, we discuss KUSHARE Transform for Decay problem which is given as follows.

The Decay problem of the substance is defined by the first order linear ordinary differential equation

$$\frac{dN}{dt} = -PN \qquad \dots\dots(3)$$

with initial condition as,

$$N(t_0) = N_0 \qquad \dots \dots (4)$$

Where N is the amount of substance at time t, P is a positive real number and N_0 is the initial amount of the substance at time t_0.

In equation (3), the negative sign in the R.H.S is taken because of the mass of the substance is decreasing with time and so the derivative $\frac{dN}{dt}$ must be negative.

Applying KUSHARE transform on the both sides of (3)

$$K\left\{\frac{dN}{dt}\right\} = -PK\{N(t)\}$$

Now applying the property, KUSHARE Transform of derivative of function, on above equation

$$v^{\alpha}\, s(v) - v\, N(0) = -P\, s(v)$$

Since $t_0 = 0, N = N_0$,

$$(v^{\alpha} + P)\, s(v) = v\, N_0$$

$$\Rightarrow s(v) = \frac{vN_0}{v^{\alpha} + P}$$

Apply inverse KUSHARE transform on above equation

$$K^{-1}[s(v)] = K^{-1}\left[\frac{vN_0}{v^{\alpha} + P}\right]$$

$$\Rightarrow N(t) = N_0 e^{-Pt}$$

which is required amount of the substance at time t.

Now we will solve some problems on population growth and decay.

Application (1): The population of the city grows at the rate proportional to the number of people presently living in the city. If after two years, the population has doubled and after three years the population is 20000, Estimate the number of people initially in the city.

Solution:

This problem can be written in mathematical form as:

$$\frac{dN}{dt} = PN \qquad \dots \dots (5)$$

where N denote the number of people living in the city at any time t and P is the constant of proportionality. Consider N_0 is the number of people initially living in the city at $t = 0$.

Applying KUSHARE transform on the both sides of equation (5)

$$K\left\{\frac{dN}{dt}\right\} = PK\{N(t)\}$$

Now applying the property, KUSHARE Transform of derivative of function, on above equation

$$v^{\alpha}\, s(v) - v\, N(0) = P\, s(v)$$

Since $t = 0, N = N_0$,

$$(v^{\alpha} - P)\, s(v) = v\, N_0$$

$$\Rightarrow s(v) = \frac{vN_0}{v^{\alpha} - P}$$

Apply inverse KUSHARE transform on above equation

$$K^{-1}[s(v)] = K^{-1}\left[\frac{vN_0}{v^{\alpha} - P}\right]$$

$$\Rightarrow N(t) = N_0 e^{Pt} \; \ldots \ldots (6)$$

Now at $t = 2, N = 2N_0$

$$\therefore\; 2N_0 = N_0 e^{2P}$$

$$\therefore\; 2 = e^{2P}$$

$$\therefore\; P = \frac{1}{2}\log_e 2$$

$$\therefore\; P = 0.3466$$

Now $t = 3, N = 20000$

Put this value in equation (6)

$$\therefore\; 20000 = N_0 e^{3P}$$

$$\therefore\; 20000 = N_0 e^{3(0.3466)}$$

$$\therefore\; 20000 = N_0 \times 2.8287$$

$$\therefore\; N_0 = 7070.3857 \cong 7070$$

which are the required number of people initially living in the city.

Application (2): A radioactive substance is known to decay at a rate proportional to the amount present. It initially there is 100 miligrams of the radioactive substance present and after two hours it is observed that the radioactive substance has lost 10 percent of its original mass, Find the half life of the radioactive substance.

Solution:

This problem can be written in form as

$$\frac{dN}{dt} = -PN \ldots \ldots (7)$$

where N denote the amount of radioactive substance at time t and P is the constant of proportionality. Consider N_0 is the initial amount of the radioactive substance at time $t = 0$.

Applying KUSHARE transform on the both sides of (7)

$$K\left\{\frac{dN}{dt}\right\} = -PK\{N(t)\}$$

Now applying the property, KUSHARE Transform of derivative of function, on above equation

$$v^{\alpha}\, s(v) - \; vN(0) = \; -P\, s(v)$$

Since $t = 0, N = \; N_0 = 100$

$$(v^{\alpha} + P)\, s(v) = \; v\, 100$$

$$\Rightarrow s(v) = \frac{100v}{v^{\alpha} + P}$$

Apply inverse KUSHARE transform on above equation

$$K^{-1}[s(v)] = K^{-1}\left[\frac{100v}{v^{\alpha} + P}\right]$$

$$\Rightarrow N(t) = 100e^{-Pt} \quad \ldots\ldots (8)$$

Now at $t = 2,$ the radioactive substance has lost 10 percent of the its original mass 100 mg. so $N = 100 - 10 = 90$

$$\therefore \;\; 90 = 100e^{-2P}$$

$$\therefore \;\; e^{-2P} = 0.9$$

$$\therefore \;\; -2P = \log_e 0.9$$

$$\therefore \;\; P = -\frac{1}{2}\log_e 0.9$$

$$\therefore \;\; P = 0.0527$$

We required half time of radioactive substance (t)

When $N = \frac{N_0}{2} = \frac{100}{2} = 50$

Substitute this value in equation (8)

$$\therefore \;\; 50 = 100e^{-Pt}$$

$$\therefore \;\; 50 = 100e^{-0.0527t}$$

$$\therefore \;\; 0.5 = e^{-0.0527t}$$

$$\therefore \;\; -0.0527t = \log_e 0.5$$

$$\therefore \;\; -0.0527t = -0.6932$$

$$\therefore \;\; t = 13.1537\; hours$$

Which is required half time of radioactive substance.

4.CONCLUSION:

We successfully used “KUSHARE TRANSFORM” in population Growth and Decay problems.

5.REFERENCES:

[1] S. R. Kushare, D. P. Patil & A. M. Takate, The new integral transform “Kushare Transform”, International Journal of Advances in Engineering and Management, Vol.3, Issue-9, September 2019, pp. 1589-1592.

[2] R. S. Sanap & D. P. Patil, Kushare integral transform for Newton’s law of cooling, International Journal of Advances in Engineering and Management, Vol.4, Issue 1, January 2022, pp. 166-170.

[3] D. P. Patil, S. S. khakale , The new integral transform “Soham Transform”, International Journal of Advances in Engineering & Management, Vol.3, Issue 10, October 2021, pp. 126-132.

[4] D. P. Patil, Sawi transform in Bessel equations, Aayushi International Interdisciplinary Research Journal, Special Issue no. 86, pp. 171-175.

[5] D. P. Patil, Application of Sawi transform of Error function for evaluating Improper integrals, Vol.11, Issue 20, June 2021, pp. 41-45.

[6] D. P. Patil, Application of integral transform in chemical sciences, Ayushi International Interdisciplinary Research Journal, Special Issue No. 88, pp. 437-441.

[7] D. P. Patil, Sawi transform and convolution theorem for initial boundary value problems (wave equation), Journal of Research & Development, vol. 11, Issue 14, June 2021, pp. 133-136.

[8] D. P. Patil, Application of Mahgoub transform in parabolic boundary value problems, International Journal of Current Advanced Research, Vol. 9, Issue 4(c), April 2020, pp. 21949-21957.

[9] D. P. Patil, Solution of wave equation by Double Laplace and Double Sumudu transform, Vidyabharti International Interdisciplinary Reaserch Journal, Special Issue IVCIMS 2021, August 2021, pp. 135-138.

[10] D. P. Patil, Dualities between double integral transforms, International Advanced Journal in Science, Engineering and Technology, Vol. 7, Issue 6, June 2020, pp. 74-82.

[11] S. R. Kushare, D. P. Patil, A. M. Takate, Comparison between Laplace, Elzaki, Mahgoub transforms for solving system of first order and first degree diiferential equations, Vidyabharti International Interdisciplinary Research Journal, Special Issue IVIMS 2021, pp. 139-144.

[12] D. P. Patil, Aboodh & Mahgoub transform in boundary value problems of system of ordinary differential equations, International Journal of Advance Research in Science, Communication and Technology, Vol. 6, Issue 1, June 2021, pp. 67-75.