JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

Application of Kamal Transform for Solving Linear Volterra Integral Equations of Second Kind

***A. J. Ghanwat, # S. B. Gaikwad**

* MES's, Shri Dnyaneshwar Mahavidyalaya Newasa, Dist. Ahmednagar. E-mail- pushkar1967@rediffmail.com

P.G Department of Mathematics, New Arts, Commerce and Science College, Ahmednagar, Dist. Ahmednagar (Maharashtra).E-mail- sbgmathsnagar@gmail.com

Abstract:

In this paper we have used Kamal transform for solving linear Volterra equation of second kind and by determining the resolvent kernel of linear Volterra integral equation of second kind along with some of the applications for appropriateness of transform. We have also given the application of Kamal transform to solve Integro- Differential equation.

Keywords:

Convolution Theorem, Kamal transform, Inverse Kamal transform, Integro-differential equation, Resolvent kernel, Volterra Integral equation of Second kind.

1. INTRODUCTION

The Volterra Integral equation of first kind is given by [11-13]. The linear Volterra Integral equation of second kind with convolution type kernel is given by [13].

$y(x) = f(x) + \int_0^x k(x,t)\, y(t)dt$; Where the kernel $k(x,t)$ is expressed by the difference$(x-t)$.

i. e. $k(x,t) = k(x-t)$ and $f(x)$is real valued function, $y(t)$ is unknown function.

The Kamal transform of the function F(t) is defined by [13]

$F\{F(t)\} = \int_0^\infty e^{-t/v} F(x,t)dt = G(v), t \geq 0,\ k_1 \leq v \leq k_2$, provided the integral on R.H.S. exists if $F(t)$ s sectionally continuous and of exponential order. Off course these conditions are sufficient for existence of Kamal transform of $F(t)$.

The Kamal transform and inverse Kamal transform of some elementary functions is stated in [4, 6] and basic properties in [5]. The methods of solving Volterra integral equation of first kind are studied in [7, 10], [1, 6, 9], Kamal transform is used to solve ordinary differential equations with non-constant co-efficient [3], partial differential equation by [2] and linear partial Integro-differential equation by [8].

The objective of this work is to derive exact solutions for linear Volterra Integral equations of second kind using Kamal transform, obtaining resolvent kernel and then solve linear Volterra Integral equation of second kind and also derive the solution of Integro-differential equation using Kamal transform.

2. THE CONVOLUTION OF FUNCTIONS

Convolution of two functions $F(t)$ and $G(t)$ denoted by $F(t) * G(t)$; is defined by

$$F(t) * G(t) = \int_0^t F(x)G(x-t)dx = \int_0^t F(x-t)G(x)dx$$ in [2, 3].

3. THE CONVOLUTION THEOREM

If $K\{F(t)\} = I(v)$ and $K\{G(t)\} = J(v)$ then $\{F(t) * G(t)\} = K\{F(t)\}. K\{G(t)\} = I(v).J(v)$ is given by [4, 5, 8].

The inverse Kamal transform: If $K\{F(t)\} = G(v)$ then $F(t)$ is called inverse Kamal transform of $G(v)$.

$F(t) = K^{-1}\{G(v)\}$; Where K^{-1} is inverse Kamal Operator.

4. KAMAL TRANSFORM FOR LINEAR VOLTERRA INTEGRAL EQUATION OF SECOND KIND

$y(x) = (x) + \int_0^x k(x,t)\, y(t)dt$; Where the kernel $k(x,t)$ is expressed by the difference (. $x - t)$

Appling the Convolution, this equation can be written as

$$y(x) = f(x) + k(x) * y(x) \tag{1}$$

Appling Kamal transform to both sides of equation (1)

$$K\{y(x)\} = K\{f(x)\} + K\{k(x)\}.K\{y(x)\}$$

$$\therefore K\{y(x)\} = \frac{K\{y(x)\} - K\{f(x)\}}{K\{k(x)\}}$$

$$\therefore K\{y(x)\} = \left[\frac{K\{y(x)\}}{K\{k(x)\}}\right] - \left[\frac{K\{f(x)\}}{K\{k(x)\}}\right] \tag{2}$$

Appling inverse Kamal transform on both sides of equation (2)

$y(x) = K^{-1}\left\{\left[\frac{K\{y(x)\}}{K\{k(x)\}}\right]\right\} - K^{-1}\left\{\left[\frac{K\{f(x)\}}{K\{k(x)\}}\right]\right\}$. This gives solution for linear Volterra integral equation of second kind.

5. APPLICATIONS

Here, some of the applications are given for the appropriate use of Kamal transform to solve linear Volterra integral equation of second kind.

***Application 1*:** Consider the linear Volterra integral equation of second kind

$$y(x) = 1 + \int_0^x y(t)dt$$

Appling the Convolution theorem of Kamal transform to both sides of equation

We have $y(x) = 1 + (1 * y(x))$ (3)

Appling Kamal transform to both sides of equation (3), we have

$K\{y(x)\} = K\{1 + (1 * y(x)) = K\{\} \quad 1\} + K\{1 * y(x)\}$

$\therefore K\{y(x)\} = K\{1\} + K\{1\}.K\, y(x)\}$

$\therefore K\{y(x)\} = v + v.K\{y(x)\}$

$\Rightarrow (1-v)K\{y(x)\} = v$

*

$$\Rightarrow K\{y(x)\} = \frac{v}{1-v} \qquad (4)$$

Now, operating inverse Kamal transform on both sides of equation (4) we will have

$$y(x) = K^{-1}\left\{\frac{v}{1-v}\right\} \qquad (5)$$

$y(x) = e^{x}$.

Application 2**:** Consider the linear Volterra integral equation of second kind

$$y(x) = x + \int_0^x (x-t)y(t)dt$$

Appling the Convolution theorem of Kamal transform to both sides of equation

$$y\,(x) = x + x * y(x) \qquad (6)$$

Taking Kamal transform of both sides of equation (6), we get

$$K\{y(x)\} = K\{x + (x * y(x))\}$$

$$K\{y(x)\} = K\{x\} + K\{x * y\,(x)$$

$$K\{y(x)\} = K\{x\} + K\{x\}.\,K\,\{y(x$$

$$\therefore K\{y(x)\} = v^2 + v^2 K\{y\,(x)\}$$

$$\therefore (1-v^2)K\{y(x)\} = v^2$$

}

)}

$$\Rightarrow K\{y(x)\} = \frac{v^2}{1-v^2} \qquad (7)$$

Appling inverse Kamal transform operator, we have

$$y(x) = K^{-1}\left\{\frac{v^2}{1-v^2}\right\} \qquad (8)$$

$y(x) = sinht.$

Application 3**:** The linear Volterra integral equation of second kind

$$Y(t) = t + \int_0^t Y(u)\sin(t-u)\,du$$

Appling the Convolution theorem to both sides of equation

$$Y(t) = t + Y(t) * sint \qquad (9)$$

Appling Kamal transform, we have

$$K\{Y(t)\} = K\{t\} + K\{Y(t) * sint\}$$

$$K\{Y(t)\} = K\{t\} + K\{Y(t)\}.\,K\{sint\}$$

$$K\{Y(t)\} = v^2 + \left(\frac{v^2}{1+v^2}\right)K\{Y(t)\}$$

$$\therefore \left[1 - \left(\frac{v^2}{1+v^2}\right)\right]K\{Y(t)\} = v^2$$

$$\Rightarrow K\{Y(t)\} = \frac{v^2}{1+v^2} \qquad (10)$$

Appling inverse Kamal transform to equation (10)

$Y(t) = K^{-1}\left\{\frac{v^2}{1+v^2}\right\} = sint.$

Application 4: The linear integral equation

$$Y(t) = 1 + \int_0^t Y(x)\sin(t-x)\,dx$$

Appling the Convolution theorem,

$$Y(t) = 1 + Y(t) * sint \quad (11)$$

Using Kamal transform, we have

$K\{Y(t)\} = K\{1\} + K\{Y(t) * sint\}$

$K\{Y(t)\} = K\{1\} + K\{Y(t)\}.K\{sint\}$

$K\{Y(t)\} = v + \left(\frac{v^2}{1+v^2}\right)K\{Y(t)\}$

$\therefore \left[1 - \left(\frac{v^2}{1+v^2}\right)\right]K\{Y(t)\} = v$

$\Rightarrow K\{Y(t)\} = v(1+v^2) \quad (12)$

Appling inverse Kamal transform

$Y(t) = K^{-1}(v+v^3) = K^{-1}(v) + K^{-1}(v^3)$

$$Y(t) = 1 + \frac{t^2}{2!}. \quad (13)$$

Now, let us verify that solution given by equation (13) satisfies the given Volterra integral equation of second kind.

From equation (13), we have $Y(x) = 1 + \frac{x^2}{2}$

Then R. H. S. of given equation becomes

R. H. S. $= 1 + \int_0^t \left(1 + \frac{x^2}{2}\right)\sin(t-x)\,dx$

$= 1 + \left[\left(1+\frac{x^2}{2}\right)\cos(t-x)\right]_{x=0}^{x=t} - \int_0^t \cos(t-x).xdx$

$= 1 + \left[\left(1+\frac{x^2}{2}\right)\cos(t-x)\right]_{x=0}^{x=t} - \left\{[-xsin(t-x)]_{x=0}^{x=t} + \int_0^t \sin(t-x)\,dx\right\}$

$= 2 + \frac{t^2}{2} - cost - \left\{\begin{matrix}[-xsin(t-x)]_{x=0}^{x=t} \\ +[\cos(t-x)]_{x=0}^{x=t}\end{matrix}\right\}$

$= 1 + \frac{t^2}{2}$. Hence, verified.

Definition 1: (Resolvent Kernel) – Suppose that the integral equation

$y(x) = f(x) + \int_a^x k(x,t)y(t)dt$ has a solution $y(x) = f(x) + \int_a^x R(x,t)f(t)dt$ then $R(x,t)$ is called the resolution kernel or reciprocal kernel of the integral equation.[2]

Application 5: Suppose the linear Volterra integral equation is

$$Y(t) = F(t) + \int_0^t e^{t-x}\,Y(x)dx$$

By Convolution theorem, we have

$$Y(t) = F(t) + e^t * Y(t) \quad (14)$$

Taking Kamal transform of both sides of equation (14)

$K\{Y(t)\} = K\{F(t)\} + K\{e^t * Y(t)\}$

$K\{Y(t)\} = K\{F(t)\} + K\{e^t\}.K\{Y(t)$
}

$K\{Y(t)\} = K\{F(t)\} + \left(\frac{v}{1-v}\right) K\{Y(t)\}$

$\therefore \left[1 - \left(\frac{v}{1-v}\right)\right] K\{Y(t)\} = K\{F(t)\}$

$\therefore \left(\frac{1-2v}{1-v}\right) K\{Y(t)\} = K\{F(t)\}$

$$\Rightarrow K\{Y(t)\} = \left(\frac{1-v}{1-2v}\right) K\{F(t)\} \qquad (15)$$

Now, let $R(t-x)$ be the resolvent kernel then

$$Y(t) = F(t) + \int_0^t R(t-x)F(x)dx \qquad (16)$$

Convolution theorem gives

$$Y(t) = F(t) + R(t) * F(t) \qquad (17)$$

Taking Kamal transform of both sides of equation (17)

$K\{Y(t)\} = K\{F(t)\} + K\{R(t) * F(\ t)+$

$K\{Y(t)\} = K\{F(t)\} + K\{R(t)\}.K\{F$

$K\{Y(t)\} = (1 + K\{R(t)\})K\{F(t)\}$
( )}

From equation (15),
$\left(\frac{1-v}{1-2v}\right) K\{F(t)\} = (1 + K\{R(t)\})K\{F(t)\}$

$\therefore 1 + K\{R(t)\} = \frac{1-v}{1-2v}$

$$\Rightarrow K\{R(t)\} = \frac{v}{1-2v} \qquad (18)$$

Appling inverse Kamal operator to equation (18), we have

$$R(t) = K^{-1}\left\{\frac{v}{1-2v}\right\} \qquad (19)$$

$R(t) = e^{2t}$. So that the resolvent kernel is

$R(t-x) = e^{2(t-x)}$ (20) Putting this in

equation (16), we get

$$Y(t) = F(t) + \int_0^t e^{2(t-x)} F(t)dt \qquad (21)$$

which is a solution of given equation.

Application 6: Let the linear Volterra integral equation be

$$Y(t) = F(t) + \int_0^t (t-x)Y(x)dx$$

Appling Convolution theorem to this equation, we have

$$Y(t) = F(t) + t * Y(t) \tag{22}$$

Taking Kamal transform, we have

$K\{Y(t)\} = K\{F(t)\} + K\{t * Y(\quad) +$

$K\{Y(t)\} = K\{F(t)\} + K\{t\}. K\{Y$

$K\{Y(t)\} = K\{F(t)\} + v^2 K\{Y(t$

$\therefore (1 - v^2) K\{Y(t)\} = K\{F(t)\}$

$( )\}$

$)+$

$$\therefore K\{Y(t)\} = \left(\frac{1}{1-v^2}\right) K\{F(t)\} \tag{23}$$

Now, let $R(t-x)$ be the resolvent kernel then the solution is given by

$$Y(t) = F(t) + \int_0^t R(t-x) F(t) dt \tag{24}$$

By convolution theorem we can write this as

$$Y(t) = F(t) + R(t) * F(t) \tag{25}$$

Taking Kamal transform of both sides of equation (25), we have

$K\{Y(t)\} = K\{F(t)\} + K\{R(t) * F(t)\}$

$K\{Y(t)\} = K\{F(t)\} + K\{R(t)\}. K\{F(t)$

$\}$

$K\{Y(t)\} = (1 + K\{R(t)\}) K\{F(t)\}$

From equation (23),

$\left(\frac{1}{1-v^2}\right) K\{F(t)\} = (1 + K\{R(t)\}) K\{F(t)\}$

$\therefore \left(\frac{1}{1-v^2}\right) = 1 + K\{R(t)\}$

$$\therefore K\{R(t)\} = \frac{v}{1-v^2} \tag{26}$$

Appling inverse operator to both sides of equation (26)

$$R(t) = K^{-1}\left\{\frac{v}{1-v^2}\right\} \tag{27}$$

$R(t) = Sinht.$

So that the resolvent kernel is $R(t-x) = \sinh(t-x)$ (28)

Putting this in equation (24), we get the solution for given integral equation as

$$Y(t) = F(t) + \int_0^t \sinh(t-x)\, F(t) dt \tag{29}$$

Definition 2: (Integro-Differential Equation) - An equation in which the derivatives of the unknown function can also be present is called an Integro –Differential Equation. [2]

Application 7: Consider the integro-differential equation

$$y'(t) = sint + \int_0^t y(t-x) cosx\, dx \, , \, y(0) = 0$$

Appling Convolution theorem of Kamal transform to this equation, we have

$$y'(t) = sint + y(t) * cost\ ,\ \ y(0) = 0 \qquad (30)$$

Taking Kamal transform of both sides of equation (30)

$K\{y'(t)\} = K\{sint\} + K\{y(t) * cost\}$

$K\{y'(t)\} = K\{sint\} + K\{y(t)\}.K\{cost\}$. Using initial condition, we have

$$(v^{-1}K\{y(t)\} - y(0)) = \frac{v^2}{1+v^2} + \left(\frac{v}{1+v^2}\right)K\{y(t)\},$$

$\therefore \left(\frac{1}{v} - \frac{v}{1+v^2}\right)K\{y(t)\} = \frac{v^2}{1+v^2}$

$\therefore \left(\frac{1}{v(1+v^2)}\right)K\{y(t)\} = \frac{v^2}{1+v^2}$

$$\Rightarrow K\{y(t)\} = v^3 \qquad (31)$$

Appling inverse Kamal transform operator to both sides of equation (31)

$y(t) = K^{-1}\{v^3\} = \frac{t^2}{2!}$.

Application 8: Consider the integro-differential equation

$$Y'(t) = t + \int_0^t Y(t-x)cosx\,dx\ , Y(0) = 4$$

Appling Convolution theorem of Kamal transform, we have

$$Y'(t) = t + Y(t) * cost\ , Y(0) = 4 \qquad (32)$$

Taking Kamal transform of both sides of equation (32), we get

$K\{Y'(t)\} = K\{t\} + K\{Y(t) * cost\}$

$K\{Y'(t)\} = K\{t\} + K\{Y(t)\}.K\{cost\}$

$(v^{-1}K\{Y(t)\} - Y(0)) = v^2 + \frac{v}{1+v^2}K\{Y(t)\}$

Using initial condition, we get

$\therefore \frac{1}{v}K\{Y(t)\} - 4 = v^2 + \frac{v}{1+v^2}K\{Y(t)\}$

$\therefore \left(\frac{1}{v} - \frac{v}{1+v^2}\right)K\{Y(t)\} = v^2 + 4$

$\therefore \left(\frac{1}{v(1+v^2)}\right)K\{Y(t)\} = v^2 + 4$

$\Rightarrow K\{Y(t)\} = (v^2 + 4).[v(1 + v^2)]$

$$= v^5 + 5v^3 + 4v \qquad (33)$$

Appling inverse Kamal transform operator to both sides of equation (33),

$Y(t) = K^{-1}\{v^5 + 5v^3 + 4v\}$

$Y(t) = \frac{t^4}{4!} + \frac{5t^2}{2!} + 4$.

6. CONCLUSIONS

1. Used successively the Kamal transform to solve linear Volterra integral equation of second kind.
2. Used the Kamal transform to solve linear Volterra integral equation of second kind by obtaining the resolvent kernel.

3. Integro-Differential Equations can be solved by using the Kamal transform.

REFERENCES

[1] Abdelilah K. and Hassan S. (2016). The new integral transform Kamal transform, Advances in theoretical and applied mathematics, 11(4), 451-458.

[2] Abdelilah K. and Hassan S. (2017). The use of Kamal transform for solving partial differential equations, Advances in theoretical and applied mathematics, 12 (1), 7-13. [3] Abdelilah K., Hassan Sedeeg, I. Zahra and Adem Mahamoud (2017). Applications of New Transform Kamal Transform to Ordinary Differential Equations with Non- constant Co-efficients, Elixir Applied Mathematics, 103, 45727-45730. [4] Aggarwal S., Chauhan R. and Sharma N. (2018). A new application of Kamal transform for solving linear Volterra integral equations, International Journal of Latest Technology in Engineering, Management & Applied Sciences, 7 (4), 138-140.

[5] Aggarwal S. (2018). Kamal transform of Bessel's functions, International Journal of Research and Innovation in Applied Sciences, 3(7), 1-4.

[6] Aggarwal Sudhanshu, Sharma Nidhi, Chauhan Raman (2018). Application of Kamal transform for solving linear Volterra integral equations of first kind, International Journal of Research in Advent Technology, 6(8), 2081-2088.

[7] Babolian E. and Masouri Z. (2008). Direct method to Solve Volterra integral equation of first kind using operational matrix with block-pulse functions, Journal of Computational and Applied Mathematics, 220, 51-57.

[8] Gupta A. R., Aggarwal S. and Agrawal D. (2018). Solution of linear partial integro- differential equations using Kamal transform, International Journal of Latest Technology in Engineering, Management & Applied Sciences, 7(7), 88-91. [9] Maleknejad K., Roodaki M. and Almasieh H. (2009). Numerical Solution of Volterra integral equations of first kind by using a recursive scheme, Journal of Mathematical Extension, 3(2), 113-121.

[10] Mirzaee Farshid. (2012). Numerical solution for Volterra integral equation of first kind via Quadrature rule, applied Mathematical Sciences, 6 (20), 969-974. [11] Debnath Loknath and Batta D. Integral transforms and their applications, second edition, Chapman and Hall/CRC, 2006.

[12] Polyanin A. D. and Manzhirov A. V. Handbook of integral equations, Chapman & Hall/CRC, 2008.

[13] Raisinghania M. D. Integral equations and boundary value problems, S. Chand & Company, New Delhi, 2013.