JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

# SCHEDULED REPLACEMENT OF ITEMS IN DIFFERENT STAGES OF FAILURE WITH MINIMAL REPAIR AND DISCOUNTING RATE

**Wakil Kumar[1], Dr. Ashish Kumar Jha[2]**
Department of Mathematics, Ranchi University, Ranchi (Jharkhand), India
wakil.kumar@jru.edu.in

**Abstract:**

This paper discusses the replacement of item in discrete schedules, involving minor repair with discounting rate. In my study, it was observed that for some or other reason, a running system cannot be replaced at fixed time length. The ideal time of replacement of system may be some weeks, some months, some years and so on. This paper focuses on replacement of items at ideal time with discrete discount rate and summarises explicit expression of replacement of machine on discounted discrete schedules in three stages of failure. The assumption in my study is that the replacement or repair of items at scheduled time is NT (N= 1, 2, 3, --------.) for fixed T > 0 involving minimal repair with discounting rate $\beta > 0$. For theoretical result, a numerical example has been provided.

**Keywords:** Replacement, Repair, Discrete, Discounted, Optimal, Stage.

## 1. Introduction

Productivity, better quality of product and reliability technology have been playing a vital role in modern world. The reliability analysis deals with the proper functioning of the components and making the system more effective. Any operating system fails completely or temporarily at any time when it is in operating condition. Once an operating system fails completely, it can be replaced by a new one and when the operating system fails temporarily then it may be repaired. In such a situation the decision to replace or repair the operating system is made on the parameter of cost benefit. If any system fails then it has a direct effect on production, customer's relationship and revenue. In such situations, operating system is repaired or replaced periodically. i.e. some weekend, at the interval of some months or at the year ends.

Many researchers work in the field of reliability on stochastic nature with different types of failure, repair and replacement strategies. Cao et al (1989) discussed on reliability analysis of system with replaceable repair facility in the system. Nafeesa Bashir, JPS Joorel, TR Jan (2021) discussed on Reliability Analysis of Two Unit Standby Model with Controlled and Uncontrolled Failure of Unit and Replacement Facility Available in the System. Murthy, P.R (2007). Operations Research. 2nd Edition, proposed a replacement model when items fail suddenly. They calculate probability of failure through utilized relative frequency approach. Enogwe et al,(2018) modified Murthy model to calculate replacement cost of items that are in random nature. After this use goodness – of – fit test calculate the probability distribution that best describe the failure time and replacement cost. Tomohiro Kitagawa, Tetsushi Yuge, shigeru Yanagi.(2018), discuss replacement time of a single – unit one – shot system when failure system removed by minimal repair. Again they compare the two replacement policies that manage the number of failure and the number of periodic inspection. J. H. Lim, Jian Qu, Ming J.Zuo (2016), discussed replacement policy based on imperfect repair with random probability. Also incorporate expected cost rate per unit time are derived for both the infinite – horizon case and the one replacement cycle.Yusuf Bashir, Yusuf Ibrahim, Yakasai M.Bashir (2015) discussed age and random preventive maintenance policy for a system with two types of failures.

In this research paper, three stages of failure of a system have been considered, which are stage I, stage II and stage III. Stage I failure is catastrophic failure, which occur suddenly and is not repairable. Both stage II and stage III failures occur gradually and depend on time and frequency of use. The rate of failure of stage III is greater than that of stage II. Similarly, the failure rate of stage II is greater than the failure rate of stage I. Replacement action is instant, when it reaches planned replacement time NT for a fixed time T to include the discounting factor $\beta$ in the replacement model. $e^{-NT}$ will be included for each of an associated cost of replace/repair of a type of failure of the system, where $\beta$ is fixed.

**2. Model Notations and description**

Notations:

$\beta$: Discount Rate

$s_1$(t): Failure rate of unit in first stage

$s_2$(t): Failure rate of unit in second stage

$s_3$(t): Failure rate of unit in third stage

$R_1$(t): Reliability function of the unit due to first stage failure

$F_1$(t): Failure distribution of unit in first stage

$c_2$: Minimal repair cost of unit in second stage

$c_3$: Minimal repair cost of unit in third stage

$c_p$: Scheduled replacement cost of unit at NT, for N = 1, 2, 3, 4…

$c_{up}$: Un-planned replacement cost of unit in first stage failure

$N^*$: Optimal scheduled replacement time of the unit.

**2.1 *Description:***

1. Second stage and third stage failures are repairable while un-repairable failure is put in first stage.

2. For cost of replacement/repair we follow order: $c_{up} > c_p > c_2 > c_3$

3. All the three failures are detected instantaneously.

4. If any unit fails in first stage then it is replaced instantaneously with a new one.

5. If any unit fails in second and/or third stages then it is repaired and operates whenever it is not stopped.

6. The unit is replaced at a fixed scheduled time NT.

7. In First and second stage failure, repair time is negligible.

**2.2 *Methodology:***

On the basis of assumptions,

The reliability function of unit in first stage failure is

$$R_1(NT) = e^{-\int_0^{NT} s_1(t)dt} \quad (1)$$

Replacement cost of unit in first stage in one replacement cycle is

$$\int_0^{NT} c_{up} e^{-\beta t} d\, F_1(t) \quad (2)$$

Scheduled replacement cost of unit at NT time in one replacement cycle is

$$c_p e^{-\beta(NT)} R_1(NT) \quad (3)$$

Minimal repair cost of unit due to second stage failure in one replacement cycle is

$$\int_0^{NT} c_2 s_2(t) e^{-\beta t} R_1(t) dt \quad (4)$$

Minimal repair cost of unit due to third stage failure in one replacement cycle is

$$\int_0^{NT} c_3 s_3(t) e^{-\beta t} R_1(t) dt \quad (5)$$

The average of one replacement cycle is

$$\int_0^{NT} \alpha e^{-\beta t} R_1(t) dt \quad (6)$$

From (2) to (6), the average replacement of discounted cost of the unit in one replacement cycle is;

$$C_{avg}(N) = \frac{c_p e^{-\beta(NT)} R_1(NT) + \int_0^{NT} c_{up} e^{-\beta t} d\, F_1(t) + \int_0^{NT} c_2 s_2(t) e^{-\beta t} R_1(t) dt + \int_0^{NT} c_3 s_3(t) e^{-\beta t} R_1(t) dt}{\int_0^{NT} \alpha e^{-\beta t} R_1(t) dt} \quad (7)$$

$$\because \; F_1(t) = 1 - R_1(t) \Rightarrow dF_1(t) = s_1(t) R_1(t) dt$$

$$C_{avg}(N) = \frac{c_p e^{-\beta(NT)} R_1(NT) + \int_0^{NT} c_{up} e^{-\beta t} s_1(t) R_1(t) d\,(t) + \int_0^{NT} c_2 s_2(t) e^{-\beta t} R_1(t) dt + \int_0^{NT} c_3 s_3(t) e^{-\beta t} R_1(t) dt}{\int_0^{NT} \alpha e^{-\beta t} R_1(t) dt} \quad (8)$$

$$= \frac{c_p e^{-\beta(NT)} R_1(NT) + \int_0^{NT} \gamma(t) e^{-\beta t} R_1(t) dt}{\int_0^{NT} \beta e^{-\beta t} R_1(t) dt} \quad (9)$$

Where $\gamma(t) = c_{up} s_1(t) + c_2 s_2(t) + c_3 s_3(t)$ (10)

Here $C_{aveg}(N)$ is our objective function which is to be optimized, and the aim is to obtain an optimum number of replacements of item that minimize $C_{avg}(N)$.

**3. Numerical Example**

Through Weibull alternative parameterization,

Rate of stage I failure is

$$s_1(t) = \lambda_1 \alpha_1 t^{\alpha_1 - 1} \qquad \alpha_1 > 0, \quad t \geq 0 \tag{11}$$

Rate of stage II failure is

$$s_2(t) = \lambda_2 \alpha_2 t^{\alpha_2 - 1} \qquad \alpha_2 > 0, \quad t \geq 0 \tag{12}$$

Rate of stage III failure is

$$s_3(t) = \lambda_3 \alpha_3 t^{\alpha_3 - 1} \qquad \alpha_3 > 0 \quad t \geq 0 \tag{13}$$

Through illustrative example we find the repair/ replacement cost, we set the parameter as follows:

$$\alpha_1 = 2, \quad \alpha_2 = 3, \quad \alpha_3 = 4$$

$$\lambda_1 = 0.0004, \quad \lambda_2 = 0.002, \quad \lambda_3 = 0.005$$

$$c_{up} = 30, \; c_p = 20, c_2 = 5, \quad c_3 = 2.5$$

Substituting these values in (11), (12) and (13) then failure rate of different stage is obtained as follows:

$$s_1(t) = 0.0008t \tag{14}$$

$$s_2(t) = 0.006t^2 \tag{15}$$

$$s_3(t) = 0.02t^3 \tag{16}$$

Optimal replacement/Repair items that minimize cost for different values of discounting rate through assumed cost.

**Table-1**

The values of C (N) for different N(1, 2, 3, 4, …….) and different values of discounting factor ($\beta$), When T = 1.

| N | C $_{avg.}$(N) when $\beta = 0.02$ | C $_{avg.}$ (N) when $\beta$ =0.04 | C $_{avg.}$ (N) when $\beta$ =0.06 | C $_{avg.}$ (N) when $\beta$ =0.08 | C $_{avg.}$ (N) when $\beta$ =0.1 |
|---|---|---|---|---|---|
| 1 | 9914.8527 | 4907.8951 | 3239.1315 | 2404.9163 | 1904.5204 |
| 2 | 4976.4988 | 2438.8517 | 1593.4150 | 1171.0295 | 917.8652 |
| 3 | 3454.1355 | 1676.7294 | 1084.9348 | 789.5414 | 612.7094 |
| 4 | 2882.5296 | 1387.7430 | 890.4025 | 642.4235 | 494.1861 |
| 5 | 2795.2625 | 1337.0951 | 852.2547 | 610.7481 | 466.5741 |
| 6 | 3057.9780 | 1455.6151 | 923.0972 | 622.4120 | 499.9994 |
| 7 | 3630.2048 | 1720.6857 | 1086.3345 | 770.8005 | 582.8047 |
| 8 | 4505.7540 | 2126.4590 | 1336.3388 | 943.5791 | 709.8033 |
| 9 | 5692.8030 | 2673.9181 | 1671.8409 | 1174.1009 | 878.1794 |
| 10 | 7205.8883 | 3366.8951 | 2093.2972 | 1461.3037 | 1086.1102 |

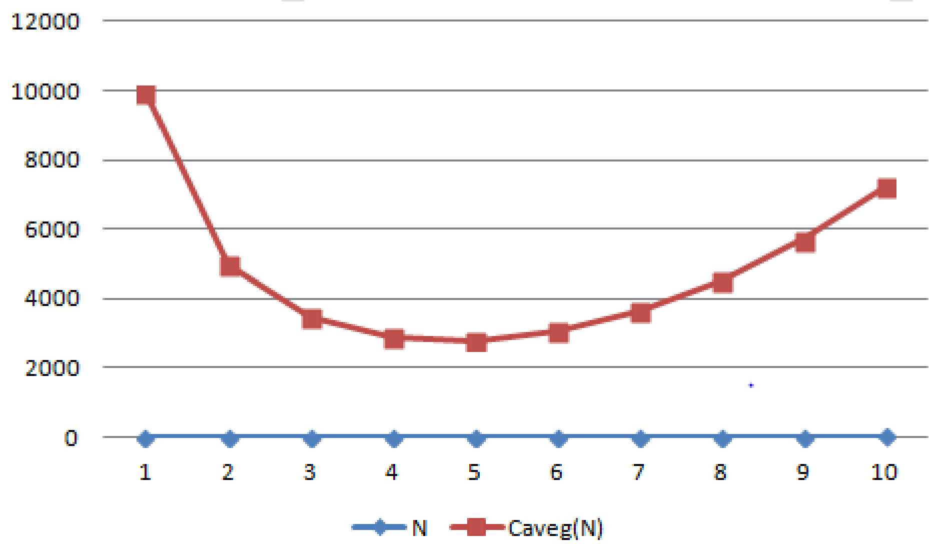


Fig. 1. Graph of C $_{avg.}$(N) for different values of N When T = 1 and $\beta = 0.02$

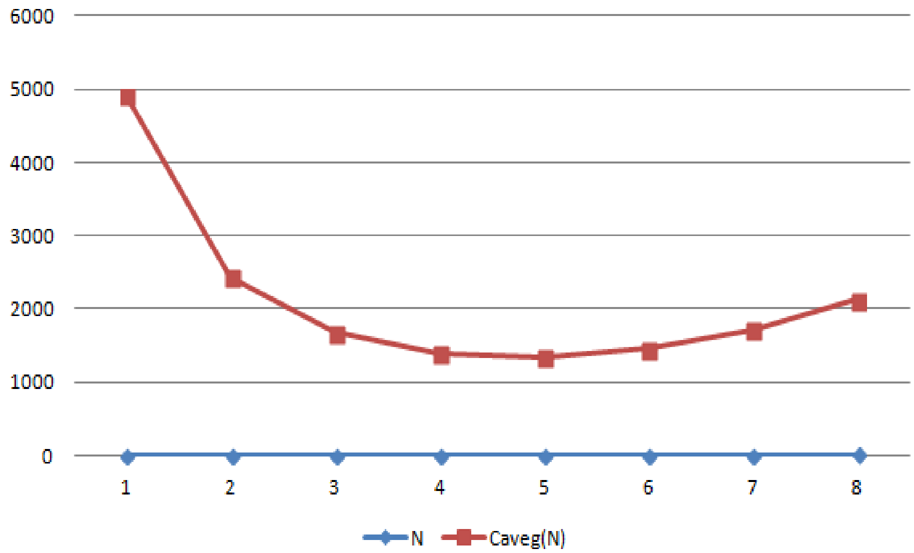


Fig.2. Graph of C $_{avg.}$ (N) for different values of N When T = 1 and $\beta = 0.04$

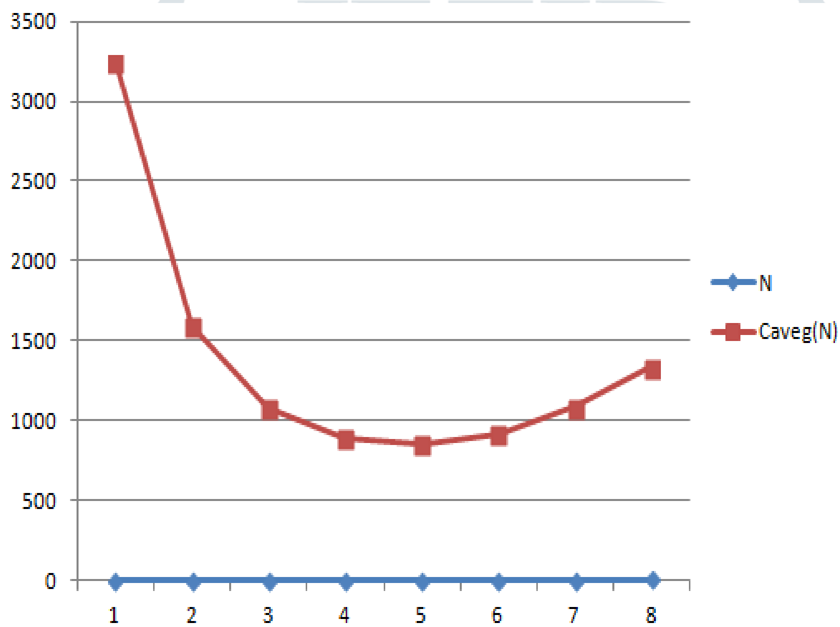


Fig.3. Graph of C $_{avg.}$ (N) for different values of N When T = 1 and $\beta = 0.06$

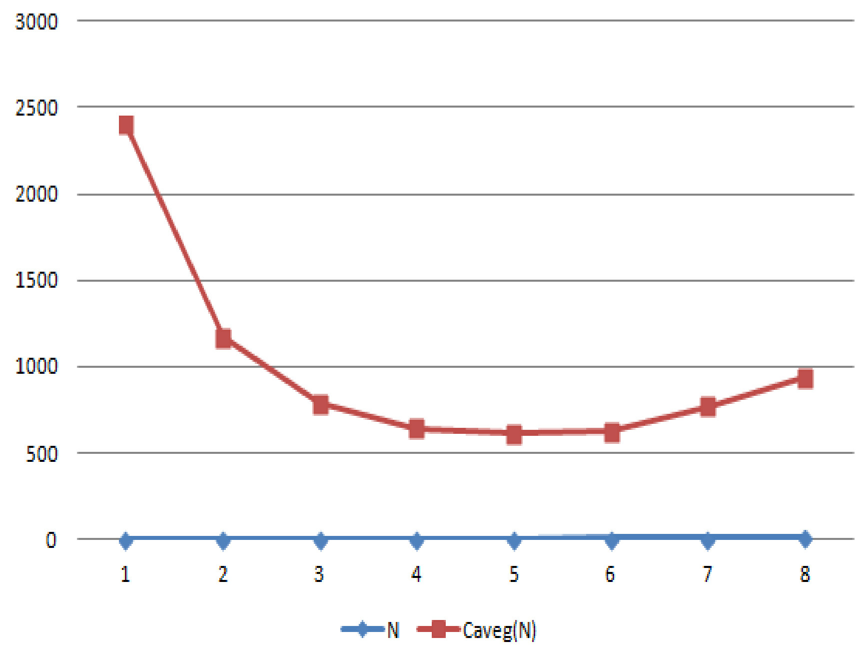


Fig. 4. Graph of C $_{avg.}$ (N) for different values of N When T = 1 and $\beta = 0.08$

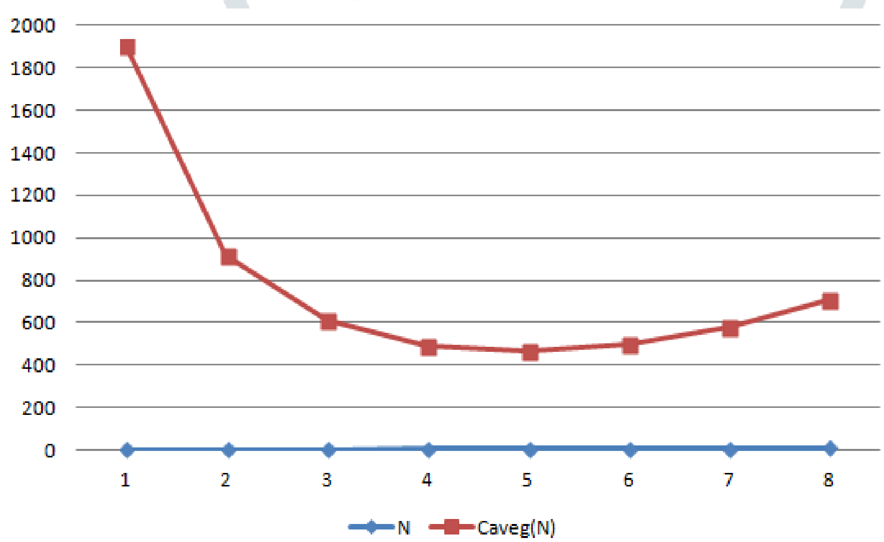


Fig. 5. Graph of C $_{avg.}$ (N) for different values of N When T = 1 and $\beta = 0.1$

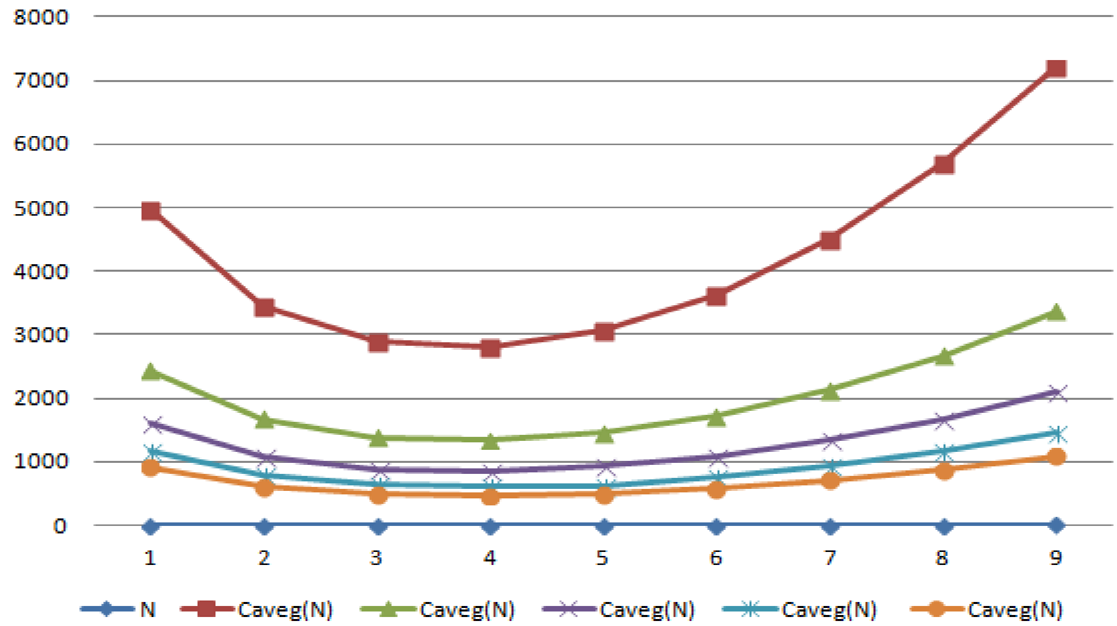


Fig. 6. Comparing the different values for $\beta$

**Table-2**

The values of C(N) for different N(1, 2, 3, 4, …….) and different values of T and fixed discounting rate 0.02

| N | $C_{avg.}$(N) when $T$ =1 | $C_{avg.}$(N) when $T$ =2 | $C_{avg.}$(N) when $T$ = 3 | $C_{avg.}$(N) when $T$ =4 | $C_{avg.}$(N) when $T$ =5 |
|---|---|---|---|---|---|
| 1 | 9914.8527 | 4976.4888 | 3454.1355 | 2882.5296 | 2795.2625 |
| 2 | 4976.4988 | 2882.5296 | 3057.9780 | 4505.7540 | 7205.8883 |
| 3 | 3454.1355 | 3057.9780 | 5692.8030 | 11279.8755 | 20274.5063 |
| 4 | 2882.5296 | 4505.7540 | 11279.8755 | 24105.7979 | 43923.9760 |
| 5 | 2795.2625 | 7205.8883 | 20274.5063 | 43923.9760 | 79286.0692 |
| 6 | 3057.9780 | 11279.8755 | 33096.8994 | 71261.1088 | 126328.3459 |
| 7 | 3630.2048 | 16869.5880 | 50043.2683 | 106160.7698 | 183877.7500 |
| 8 | 4505.7540 | 24105.7979 | 71261.1088 | 148188.0735 | 249791.5077 |
| 9 | 5692.8030 | 33096.8994 | 96742.7726 | 196474.1740 | 321231.9972 |
| 10 | 7205.8883 | 43923.9760 | 126328.3459 | 249791.5077 | 395007.5797 |

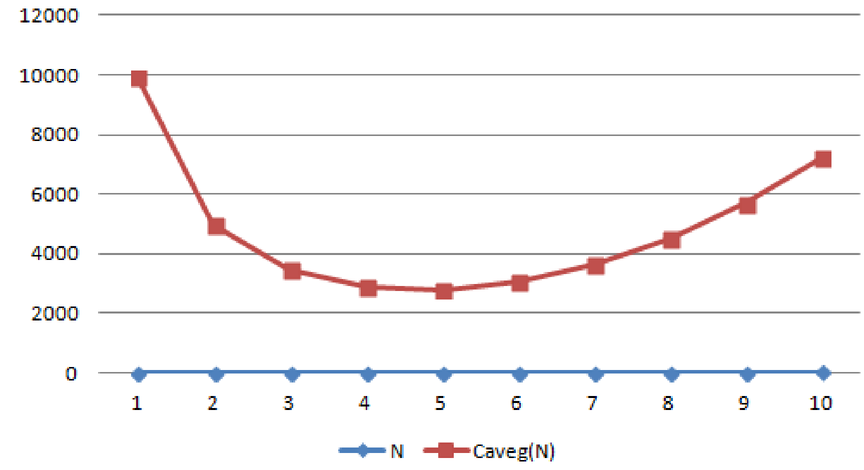


Fig. 7. Graph of C $_{avg.}$ (N) for different values of N When T = 1 and $\beta = 0.02$

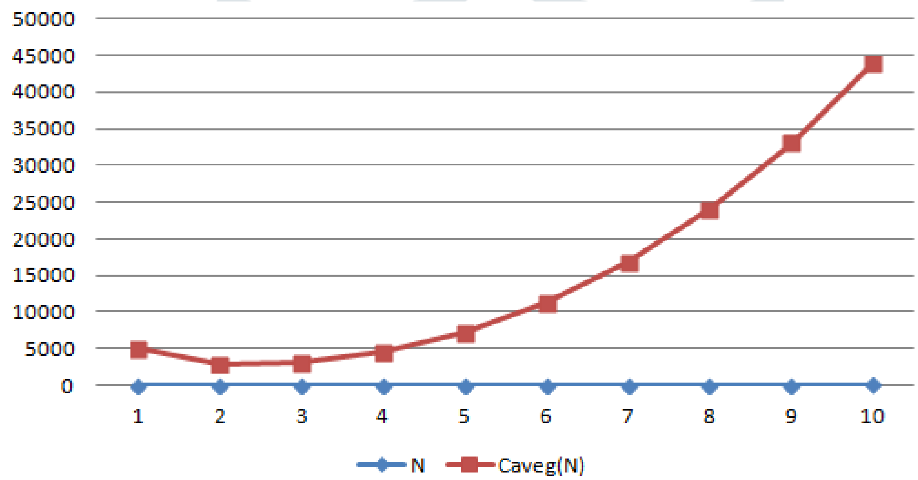


Fig. 8. Graph of C $_{avg.}$ (N) for different values of N When T = 2 and $\beta = 0.02$

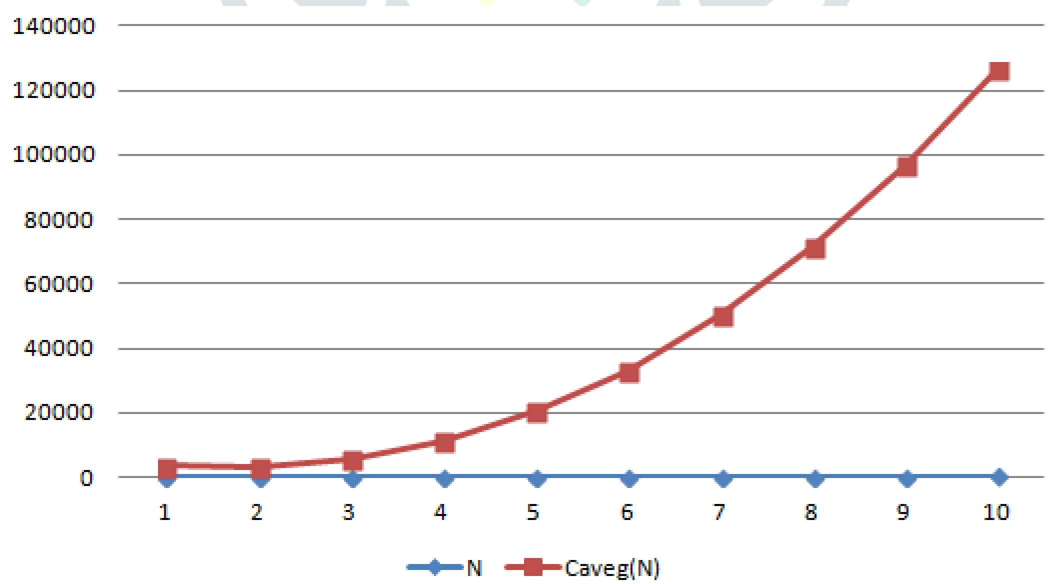


Fig. 9. Graph of C $_{avg.}$ (N) for different values of N When T = 3 and $\beta = 0.02$

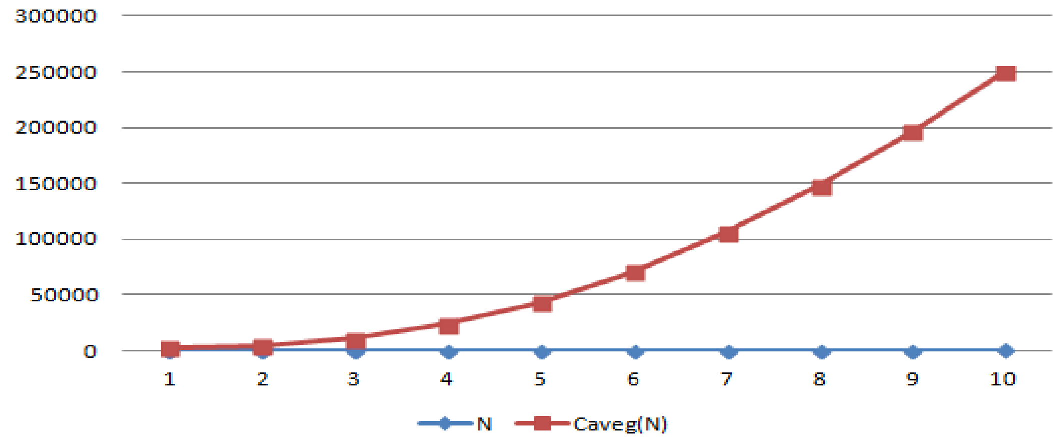


Fig. 10. Graph of C $_{avg.}$ (N) for different values of N When T = 4 and $\beta = 0.02$

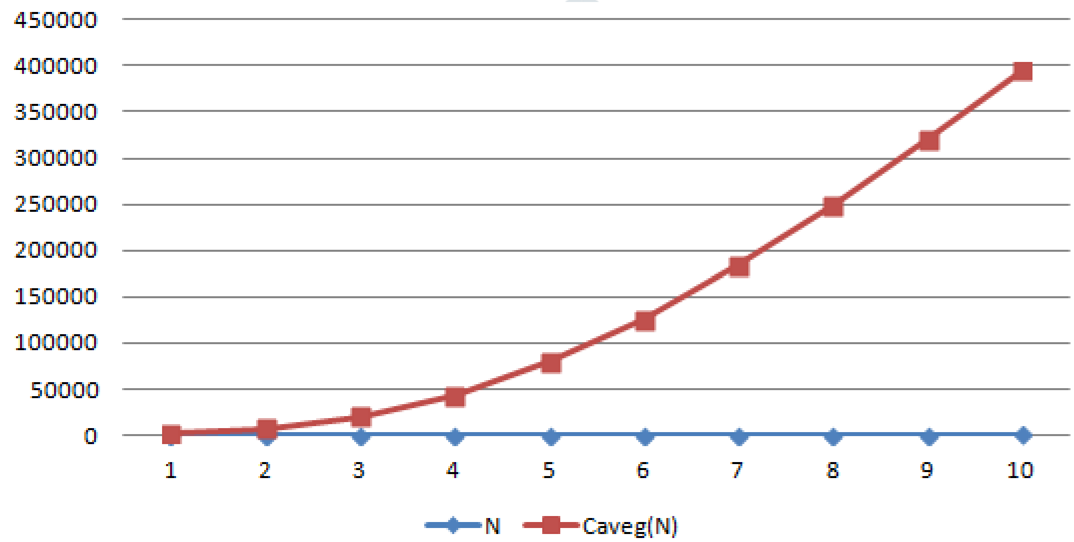


Fig. 11. Graph of C $_{avg.}$ (N) for different values of N When T = 5 and $\beta = 0.02$

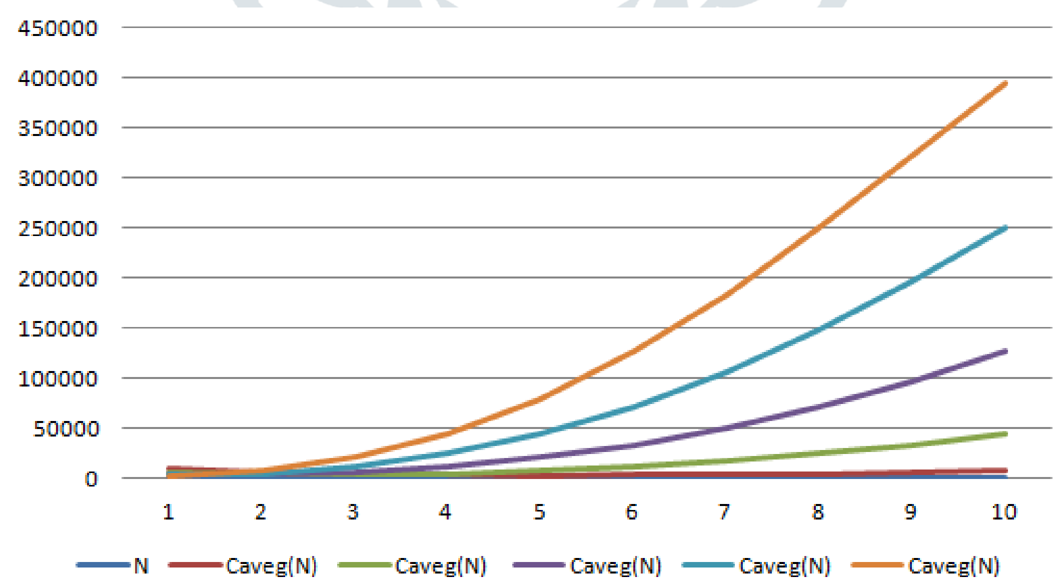


Fig. 12. Comparing the different values for T

**3.1** ***Result:***

From Table-1 we observed that optimal replacement time of unit is N* = 5, with average cost rate $C_{avg.}(N^*)$ = 2795.2625, when T = 1, and discounting rate $\beta = 0.02$

- From Table-1 we observed that optimal replacement time of unit is N* = 5, with average cost rate $C_{avg.}(N^*)$ = 1337.0951, when T = 1, and discounting rate $\beta = 0.04$
- From Table-1 we observed that optimal replacement time of unit is N* = 5, with average cost rate $C_{avg.}(N^*)$ = 852.2547, when T = 1, and discounting rate $\beta = 0.06$
- From Table-1 we observed that optimal replacement time of unit is N* = 5, with average cost rate $C_{avg.}(N^*)$ = 610.7481, when T = 1, and discounting rate $\beta = 0.08$
- From Table-1 we observed that optimal replacement time of unit is N* = 5, with average cost rate $C_{avg.}(N^*)$ = 466.5741, when T = 1, and discounting rate $\beta = 0.1$
- From Table-2 we observed that optimal replacement time of unit is N* = 5, with average cost rate $C_{avg.}(N^*)$ = 2795.2625, when T = 1, and discounting rate $\beta = 0.02$
- From Table-2 we observed that optimal replacement time of unit is N* = 2, with average cost rate $C_{avg.}(N^*)$ = 2882.5296, when T = 2, and discounting rate $\beta = 0.02$
- From Table-2 we observed that optimal replacement time of unit is N* = 2, with average cost rate $C_{avg.}(N^*)$ = 3057.9780, when T = 3, and discounting rate $\beta = 0.02$
- From Table-2 we observed that when T = 4, 5 cost rate is increase continuous when discounting rate $\beta = 0.02$

**4. Conclusion:**

In this research work, the scheduled replacement of items in different stages of failure was investigated. It was taken into consideration that the failure of items occurs in three stages. First stage failure is non- repairable while second and third stage failures are repairable with time and subject to use. Few numerical examples have been provided that display the effect of discounting rate on scheduled replacement time. The numerical example has also highlighted the characteristic of the model. This research work is crucial to engineers, maintenance managers and plant management in replacing units at ideal time intervals such as weekends, month end or year-end respectively.

**References:**


1. Cao, Jinhua & Hong, W.Y. (1989), "Reliability of two-unit cold standby system with replaceable repair facility", Microelectronics Reliability, Vol. 29(2), pp. 145-150.
2. Nafeesa Bashir, JPS Joorel, TR Jan (2021) "Reliability Analysis of Two Unit Standby Model with Controlled and Uncontrolled Failure of Unit and Replacement Facility Available in the System" Pak.j.stat.oper.res. Vol.17 No. 3 2021 pp 625-632.
3. Sureria, J.K.Malik, S.C & Anand J. (2012). "Cost – benefit analysis of a computer system with priority to S/W replacement over H/W repair" Appl. Math. Sci., Vol.6 (75), 3723 – 3734.
4. Gupta, P.K and Hira, D.S (2013). Operation Research Revised edition. S. Chand and Co Ltd, New Delhi, India.
5. Murthy, P.R (2007). Operation Research. 2nd Edition, New Age International Ltd., Publisher, New Delhi, India.
6. Enogwe, S.U., Oruh, B.I. and Ekpenyong, E.J. (2018). A Modified Replacement Model for Items That Fail Suddenly with Variable Replacement Costs. American Journal of Operations Research, 8. 457-473.
7. Tomohiro Kitagawa, Tetsushi Yuge, shigeru Yanagi.(2018). Replacement timing for a one – shot system with minimal repair. Malaya Journal of Matematik, Vol. S, No. 1, 84 – 89.
8. Lim, J.H., Qu, j & Zou, J.M. (2016). Age replacement policy based on imperfect repair with random Probability. Reliability Engineering and System Safety, Vol(149), 24 – 33.
9. Yusuf Bashir, Yusuf Ibrahim, Yakasai M.Bashir(2015). Minimal Repair-Replacement Model for a System with Two Types of Failure. Applied Mathematical Sciences, Vol. 9, 2015, no. 138, 6867 - 6875