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A NEW APPROCH ON CONNECTIVITY OF ANTI FUZZY GRAPHS USING ANTIFUZZY PATHS

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Abstract: Graphs are one of the simplest models to solve real life problems using network theory. The concept of fuzzy graph theory and anti fuzzy graph theory are used to solve complexities, imprecisions and fuzziness in networks. The well-known fuzzy paths in anti fuzzy graph theory answers to max-min problems. But it is incapable of defining min-max problems. So to develop a new optimization technique we introduced a new concept called anti fuzzy path. By using this concept a new structure of anti fuzzy graphs is constructed and based on that some definitions and theorems of anti fuzzy graphs are established to study different dimensions of connectedness.

***Index Terms* - anti fuzzy path, constraint of connectedness, anti fuzzy bridge, anti fuzzy cut vertex, complete anti fuzzy graph.**

I. Introduction

Most of the problems in real life can be visualised as graphs in the nature of vertices (objects) and edges (relationships), therefore is a worthy topic for problem solving strategy. While defining the problems, many of the objects, relationships or both may not be clear cut or difficult to handle. ie.they are fuzzy in nature. According to Blue et al ([1], [2]) there are different types of uncertainties, fuzziness or vagueness can occur in Graphs. Zadeh's [14] fuzzy sets and the incorporation of this fuzzy sets to graph theory i.e. Fuzzy graph theory eradicates these situations in a better way. Studies of Kauffman [3] Rosenfeld [12], Yeh & bang [13] are the stepping stones in the development of fuzzy graph theory. Later Sunil Mathew, Modeson and Malik ([5], [6]) put forward an elaborate study about Fuzzy graphs. Meanwhile in 2012 Akram [7] defined a new area in fuzzy graphs known as anti fuzzy graphs. One of the main topics in anti fuzzy graph theory is fuzzy paths and using this concept we can solve maximum bandwidth problems and widest path problems. It's inefficiency in defining min-max problems forced the development of a new concept called anti fuzzy path and related concepts of anti fuzzy graph.

In this paper we introduced anti fuzzy paths and using that we structure anti fuzzy graphs in a different direction. Then we defined the connectivity in anti fuzzy graphs and established some important theorems, examples and also try to analyse some real life situations.

2. Preliminaries

Definition 2.1. Crisp Graph $G^* = (V, E)$ is a pair, where V is the nonempty set of vertices and $E \subseteq V \times V$ is the set of edges.
Definition 2.2. A fuzzy subset σ on a set X is a map $\sigma: X \to [0,1]$. A fuzzy relation μ on X is a fuzzy binary relation given by $\mu: X \times X \to [0,1]$. Also μ is called a fuzzy relation on σ if $\mu\,(a, b) \leq \sigma(a) \wedge \sigma(b)$ ∀ a, b ∈ X *and* ∧ stands for minimum. Also μ is called an anti fuzzy relation on σ if μ (a, b) ≥ σ (a) ∨ σ (b), ∀ a, b ∈ V *and* ∨ stands for maximum.

Definition 2.3. A fuzzy graph $G = (V, \sigma, \mu)$ of a (crisp) graph G^* is a triple with a pair of functions $\sigma: V \to [0,1]$, $\mu: V \times V \to [0,1]$ where V is a nonempty set of vertices, σ is a fuzzy subset of V and μ is a fuzzy relation on $V \times V$ such that μ (a, b)≤ $\sigma(a) \wedge \sigma(b)$, ∀ a, b ∈ V.

Definition 2.4 An anti fuzzy graph $G_A = (V, \sigma, \mu)$ of a (crisp) graph $G*$ is a triple with a pair of functions $\sigma{:}V \to [0,1]$ and μ: $V \times V \to [0,1]$ where V is a non- empty set of vertices, σ is a fuzzy subset of V and μ is an anti fuzzy relation on $V \times V$ such that μ (a, b) ≥ σ (a) ⋁ σ (b), ∀ a, b ∈ V.

Example 2.5 Let $V = \{u, v, w\}$, define σ as $\sigma\ (u) = 0.4$, $\sigma\ (v) = 0.1$, $\sigma\ (w) = 0.8$ and μ as $\mu\ (uv) = 0.5$, $\mu\ (vw) = 0.8$, $\mu\ (uw) = 0.9$. i.e. $G_A = (V, \sigma, \mu)$ is an anti fuzzy graph.

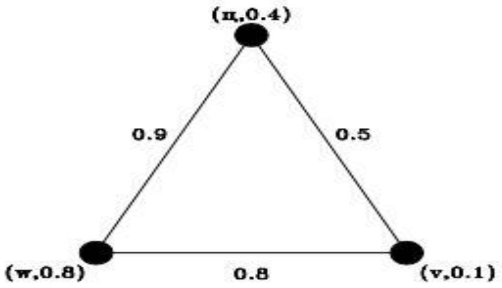

Figure 1.Anti fuzzy graph

Definition 2.6. Let G_A= (V, σ, μ) be an anti fuzzy graph. The order and size of G_A is defined as $\sum_{a\in V} \sigma(a)$, $\sum_{ab\in E} \mu(ab)$ respectively.

Definition 2.7. An anti fuzzy graph H_A= (V, τ, ρ) is a partial anti fuzzy sub graph of G_A= (V, σ, μ) if $\tau \subseteq \sigma$ and $\rho \subseteq \mu$. Also $\tau\ (a) \leq \sigma\ (a)\ \forall u \in V$ and $\rho\ (a, b) \leq \mu\ (a, b)\ \forall a, b \in V$.

Definition 2.8. An anti fuzzy graph H_A= (V, τ, ρ) is an anti fuzzy sub graph of G_A induced by P if $P \subseteq V$, $\tau\ (a) = \sigma\ (a)\ \forall u \in P$ and $\rho\ (a, b) = \mu\ (a, b)\ \forall a, b \in P$.

Example 2.9. Let $H_A = (V, \tau, \rho)$ where $\tau = \{u, v, w\}$ and $\rho = \{uv, vw\}$ with $\tau\ (u)$ =0.4, $\tau(v) =$ 0.1, $\tau(w)$ = 0.8 and ρ as $\rho(uv)$ = 0.5, $\rho(vw)$ = 0.8 , Then H_A= (V, τ, ρ) is an anti fuzzy sub graph of G_A (as in **example 2.5**).Also if $P = \{u, v\}$ and let $H_A=(V,\tau,\rho)$ where $\tau(u) = 0.4$, $\tau(v) = 0.1$ and $\rho(uv)$ = 0.5 then H_A is an induced sub graph of G_A (as in **example 2.5**).

Definition 2.10. An anti fuzzy sub graph H_A= (V, τ, ρ) of $G_A = (V, \sigma, \mu)$ is said to span G_A if $\sigma = \tau$ and is called a spanning sub graph of G_A.

Definition 2.11. An anti fuzzy path P in an anti fuzzy graph of $G_A = (V, \sigma, \mu)$ is a sequence of distinct vertices P: $a_0\ a_1$ .... a_n (except possibly a_0 and a_n) such that $\mu\ (a_{i-1}a_i) > 0$, i = 1, 2…, n. n is called length of the anti fuzzy path P. A single vertex is a null length path. The consecutive pairs are the edges of P. The longest path joining is called diameter of P and is denoted by diam (a, b). The constraint of P is denoted by $\bigvee_{i=1}^{n} \mu(a_{i-1}a_i)$.i.e. The highest membership grade of the edge in that path.

Definition 2.12. Constraint of connectedness between two vertices a& b is $\mu^{\infty}(a, b) = inf\{\ \mu^k(a, b) / k = 1, 2, 3\cdots\}$, where $\mu^k(a, b) = \{\mu\ (aa_1) \vee \mu\ (a_1a_2) \vee \mu\ (a_2a_3) \vee \ldots \mu\ (a_kb)\}$ i.e. The constraint of connectedness of all paths between a&b is defined as the minimum of the constraint of all paths between a& b and is denoted by $CONN_{G_A}(a, b)$.

Remark. The constraint of connectedness plays the most significant role in anti fuzzy graphical structures. The ideal path between two vertices is the path containing the edge whose membership grade coincides with the constraint of connectedness.
In **example 2.5** constraint of connectedness between each pair of vertices as follows.

a	b	$CONN_{G_A}(a, b)$
u	v	0.5
u	w	0.8
v	w	0.8

Definition 2.13. An anti fuzzy graph G_A is connected if any two vertices are joined by an anti fuzzy path.i.e. An anti fuzzy graph G_A is connected if is $\mu^{\infty}(a, b) > 0$, $\forall a, b \in V$.

Remark. In the case of crisp graphs, if the membership grade is 1 for an edge that edge is called the weakest edge of that G_A.

Definition 2.14. Let $G_A = (V, \sigma, \mu)$ be a connected anti fuzzy graph and $G_A' = (V, \sigma', \mu')$ be the partial fuzzy sub graph of G_A obtained by deleting edge ab, where $\mu'(ab) = 0$ and $\mu' = \mu$ for all other edges. If deletion of *ab* increases the constraint of connectedness between some pair of vertices in G_A then *ab* is called anti fuzzy bridge.
In the above **example 2.5,** removal of edges uv & vw, increases the constraint of connectedness between u and v from 0.5 to 0.9, v and w from 0.8 to 0.9. Hence uv, vw are anti fuzzy bridges.

Definition 2.15. Let $G_A = (V, \sigma, \mu)$ be a connected anti fuzzy graph and $G_A' = (V, \sigma', \mu')$ be the partial fuzzy sub graph of G_A obtained by deleting a vertex z, where $\sigma'(z) = 0$, $\sigma = \sigma'$ for all vertices except z, $\mu'(z, c) = 0\ \forall\ c \in V$ and $\mu' = \mu$ for all other edges. If deletion of c reduces the constraint of connectedness between any pair of vertices in G_A then c is called an anti fuzzy cut vertex.
In the above **example 2.5**, removal of vertex v increases the constraint of connectedness between u and w from 0.8 to 0.9. Hence v is an anti fuzzy cut vertex.

Definition 2.16. A connected anti fuzzy graph $G_A = (V, \sigma, \mu)$ is an anti fuzzy tree if it has a partial anti fuzzy spanning sub graph $H_A = (V, \sigma, \rho)$**,** which is a tree, where all edges ab not in H_A, $\mu(ab) > \rho^{\infty}(a, b)$.

Definition 2.17. Let $G_A = (V, \sigma, \mu)$ is a connected anti fuzzy graph. A minimum spanning tree of G_A is an anti fuzzy spanning sub graph $T_A = (V, \sigma, \rho)$ which is a tree, such that *(a, b)* is the constraint of the unique ideal path *a-b* for all $a, b \in V$.

Theorem 2.18. *Let G_A be an anti-fuzzy graph. Then the following statements are equivalent.*

1. *ab is an anti-fuzzy bridge.*
2. *$\mu'^{\infty}(a, b) > \mu(ab)$.*
3*. ab is not the highest edge of any cycle.*

Proof:
2 → 1.We have $\mu'^{\infty}(a, b) > \mu(ab)$. Let ab is not an anti fuzzy bridge, then $\mu'^{\infty}(a, b) = \mu(a, b) \leq \mu(ab)$, a contradiction.
1 → 3. Let ab is the highest edge of a cycle. Consider that cycle as *PP*' where *P* is the path containing the edge *ab* and *P'* is the path not containing *ab* but at least as strong as *P* by using the remaining part of the cycle as a path from *a* to *b*. Thus *ab* can't be a fuzzy bridge.
3 → 2. Let *ab* is not the highest edge of any cycle. If $\mu'^{\infty}(a, b) \leq \mu(ab)$, then the constraint of *ab* is greater than the constraint of the path P not involving ab. This P and ab forms a cycle with the highest edge ab.a contradiction.

Theorem 2.19. *A vertex c of an anti fuzzy graph $G_A = (V, \sigma, \mu)$ is an anti fuzzy cut vertex iff c is an internal vertex of every minimum spanning tree of G_A.*
Proof:
Let G_A be an anti fuzzy graph and *c* be an anti fuzzy cut vertex. Then *a, b* distinct from *c* such that *c* is in every ideal path *a-b*. Now all the minimum spanning trees of G_A contains a unique ideal path *a-b* and hence *c* is an internal vertex of **every** minimum spanning tree**.**
Conversely let *c* be an internal vertex of every minimum spanning tree. Let *T* be a minimum spanning tree and let *ac* and *cb* be edges in *T*. Note that *a, c, b* is the ideal path *a-b* in *T*. If possible assume that *c* is not an anti fuzzy cut vertex. Consider all the paths between every pair of vertices *a, b*. It is clear that there exists at least one ideal path *a-b* not containing *c*. Take one such *a-b* path *P* which clearly contains the edges not in *T*. Now without loss of generality, let $\mu^{\infty}(a, b) \leq \mu(ab)$ in *T*. Then edges in *P* have constraint $\leq \mu(ab)$. Removal of ac and adding *P* in *T* will result another minimum spanning tree of G_A of which *w* is an end vertex, which is a contradiction to our assumption.

Corollary 2.20*. Every anti fuzzy graph G_A has at least two vertices which are not anti fuzzy cut vertices.*

Proof:
From the above theorem **2.19** the end vertices of a minimum spanning tree *T* of G_A cannot be an anti fuzzy cut vertex. Hence the result.

Theorem 2.21.*An edge ab of an anti fuzzy graph $G_A = (V, \sigma, \mu)$ is an anti fuzzy bridge iff ab is in every minimum spanning tree of G_A*
Proof:
Let ab be an anti fuzzy bridge of G_A .Then the edge ab is in the unique ideal path *a-b* and hence is in every minimum spanning tree of G_A.
Conversely, let *ab* be in every minimum spanning tree of G_A and assume that ab is not an anti fuzzy bridge. Then *ab* is the highest edge of some cycle in G_A and $\mu^{\infty}(a, b) \leq \mu(ab)$, which implies that there is at least one minimum spanning tree of G_A not containing *ab*.a contradiction.

Theorem 2.22. *Let $G_A = (V, \sigma, \mu)$ be an anti fuzzy graph such that (V, σ^*, μ^*) is a cycle. Then a vertex of G_A is an anti fuzzy cut vertex iff it is a common vertex of two anti fuzzy bridges.*
Proof:
Let *c* be an anti fuzzy cut vertex of G_A.Then there exists *a* and *b* other than *c* such that *c* is on every ideal path *a-b*. Because

G_A* = (V, σ*, μ*) is a cycle there exists only one ideal path *a-b* containing c and all its edges are anti fuzzy bridges. Thus c is a common vertex of two anti fuzzy bridges.

Conversely suppose *c* be a common vertex of two anti fuzzy bridges *ac* and *cb*. Then both ac and cb are not the highest edges of G_A .Also the path from *a* to *b* not containing edges *ac* and *cb* has constraint greater than μ *(ac)* ∨ μ *(cb*). Hence the ideal path *a-b* is the path *a, c, b* and $\mu^{\infty}(a, b) = \mu\ (ac) \vee \mu\ (cb)$. Thus *c* is an anti fuzzy cut vertex.

Theorem 2.23. *If c is a common vertex of at least two anti fuzzy bridges, then c is an anti fuzzy cut vertex.*

Proof:

Let a_1c and ca_2 be two anti fuzzy bridges. Then there exists *a, b* such that a_1c is on every ideal path *a-b*. If *c* is distinct from *a* and *b*, then it follows that *c* is an anti fuzzy cut vertex. Next suppose one of *a, b* is *c* so that a_1c is on every ideal *a-c* path or ca_2 is on every ideal path *c-b*. Suppose that *c* is not an anti fuzzy cut vertex. Then between every two vertices there exists at least one ideal path joining a_1& a_2 not containing c. This path together with a_1c & ca_2 forms a cycle. We now consider two cases .First suppose thata_1, c, a_2 is not the ideal path. Then clearly one of a_1c, ca_2 or both become the highest edges of a cycle which contradicts that a_1c and ca_2 are anti fuzzy bridges. Second suppose thata_1, *c,* a_2 is also an ideal path joining $a_1$toa_2. Then $\mu(a_1, a_2) = \mu(a_1c) \vee \mu(ca_2)$ the constraint of *P*. Thus edges of *P* are at least equals $\mu(a_1c)$ *and* $\mu\ (ca_2)$, which implies that a_1c, ca_2 are both the highest edges of a cycle, which is a contradiction.

Example 2.24. This example shows that the condition in theorem 2.23 is not necessary. Let *V = {u, v, w, z}* and *X = {uv, vw, wz, zu, uw, vz*}with $\sigma(x)$= 0.1 for all $x \in V$, and is defined by $\mu(uv)$ = 0.2, $\mu(uw)$ = 0.1, $\mu(wz)$ = 0.7, $\mu(vz)$ = 0.1, $\mu(vw)$ = 0.2, $\mu(uz)$ = 0.7 and . Clearly v is an anti cut vertex. However *uw* and *vz* are the only anti-fuzzy bridges. [**Figure 2].**

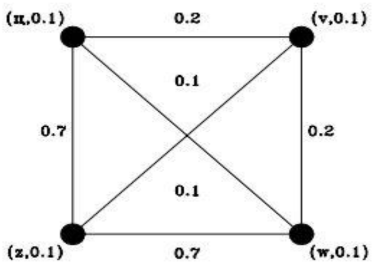

Figure 2

Theorem 2.25. *If ab is an anti fuzzy bridge, then* $\mu^{\infty}(a, b) = \mu\ (ab)$.

Proof: Suppose that *ab* is an anti fuzzy bridge and that $\mu^{\infty}(a, b) < \mu\ (ab)$. Then there exists an ideal path a-b with constraint less than $\mu\ (ab)$ and all edges of this ideal path have constraints less than $\mu\ (ab)$. Also this path together with the edge *ab* forms a cycle in which *ab* is the highest edge, contradicting that *ab* is an anti fuzzy bridge.

3. Complete Anti fuzzy Graph (CAFG)

Definition 3.1. A complete anti fuzzy graph is an anti fuzzy graph G_A= (V, σ, μ) such that μ (ab) = σ (a) ∨ σ (b), for all a, b∈σ *.

Example 3.2: Let A = {u, v, w, z} and X = {uv, vw, wz, zu, uw, vz} with σ (u) =0.6, σ (v) =0.5, σ (w) =0.1 and σ (z) =0.7. Let μ X is defined by μ (uv) = μ (uw) = 0.6, μ (vz) = μ (wz) = μ (uz) = 0.7 and μ (vw) = 0.5. Clearly (V, σ, μ) is a complete anti fuzzy graph. [**Figure 3**].

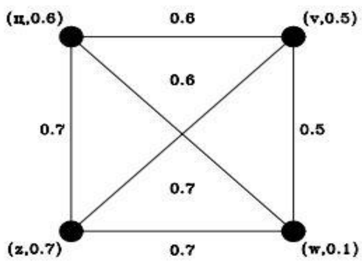

Figure 3.CAFG

Theorem 3.3. *If* G_A= *(V, σ, μ) is a complete anti fuzzy graph, then for any edge ab* $\in \mu^*$, $\mu^{\infty}(a, b) = \mu\ (a, b)$.

Proof:

By definition $\mu^2(a, b) = \wedge_{c\in\sigma*}\{\mu\,(a\,c) \vee \mu(c\,b)\}$

$= \wedge\,(\sigma\,(a) \vee \sigma\,(b) \vee \sigma(c))$

$= \sigma\,(a) \vee \sigma\,(b) = \mu\,(ab).$

Similarly, $\mu^3(a, b) = \mu(ab)$ and in the same way we can show that $\mu^k(a,b) = \mu(ab)$ for all positive integers k thus $\mu^\infty(a, b) = inf\,(\mu^k(a,b)$ / for all integers $k \geq 1) = \mu\,(a, b)$.

Corollary 3.4. *A complete anti fuzzy graph G_A has no anti fuzzy cut vertices.*

Example 3.5: This example shows that condition in **Corollary 3.4** is not necessary. Let $V = \{u, v, w, z\}$ and X = $\{uv, vw, wz, zu, uw, vz\}$ with $\sigma\,(u)$ =0.3, *If* $\sigma\,(v) = \sigma\,(w) = \sigma\,(z) = 0.1$. Let μ is defined by $\mu(uv) = \mu(zu) = \mu(uw) = 0.5$, $\mu(vw) = 0.1$ and $\mu(wz)=\mu(vz)=0.3$.Then μ^∞=μ and G_A has no antifuzzy cut vertices, but is not complete.[Figure4].

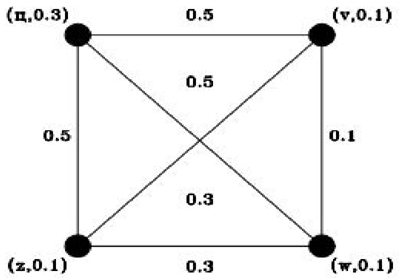

Figure 4.

Theorem 3.6. *Let G_A= (V, σ, μ) be a CAFG with | σ*|=n. Then G_A has an anti fuzzy bridge iff there exists a decreasing sequences $\{t_1, t_2 \ldots\ldots\}$ such that $t_{n-2} > t_{n-1} \geq t_n$, where $t_i = \sigma\,(a_i)$ for i = 1, 2… n. Also the edge $a_{n-1}a_n$ is the anti fuzzy bridge of G_A .*

Proof:

Assume that $G_A = (V, \sigma, \mu)$ is a complete anti fuzzy graph and G_A has an anti fuzzy bridge ab. Now $\mu\,(ab) = \sigma\,(a) \vee \sigma\,(b)$ without loss of generality let $\sigma\,(a) \geq \sigma\,(b)$, so that $\mu\,(ab) = \sigma\,(a)$. Note that ab is not a highest edge of any cycle in G_A . It is required to prove that $\sigma\,(a) < \sigma(c)$ for all c ≠ b. On the contrary, assume that there is at least one vertex $c \neq b$ such that $\sigma\,(a) \geq \sigma(c)$. Now consider the cycle C: a, b, c, a. Then $\mu(ab) = \mu(ac) = \sigma(a)$ and $\mu(bc) = \sigma(b)$ if $\sigma(a) = \sigma(b)$ or $\sigma(a) > \sigma(b) \geq \sigma(c)$ and $\mu(bc) = \sigma(c)$ if $\sigma(a) > \sigma(c) > \sigma(b)$. In either case the edge ab becomes the highest edge of a cycle and by theorem 3.15, *ab* cannot be an anti fuzzy bridge. Conversely let $t_1 \geq t_2 \geq \ldots\ldots. \geq t_{n-2} \geq t_{n-1} \geq t_n$ and $t_i = \sigma\,(u_i)$ for all *i*.

Claim 1: Edge is the anti fuzzy bridge of G_A .

We have $\mu\,(a_{n-1}a_n) = \sigma\,(a_{n-1}) \vee \sigma\,(a_n) = \sigma\,(a_{n-1})$ and by hypothesis, all other edges of G_A will have constraint strictly greater than that of $\sigma\,(a_{n-1})$. Thus the edge $a_{n-1}a_n$ is not the highest edge of any cycle in G_A and by theorem 3.15, $a_{n-1}a_n$ is an anti fuzzy bridge.

Conclusion

This paper has investigated and explored the realm of anti fuzzy graphs and its properties in a new direction. We have analysed some concepts like anti fuzzy paths, anti fuzzy bridges, anti fuzzy cut vertices and complete anti fuzzy graphs. By using the constraint of connectedness property some important results are proved. We can extend these findings as a future work for the google map problems in traffic system. .

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