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Moment-Rotation Relationship for Semi-Rigid Connection

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Abstract: Generally, structure frameworks are designed for vertical loads. Full strength of the beam section is never fully utilized due to the fact that support moments are always higher than the span moments and hence selection of a suitable section depends on support moments. By providing connection flexibility, the support moments can be reduced and span moment increased hence leading to smaller sections resulting in economy. This can be achieved by providing semi-rigid connections. Initial stiffness of the semi-rigid connection is to be found out first and moment-rotation relationship curve is developed for semi-rigid connection. Using available literature, moment-rotation relationship for semi-rigid joints are reviewed for different loading and further work will be extended based on gap in research. By reading the available literature, the equation for finding the initial rotational stiffness of unstiffened top and seat angle connection is reviewed. The equation is used for finding the initial rotational stiffness of unstiffened top and seat angle connection considering vertical loading only.

***Index Terms* -** semi-rigid connection, moment-rotation relationship, unstiffened top and seat angle connection, initial rotational stiffness**.**

I. Introduction

Moment required to cause unit rotation is known as "Rotational Stiffness" of a connection. In pinned connection, at supports, moment is always zero so that stiffness of a pinned connection becomes zero. Similarly, in rigid connection, at supports, angle of rotation is always zero so that stiffness of a rigid connection becomes infinity. But in semi-rigid connection, at supports, moment and angle of rotation will not be equal to zero. This means that moment and angle of rotation both will be exist together with having some values.

Semi-rigid connections are basically categorized into seven types. These types are mentioned in chapter number 39 of INSDAG (Indian Institute for Steel Development and Growth). The paper named as "Moment-Rotation Relation of Top and Seat Angle Connections" is published by Wai-Fah Chen and Sumio G. Nomachi. In this paper, they have developed an equation by simple analytical procedure for initial rotational stiffness and ultimate moment capacity of top and seat angle connection. Another paper named as "Moment-Rotation Relations of Semi-rigid Connections with Angles" is published

by N. Kishi and Wai-Fah Chen. In this paper, they have developed moment-rotation relationships of semi-rigid steel beam-to-column connections. They have considered single and double web-angle connection and top and seat angle connection with or without double web angle. The paper named as "Practical Advanced Analysis for Semi-rigid Space Frames" is published by Seung-Eock Kim and Se-Hyu Choi. In this paper, they have developed practical advanced analysis of semi-rigid space frame. They have discussed nonlinear behaviour of beam-to-column connection and also introduced practical modeling of this connection. The paper named as "Analytical Models for the Initial Stiffness and Plastic Moment Capacity of an Unstiffened Top and Seat Angle Connection under a Shear Load" is published by Jae-Guen Yang and Seong-Sam Jeon. In this paper, they have conducted a study to propose analytical models which can predict the initial stiffness and plastic moment capacity of unstiffened top and seat angle connection. For developing these analytical models, they have considered the moment-rotation curves, hinge lines and failure modes of an unstiffened top and seat angle connection.

The equations for finding the initial rotational stiffness and ultimate moment capacity of unstiffened top and seat angle connection are reviewed. Based on relation of these two equations, "Moment-Rotation Relationship Curve for Semi-rigid Connection" is verified.

II. Review of moment-rotation relationship for semi-rigid connection

A. General

To understand the connection flexibility, let us select a single bay two storey structure as shown in "Fig. 1". The beam BC is connected to the columns AE and DF by bolted or riveted connection. The joint B in "Fig. 1" is made up of "Top and Seat angle connection.

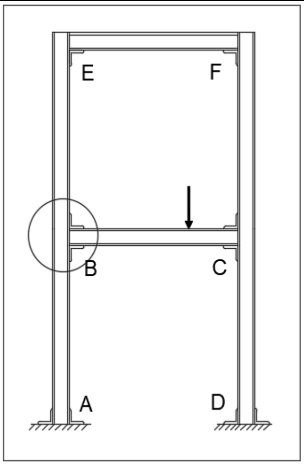

Fig. 1 Modeling of steel frame connection[1]

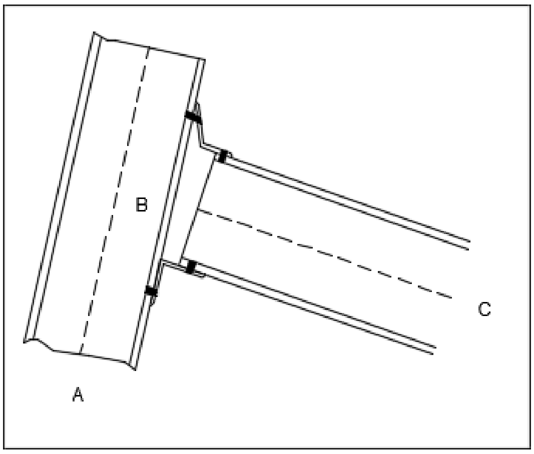

Fig. 2 (a) Deformed shape of joint B in large scale[1]

Now let us focus on "Fig. 2". The deformed shape of joint B is shown in "Fig. 2 (a)" and centre line diagram of the same joint after deformation is shown in "Fig. 2 (b)".

After deformation, beam BC will rotate with larger clockwise angle than column BA. In "Fig. 2 (b)" we can see that if the beam column joint were to be perfectly rigid, the beam BC should be rotated along the line BR_1. But beam has rotated along the line BR_2. This means that, after deformation, the angle between beam-column joint is more than 90 degree with some extra angle of rotation. This extra angle of rotation is known as "relative angle of rotation (θr)". This relative angle of rotation comes due to connection flexibility. So, if we want to give connection flexibility in the analysis, the relation between the relative angle of rotation (θr) and the moment at the joint (M) is very important for us.

The various types of structural steel connections are given in INSDAG. Based on the flexibility, these connections are categorized into schematic classification as shown in "Fig. 3".

Ideally pinned connection is shown on horizontal axis along with relative rotation and ideally rigid connection is shown on vertical axis along with joint moment. Web angle, header plate, top and seat angle, top and seat angle along with web angle, end plate, T-stub and beam to column welded connection all these structural steel connections are classified from pinned connection to rigid connection. By referring this classification as shown in "Fig. 3", "Top and Seat angle connection" is exactly in between the pinned and rigid connection. Addition to this, it is towards flexible side. Therefore, "Top and Seat angle connection" should be selected as semi-rigid connection.

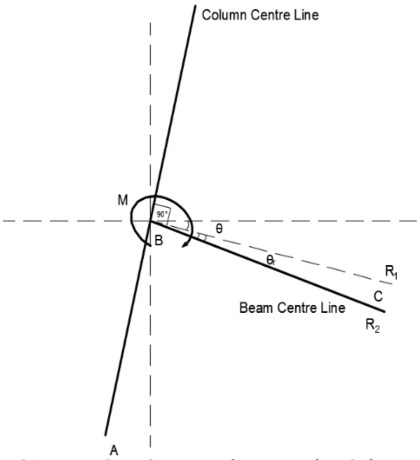

Fig. 2 (b) Centre line diagram of joint B after deformation[1]

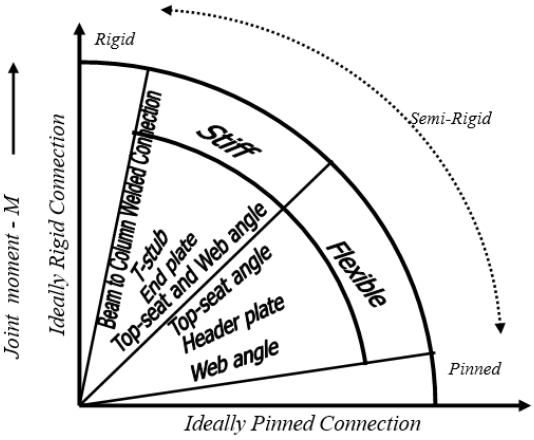

Fig. 3 Classification of structural steel connections[1]

B. Initial rotational stiffness

To find out the initial rotational stiffness, some assumptions are made as follows:

- Materials used for top and seat angle connections are linearly elastic and displacements are very small.
- In top angle fixed support is assumed near the fastener hole edge as shown in "Fig. 4" so that the top angle acts as a cantilever beam.
- The resisting moment is very small which is at the center of rotation, so that it can be neglected.

Based on the above assumptions and corresponding force P as shown in "Fig. 4", the horizontal displacement (Δ)[2] of heel of top angle is,

$$\Delta = \frac{P\,(g_1)^3}{3\,(EI_t)}\left(1 + \frac{0.78\,(t_t)^2}{(g_1)^2}\right) \tag{1}$$

g_1 is the distance between the center line of a top angle leg and the edge of a washer used. It can be calculated as,
$g_1 = g_t' - D/2 - t_t/2$ (Fig. 4) where, D = diameter of fastener (d_b) in case of rivets as fasteners and D = width of nut across the flat (W) in case of bolts as fasteners. g_t' is the gauge distance from top angle's heel to the center of fastener hole.
EI_t is the bending stiffness of angle's leg adjacent to column face. t_t is the thickness of top angle.
The relationship between horizontal displacement of top angle (Δ)[2] and relative angle of rotation (θr) is,

$$\Delta = d_1 \theta r \tag{2}$$

Also the relationship between moment (M)[2] and force at the leg of top angle (P) is,

$$M = d_1 P \tag{3}$$

d_1 is the distance between the center lines of a top angle leg and a seat angle leg. It can be calculated as,
$d_1 = d + t_t/2 + t_s/2$ where, d is the total depth of beam and t_s is the thickness of seat angle.
Substituting the equation (1) into equation (3) and using the relation of equation (2), the equation for the bending moment M becomes,

$$M = \frac{3\,(EI_t)}{(1+\frac{0.78\,(t_t)^2}{(g_1)^2})}\,\frac{(d_1)^2}{(g_1)^3}\,\theta_r \tag{4}$$

Definition of the rotational stiffness is, "moment required to cause unit rotation (M/θ)". So, by rearranging the terms in equation (4), the initial rotational stiffness (Rki)[2] is determined as,

$$Rki = \frac{3\,(EI_t)}{(1+\frac{0.78\,(t_t)^2}{(g_1)^2})}\,\frac{(d_1)^2}{(g_1)^3}$$

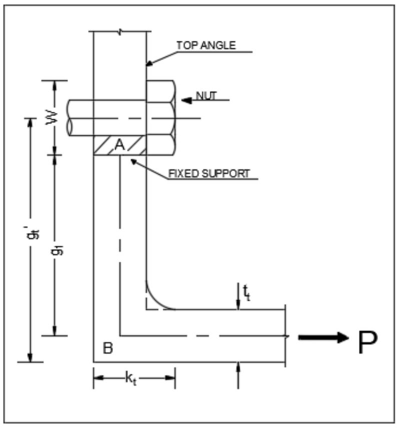

Fig. 4 Cantilever model of the top angle[2]

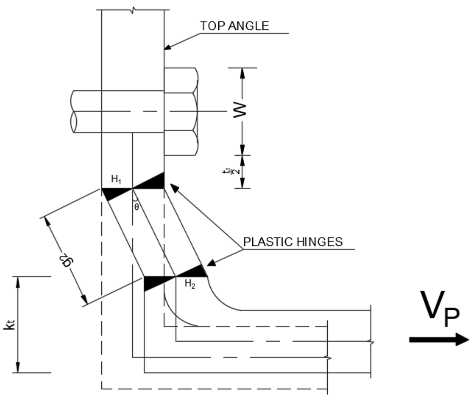

Fig. 5 Mechanism of the top angle at the ultimate condition[2]

C. Ultimate bending capacity

Assume the collapse mechanism of the top and seat angle connection as shown in "Fig. 5". The distance between two plastic hinges H_1 and H_2 are shorter as compared to the top angle's thickness. Therefore we should take into account the effect of shear force of the yielding of material. The equation for the plastic moment (Mp) and the shear force in the top angle's leg (Vp) is,

$$2\,Mp\,\theta = Vp\,g_2\,\theta$$

Using the Drucker's yield criteria,

$$\left(\frac{Mp}{M_0}\right) + \left(\frac{Vp}{V_0}\right)^4 = 1 \tag{7}$$

M_0 is the plastic bending moment capacity and V_0 is the plastic shear force capacity. It can be calculated as,

$$M_0 = fy\, l_t\,(t_t)^2/4 \tag{8}$$
$$V_0 = fy\, l_t\, t_t/2 \tag{9}$$

Where, fy is the yield strength of steel.
Substituting equations (6), (8) and (9) into equation (7) and rearranging the terms, we get,

$$\left(\frac{Vp}{V_0}\right)^4 + \frac{g_2}{t_t}\left(\frac{Vp}{V_0}\right) - 1 = 0 \tag{10}$$

g_2 is the distance between two plastic hinges. It can be calculated as,
$g_2 = g_t' - k_t - (D/2) - (t_t/2)$
where, k_t is the distance from the top angle heel to the toe of the fillet. This means, it is the addition of thickness of the angle and root radius of the angle.
Taking the moment about the center of rotation (about the point C in "Fig. 6"), the ultimate moment capacity (Mu)[2] is,

$$Mu = M_{0s} + Mp + Vp\ d_2 \quad (11)$$

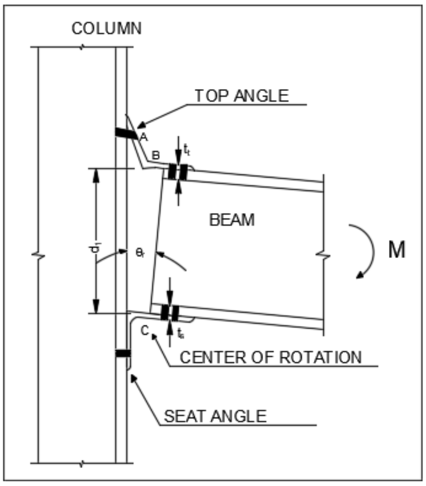

Fig. 6 Deflected configuration of top and seat angles at the elastic condition[2]

M_{0s} is the plastic bending moment capacity at corner point of seat angle. It can be calculated as,

$$M_{0s} = fy\ l_s\,(t_s)^2/4 \quad (12)$$

Mp is the plastic moment capacity at corner point of top angle. By rearranging the terms in equation (6), we can get the value of Mp. Vp is ultimate shear force in the top angle's leg and it can be determined by solving the equation (10).

d_2 is the distance between the center line of a seat angle and the plastic hinge developed at a top angle leg. It can be calculated as,

$$d_2 = d + t_s/2 + k_t \quad (13)$$

D. Modeling of the moment-rotation relationship

Using the initial rotational stiffness (Rki) and the ultimate moment capacity (Mu), the moment-rotation (M-θr) relationship of the semi-rigid connection can be represented by,

$$M = \frac{R_1\,\theta_r}{[1+(\theta_r/\theta_0)^n]^{1/n}} + Rkp\ \theta_r \quad (14)$$

Rkp is the plastic rotational stiffness. R_1 is the difference of initial rotational stiffness and plastic rotational stiffness. θ_0 is a reference plastic rotation, can be calculate as (Mu/Rki). n is the shape parameter.

For an elastic perfectly moment-rotation cure, plastic rotational stiffness is zero, Rkp = 0. So the equation (14) reduces to,

$$M = \frac{Rki\ \theta_r}{[1+(\theta_r/\theta_0)^n]^{1/n}} \quad (15)$$

Equation (15) represents the moment-rotation relationship for semi-rigid connection.

III. Literature Relation Verification

To verify the moment-rotation relationship curve of the top and seat angle connection, following data was assumed:

ISA 0.15 x 0.115 x 0.012 meter is used as top and seat angle. ISMB 350 is used for beam section. Yield strength of steel is taken as 250000 kN/m^2. Modulus of elasticity (E) is taken as 200 x 10^6 kN/m^2. M16 bolts are used as fasteners having nut diameter (D) 0.024 meter across the flat. The values of relative angle of rotation (θr) is taken as ranging from 0 to 4 degrees and converted in radians. The values of shape parameter (n) is taken as ranging from 0.8 to 1.8. Using the equation (15) and all above data, moment-rotation relationship curve of the top and seat angle connection is verified and the curve is shown below in "Fig. 7".

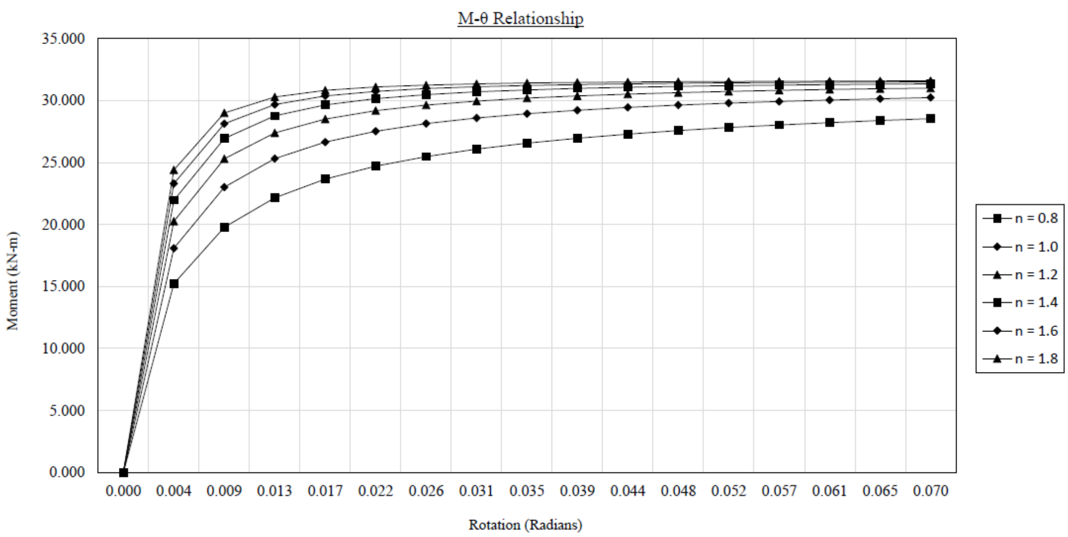

Fig. 7 Moment-rotation relationship curve for semi-rigid connection

After finding the moment rotation relationship for introduced beam section and using top and seat angle connection, it is further used in structure model to check the practical feasibility and following results are extracted.

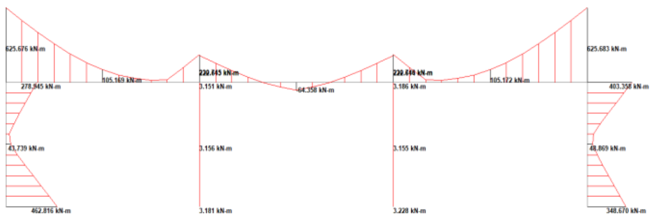

Fig. 8 Moment diagram of rigid frame

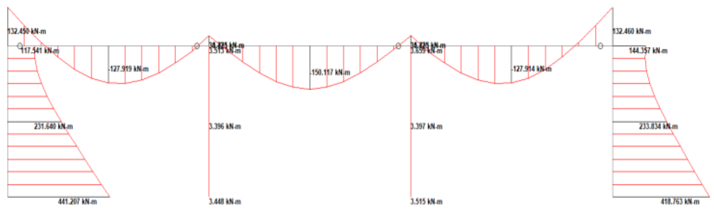

Fig. 9 Moment diagram of Semi rigid frame

Fig. 8 and 9 shows the bending moment diagram for same frame in structure and for same load combination of governing seismic case.

Fig. 8 is showing bending moment diagram for rigid connection frame in which support moment is very high than span moment hence beam member will be designed for heavy support moment. It can be observed that rigid connection attracts more moment and make the joint moment heavy. On opposite side fig. 9 shows the bending moment diagram for semi rigid frame. After providing connection flexibility support moment can be transferred to the span of beam. Hence the beam member will be designed for evenly distributed moment, hence economy of section can be achieved.

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