JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

THE HY INTEGRAL TRANSFORM FOR HANDLING EXPONENTIAL GROWTH AND DECAY PROBLEMS

D. P. Patil[1], Shaikh Areen Fatema GulamGaus[2], More Neha Kiran[3], Shaikh Jaweria Rashid[4]

[1]Professor, Department of Mathematics, K.R.T. Arts, B.H. Commerce and A.M. Science College, Nashik.

[2,3,4] M.Sc. Student, Department of Mathematics, K.R.T. Arts, B.H. Commerce and A.M. Science College, Nashik.

Abstract:

One of the most important applications of exponential functions involves growth and decay models. These equations have been solved by using many methods. Same has been solved by using Laplace Transform, Sumudu Transform, Sawi Transform, Tarig Transform, Aboodh Transform and Kamal Transform, Kushre transform, etc. In this paper we use recently developed HY Transform for solving exponential growth and decay problems. Some problems are solved by using HY transform, this shows that HY transform is useful and effective for solving such equations.

Keywords: Integral transform, HY transform, Exponential growth and decay problems.

1. Introduction:

Many quantities in the universe grow or decay at a rate proportional to their size. For example a colony of bacteria may double every hour. If the size of the colony after t hours is given by $y(t)$, then we can express this information in the mathematical language in the form of a first order differential equation

$$\frac{dy}{dt} = 2y$$

The quantity y that grows or decays at a rate proportional to its size is governed by a first order differential equation,

$$\frac{dy}{dt} = ky$$

If $k < 0$ then the above equation is called the law of natural decay and if If $k > 0$ then the above equation is called the law of natural growth. This equation is solved by separation of variable method. Integral transforms plays an important role in solving differential equations.

Nowadays, many researchers are engaged in introducing various types of integral transforms. Recently, in October 2021 S. S. Khakale and D. P. Patil [1] introduced Soham transform, which is defined for the function as

$$S[f(t)] = P(v) = \frac{1}{v}\int_0^{\infty} e^{-v^{\alpha}t} f(t)dt$$

where α is non zero real number.

Recently, in September 2021 Kushare and Patil [2] introduced Kushare transform, for simplifying the process of obtaining solution of ordinary and partial differential equations in the time domain. Many researchers are interested to use newly developed integral transform in various fields. Some of them are as follows, Recently, in January 2022 R. S. Sanap and D. P. Patil [3] obtain the solution of Newton's Law of Cooling by using Kushare transform. In October 2021 Sawi transform used in Bessel function by D. P. Patil [4] further Sawi transform of Error function is used to evaluate improper integral by D. P. Patil [5]. Further D. P. Patil [6] used Laplace and Shehu transform in Chemical Sciences, Sawi transform and its convolution theorem is applied to solve wave equation by Patil [7]. D. P. Patil also [8] used Mahgoub transform for getting the solution OR Parabolic boundary value problems. Author Patil [9] also used double Laplace and double Sumudu transform to solve the wave equation. Dualities between various double integral transforms are obtained by D. P. Patil [10]. Laplace, Elzaki and Mahgoub transforms are used for solving system of first order and first-degree differential equations by Kushare and Patil [11]. Dr. Patil [12] also used Aboodh and Mahgoub transform in boundary value problems of system ordinary differential equations recently in April 2022, Nikam, Shirsath, Aher and Patil [13] used Kushare transform in growth and decay problems. Patil [14] compared Laplace, Sumudu, Aboodh, Elzaki and Mahgoub transform in evaluating boundary value problems. Further Patil [15] used double Mahgoub transform for solving parabolic boundary value problems. D. P. Patil and Rathi sisters [16] used Soham theansform for solving system of differential equations. Soham transform is also used for solving volterra integral equations of first kind by D. P. Patil etal [17]. In May 2022, D. P.Patil et al solved Volterra integral equation of first kind by using Anuj transform[18]. Recently in June 2022, D. P.Patil et al applied general integral transform of error function for evaluating improper integrals[19].

In this paper we use HY transform for solving the exponential growth and decay problems. Paper is organized as follows; in second section definition of HY transform and some preliminary results are given. Third section is reserved for handling growth and decay problems. Applications are in fourth section and conclusion is drawn in last section.

2. Preliminary:

In this section we state preliminary concepts required to solve exponential growth and decay problems.

2.1. Definition of the HY Transform,[20]

The set A of functions for exponential order is defined as:

$A = \{f(t): \exists N, \lambda_1, \lambda_2 > 0, |f(t)| > Me^{i\lambda j|t|}, t \in (-1)^{j-i} \times (0, \infty], i \in \mathbb{C}, j = 1,2\}$. (1)

For set A in equation (1). M is a finite random constant, λ_1, λ_2 could be finite or infinite and $f(t)$ is a function that is defined for all $t \geq 0$.

The new HY integral transform is defined as:

$$HY\{f(t)\} = \frac{s}{v}\int_{t=0}^{\infty} f(t)\, e^{-i\left(\frac{s}{v}\right)t} dt \qquad (2)$$

2.2. Some useful formulae:[20]

1. $HY[1\} = \frac{1}{v}$
2. $HY[t^n] = \frac{n!}{v^{2n+1}}, \quad n = 1, 2, 3, \ldots \ldots$
3. $HY[t^p] = \frac{\Gamma(p+1)}{v^{2n+1}}, p > -1$
4. (linearity property) $HY[f(t) + g(t)] = HY[f(t)] + HY[g(t)]$
5. $HY[e^{kt}] = \frac{s}{is - kv}$

2.3. HY transform of derivatives [20]

Let $HY[f(t)] = F(v)$ then

$$HY\left[f^{(n)}(t)\right] = v^{2n} F(v) - \sum_{k=0}^{n-1} v^{2(n-k)-1} f^{(k)}(0) \quad , n \geq 1.$$

3. HY transformation for handling population growth and decay problem

Now we apply HY transform to handle growth and decay problems.

3.1. HY transform for handling population growth problem

The growth of the population (a plant, or a cell, or an organ, or a species) is mathematically expressed in terms of a first order ordinary linear differential equation as

$$\frac{dN}{dt} = KN \qquad \textbf{(3)}$$

With initial condition $N(t_o) = N_o$. (4)

Where K is a positive real number, N is the amount of population at time t and N_o is the initial population at time $t = t_o$

Equation (3) is known as the Malthusian law of population growth.

Taking HY transform on both sides of (3), we have

$$HY\left\{\frac{dN}{dt}\right\} = HY\{\mathrm{K\,N(t)}\}$$

$$\therefore HY\left\{\frac{dN}{dt}\right\} = \mathrm{K}HY\{\mathrm{N(t)}\} \qquad (5)$$

Now applying the property, HY transform of derivative of function, on (5), we have

$$-\frac{SN(0)}{V} + i\frac{S}{V}HY\{(\mathrm{N(t)}\} = \mathrm{K}HY\{\mathrm{N(t)}\}$$

$$i\frac{S}{V}HY\{\mathrm{N(t)}\} - \mathrm{K}HY\{\mathrm{N(t)}\} = \frac{SN(0)}{V}$$

$$\left(i\frac{S}{V} - k\right)HY\{\mathrm{N(t)}\} = \frac{SN(0)}{V}$$

$$\left(\frac{iS-vk}{V}\right)HY\{\mathrm{N(t)}\} = \frac{SN(0)}{V}$$

Using (4) in above condition gives

$$(iS - vk)HY\{\mathrm{N(t)}\} = S\mathrm{N_o}$$

$$HY\{\mathrm{N(t)}\} = \frac{S\mathrm{No}}{(iS-vk)} = N_0\left[\frac{S}{(iS-vk)}\right]$$

Applying inverse of HY transform on above equation, gives

$$HY^{-1}\{\,HY\{\mathrm{N(t)}\}\} = HY^{-1}\mathrm{N_0}\ \frac{S}{(iS-vk)}$$

$$\mathbf{N(t)} = N_0\boldsymbol{e^{kt}} \qquad \textbf{(6)}$$

which is the required amount of the population at time t.

3.2. HY integral Transform for handling decay problem.

The decay problem of the substance is defined mathematically by the following first order ordinary linear differential equation as

$$\frac{dN}{dt} = -KN \qquad (7)$$

With initial condition $N(t_o) = N_o$. (8)

Where K is a positive real number is the amount of substance at time t and No is the initial amount of substance at time $t = t_o$

In equation (7), the negative sign in the R.H.S. is taken because the mass of the substance is decreasing with time and dNso the derivative $\frac{dN}{dt}$ must be negative.

Taking HY transform on both sides of (7), we have

$$HY\left\{\frac{dN}{dt}\right\} = HY\{-\mathrm{K}\,\mathrm{N(t)}\}$$

$$HY\left\{\frac{dN}{dt}\right\} = -\mathrm{K}HY\{\mathrm{N(t)}\} \tag{9}$$

Now applying the property, HY transform of derivative of function, on (7), we have

$$-\frac{SN(0)}{V} + i\frac{S}{V}HY\{\mathrm{N(t)}\} = -\mathrm{K}HY\{\mathrm{N(t)}\}$$

$$i\frac{S}{V}HY\{\mathrm{N(t)}\} + \mathrm{K}HY\{\mathrm{N(t)}\} = \frac{SN(0)}{V}$$

$$\left(i\frac{S}{V} + k\right)HY\{\mathrm{N(t)}\} = \frac{SN(0)}{V}$$

$$\left(\frac{iS+vk}{V}\right)HY\{\mathrm{N(t)}\} = \frac{SN(0)}{V}$$

Using (8) in above condition gives

$$(iS + vk)HY\{\mathrm{N(t)}\} = SN_o$$

$$HY\{\mathrm{N(t)}\} = N_0\left[\frac{S}{(iS+vk)}\right]$$

Applying inverse of HY transform to above equation, gives

$$HY^{-1}\{HY\{\mathrm{N(t)}\} = N_0\mathrm{HY}^{-1}\left[\frac{S}{(iS+vk)}\right]$$

N(t) = $N_0\boldsymbol{e^{-kt}}$ **(9a)**

which is the required amount of the substance at time t.

4. Applications:

In this section, some applications are given in order to Demonstrate the effectiveness of HY transform for population Growth and decay problems.

Application: 1] The population of a city grows at a rate Proportional to the number of people presently living in the city. If after four years, the population has tripled, and after five years
The population is 50,000, estimate the number of people initially living in the city.
Solution: this problem can be written in a mathematical form as,

$$\frac{dN(t)}{dt} = \mathbf{KN(t)}$$

WhereN denotes the number of people presently living in the city at time t and K is constant of proportionality, consider N_o is the number of people initially living in the cityat time $t = t_o$

$$HY\left\{\frac{dN}{dt}\right\} = \mathrm{K}HY\{\mathrm{N(t)}\} \quad (10)$$

Now applying the property, HY transform of derivative of function, on (4.1), we have

$$-\frac{SN(0)}{V} + i\frac{S}{V}HY\{(\mathrm{N(t)}\} = \mathrm{K}HY\{\mathrm{N(t)}\}$$

$$\left(i\frac{S}{V} - k\right)HY\{\mathrm{N(t)}\} = \frac{SN(0)}{V}$$

Using (4) in above condition gives

$$(iS - vk)HY\{\mathrm{N(t)}\} = S\mathrm{N_o}$$

$$HY\{\mathrm{N(t)}\} = N_0\left[\frac{S}{(iS-vk)}\right]$$

Applying inverse of HY transform on above equation, gives

$$\mathrm{N(t)} = N_0e^{kt}$$

Now at t=4, N =3No, using in above equation gives,

$$3N_0 = N_0e^{4k}$$

$$\Rightarrow e^{4k} = 3$$

Applying $\log_e$on both side, $\Rightarrow\log_e(4k) = \log_e(3)$

$\Rightarrow$4k = 1.09861

$\Rightarrow$K= $\frac{1}{4}$(1.0981)

$\Rightarrow$K= 0.2746 **(11)**

Now using t=5, N=50,000 in equation (6) gives,

$\Rightarrow$50000 = N_0e^{5k}

Using equation (10) we have,

$\Rightarrow$50000 = $N_0e^{5(0.2746)}$

$\Rightarrow$50000 = $N_0e^{1.375}$

$\Rightarrow$50000 = $N_0(3.955)$

$\Rightarrow N_0 = \frac{50000}{3.955}$

$\Rightarrow N_0 =$ **12642**

Which, are the required number of people initially living in the city.

Application: 2] The population of a city grows at a rate Proportional to the number of people presently living in the city. If after two years, the population has doubled, and after three years
the population is 20,000, estimate the number of people initially living in the city.
Solution: this problem can be written in a mathematical form as,

$$\frac{dN(t)}{dt} = \mathbf{KN(t)}$$

where, N denotes the number of people presently living in the city at time t and K is constant of proportionality, consider N_o is the number of people initially living in the cityat time t = t_o

$$HY\left\{\frac{dN}{dt}\right\} = \mathrm{K}HY\{\mathrm{N(t)}\} \qquad (5)$$

Now applying the property, HY transform of derivative of function, on (5), we have

$$-\frac{SN(0)}{V} + i\frac{S}{V}HY\{(\mathrm{N(t)}\} = \mathrm{K}HY\{\mathrm{N(t)}\}$$

$$\left(i\frac{S}{V} - k\right)HY\{\mathrm{N(t)}\} = \frac{SN(0)}{V}$$

Using (4) in above condition gives

$$(iS - vk)HY\{\mathrm{N(t)}\} = SN_o$$

$$HY\{\mathrm{N(t)}\} = N_0\left[\frac{S}{(iS-vk)}\right]$$

Applying inverse of HY transform on above equation, gives

$N(t) = N_0e^{kt}$

⇒Now at t=2, N =$2N_0$, using in above equation gives,

$\Rightarrow 2N_0 = N_0e^{2k}$

$\Rightarrow e^{2k} = 2$

Applying $\log_e$on both side,

$\Rightarrow \log_e(2k) = \log_e(2)$

⇒2k = 0.693

$\Rightarrow K= \frac{1}{2}(0.693)$

⇒K= 0.347 **(12)**

Now using t=3, N=20,000 in equation (6) gives, 20000 = N_0e^{3k}

Using equation (12)

$\Rightarrow 20000 = N_0e^{3(0.347)}$

$\Rightarrow 20000 = N_0e^{1.041}$

$\Rightarrow 20000 = N_0(2.832)$

$\Rightarrow N_0= \frac{20000}{2.832}$

$\Rightarrow N_0= 7062$

Which are the required number of people initially living in the city.

Application: 3] A radioactive substance is known to decay at a Rate proportional to the amount present. If initially there is 100 Milligrams of the radioactive substance present and after six hours it is observed that the radioactive substance has lost 30 percent of its original mass, find the half-life of the radioactive substance.
Solution: This problem can be written in mathematical form as:

$$\frac{dN(t)}{dt} = -KN(t)$$

Where N denotes the number of radioactive substances at time t and K is the constant of proportionality.
Consider $N_0$is the initial amount of radioactive substances at time t=0
Taking HY transform on both sides of (7), we have

$$HY\left\{\frac{dN}{dt}\right\} = -KHY\{N(t)\}$$

Now applying the property, HY transform of derivative of function, on (7), we have

$$-\frac{SN(0)}{V} + i\frac{S}{V}HY\{N(t)\} = -KHY\{N(t)\}$$

$\left(i\frac{S}{V} + k\right)HY\{N(t)\} = \frac{SN(0)}{V}$

Using (8) in above condition gives

$(iS + vk)HY\{N(t)\} = SN_o$

$HY\{N(t)\} = N_0\left[\frac{S}{(iS+vk)}\right]$

Applying inverse of HY transform on above equation, gives

$\Rightarrow N(t) = N_0 e^{-kt}$

AtN_0= 100 mg

$\Rightarrow N(t) = 100e^{-kt}$

Now at t=2, the radioactive substance has lost 10% of its original mass 100mg. so N=100 – 10= 90using in above equation gives,

$90= 100e^{-2k} \quad \Rightarrow e^{-2k} = 0.9$

Applying $\log_e$on both side,

$(-2k) = \log_e(0.9) \quad \Rightarrow -2k = -(0.1053) \qquad \Rightarrow K= \frac{1}{2}(0.1053)$

$\Rightarrow K= 0.05268$ (13)

We required t when N= $\frac{N_0}{2} = \frac{100}{2} = 50$

So from equation (9) we have $\Rightarrow 50 = 100e^{-kt}$

Using equation (13)

$\Rightarrow 50 = 100\, e^{-t(0.05268)} \quad \Rightarrow 0.5 = e^{-0.05268t}$

Applying $\log_e$on both side,

$\Rightarrow -0.05268t = \log_e(0.5)$

$\Rightarrow t = \frac{-0.6931}{-0.05268} \quad \Rightarrow t = 13.157$ hours

Which are the required half time of the radioactive substance.

Application: 4] A radioactive substance is known to decay at a Rate proportional to the amount present. If initially there is 100 Milligrams of the radioactive substance present and after six hours it is observed that the radioactive substance has lost 30 Percent of its original mass, find the half-life of the radioactive substance.

Solution: This problem can be written in mathematical form as:

$$\frac{dN(t)}{dt} = -KN(t)$$

Where N denotes the number of radioactive substances at time t and K is the constant of proportionality. Consider $N_0$is the initial amount of radioactive substances at time t=0

Taking HY transform on both sides of (7), we have

$HY\left\{\frac{dN}{dt}\right\} = -K HY\{N(t)\}$

Now applying the property, HY transform of derivative of function, on (7), we have

$-\frac{SN(0)}{V} + i\frac{S}{V} HY\{N(t)\} = -K HY\{N(t)\}$

$\left(i\frac{S}{V} + k\right) HY\{N(t)\} = \frac{SN(0)}{V}$

Using (8) in above condition gives

$(iS + vk)HY\{N(t)\} = SN_o$

$HY\{N(t)\} = N_0\left[\frac{S}{(iS+vk)}\right]$

Applying inverse of HY transform on above equation, gives

$N(t) = N_0 e^{-kt}$

At N_0= 100 mg $\Rightarrow N(t) = 100e^{-kt}$

Now at t=6, the radioactive substance has lost 30% of its original mass 100mg. so N=100 – 30= 70using in above equation gives,

$\Rightarrow 70= 100e^{-6k} \Rightarrow e^{-6k} = 0.7$

Applying $\log_e$ on both side,

$\Rightarrow(-6k) = \log_e(0.7) \Rightarrow -6k = -(0.3567) \Rightarrow K= \frac{1}{6}(0.3567)$

$\Rightarrow K= 0.059$ (14)

We required t when $N= \frac{N_0}{2} = \frac{100}{2} = 50$

So from equation (9) we have $\Rightarrow 50 = 100e^{-kt}$

By substituting value of k in above equation we get,

$\Rightarrow 50 = 100\, e^{-t(0.059)} \Rightarrow 0.5=e^{-0.059t}$

Applying $\log_e$ on both side,

$\Rightarrow -0.059t = \log_e(0.5) \quad \Rightarrow t = \frac{-0.693}{-0.059}$

$\Rightarrow t = 11.75$ hours

which is the required half time of the radioactive substance.

5. CONCLUSION

In this paper, we have successfully discussed the HY transform for population growth and decay problems. Results show that HY transform is very useful integral transform for handling the population growth and decay problems. The scheme defined in this paper can be applied for finding the solution of continuous compound interest problems, heat conduction problems, vibrating beam problems, mixture problems and electric circuit problems.

REFRENCES

[1] D. P. Patil, S. S. Khakale, The new integral transform, "Soham transform", International Journal of Advances in Engineering and Management, Vol.3, Issue 10, Oct 2021, pp.126-132.

[2] S. R. Kushare, D. P. Patil and A. M. Takate, The new integral transform, "Kushare transform", International Journal of Advances in Engineering and Management, vol.3, Issue 9, Sept. 2019, pp. 1589-1592.

[3] R. S. Sanap and D. P. Patil, Kushare integral transform for Newton's law of Cooling, International Journal of Advances in Engineering and Management, vol.4, Issue 1, Jan 2022, pp. 166-170

[4] D. P. Patil, Application of Sawi transform in Bessel functions, Aayushi International Interdisciplinary Research Journal, Special Issue No. 86, pp. 171-175.

[5] D. P. Patil, Applications of Sawi transform of error function evaluating improper integral, Journal of Research and Development, Vol.11, Issue 20, June 2021, pp.41-65.

[6] D. P. Patil Applications of integral transform in Chemical Sciences, Aayushi International, Interdisciplinary Research Journal, special Issue No.88, pp.437-441.

[7] D. P. Patil, Sawi transform and Convolution theorem for initial boundary value problems (wave equations), Journal of Research and Development Vol.11, Issue 14, June 2021, pp.133-136.

[8] D. P. Patil, Applications of Mahgoub transform in parabolic boundary value problems, International Journal of current Advanced Research, Vol.9, Issue 4(c), April 2020, pp.21949-21957.

[9] D. P. Patil, Solution of wave equation by double Laplace and double Sumudu transform, Vidyabharati International Interdisciplinary Research Journal, Special Issue IVCIMS 2021, Aug 2021, pp.135-138.

[10] D. P. Patil, Dualities between double integral transforms, International Advanced Journal in Science, Engineering and Technology, Vol.7, Issue 6, June 2020, pp.74-82.

[11] S. R. Kushare, D. P. Patil, A. M. Takate, comparison between Laplace, Elzaki and Mahgoub transform for solving system of first order and first degree differential equations, Vidyabharati International Interdisciplinary Research Journal special Issue IVCIMS, Aug 2021, pp. 139-144.

[12] D. P. Patil, Aboodh and Mahgoub transform in boundary value problem of system of ordinary differential equations, International Journal of Advanced Research in Science, Communication and Technology, Vol.6, Issue 1, June 2021, pp.67-75.

[13] D. P. Patil, P. S. Nikam, S. D. Shirsath, A. T. Aher, Kushare Transform for solving the problem on growth and decay, Journal of Engineering Technologies and Innovative Research(JETIR), Vol.9, Issue 4, April 2022, pp.4317-4323.

[14] D. P. Patil, comparative study of Laplace, Sumudu, Aboodh, Elzaki and Mahgoub transform and Applications in boundary and value problems, International Journal of research and analytical reviews, Vol.5, Issue 4, Dec 2018, pp.22-26.

[15] D. P. Patil, Double Mahgoub transform for the solution of parabolic boundary value problems, Journal of Mathematics and Statistics, Vol.4, Issue 2, pp.28-36.

[16] D. P. Patil, Shweta Rathi and Shrutika Rathi, The new integral transform Soham thransform for system of differential equations, International Journal of Advances in Engineering and Management, Vol. 4, Issue 5 , May 2022, PP. 1675- 1678.

[17] D. P. Patil, Y. S. Suryawanshi, M. D. Nehete, Application of Soham transform for solving Volterra integral equations of first kind, International Advanced Research Journal in Science , Engineering and Technology, Vol. 9, Issue 4, April 2022.

[18] D. P. Patil, P. D. Shinde, G. K. Tile, Volterra integral equation of first kind by using Anuj transform, , International Journal of Advances in Engineering and Management, Vol. 4, Issue 5 , May 2022, PP. 917- 920.

[19] D. P. Patil, K. S. Kandekar and T. V. Zankar, Application of general integral transform of error function for evaluating improper integrals, International Journal of Advances in Engineering and Management, Vol. 4, Issue 6 , June 2022.

[20] Arya Wiwatwanich and Duangkamol poltem, On convolution property of HY transform and its applications, J. Math. Comput. Sci. 2022.