JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

BONDAGE NUMBERS FOR INDEPENDENCE IN GRAPHS

Dr. D.D. PANDYA

G.E.C, Rajkot, GUJARAT (INDIA)

Email: drddp.gec@gmail.com

Abstract

Fink et al. [1] introduced the bondage number of a graph in 1990 with some applications in mind. Several results related to bondage number have been appeared in an article in [2],[3],[4]. We introduce the Concept of Bondage Number for the Independence Number of the Graph. We call them Indepen- dence Bondage Number . We proved several theorems regarding these param- eters and deduced that the Independence Bondage Number of a Graph G does not exceed than the Independence Number of graph G. Some examples have also been given.

Keywords: Independent Set, Maximum Independent Set, Independence Num-ber, Independence Bondage Number, Independence Reinforcement Number.

AMS subject classification number: 05C69.

1. Introduction

Independent sets plays an important role in Graph theory and areaslike discrete optimization. they appear in matching theory, coloring of graphs and in trees. Independent sets were introduced into the communication theory on noisy channels. Bondage Number and Reinforcement Number of graph has been first Introduced by Fink[1]. Several Research papers have been published regarding these two parameters. In this paper we consider a similar aspect for Independence Number of a given graph. To do this we use those vertices whose removal does not change the Independence number. We also take into account the number of maximum Independent sets to find an upper bound forIndependence Reinforcement number of a given graph.

1. Definitions of Independent sets in Graph

Definition 1.1 Independent Set: Let G be a graph and S be a subset of V(G), then S is said to be an independent set, if any two distinct vertices of S are non adjacent.

Definition 1.2 Maximal Indepedent Set: An Independent set S is said to be a maximal independent if it is not properly contain in any independent set.

Definition 1.3 Maximum Indepedent Set: An Independent Set of Maximum size is called a maximum Independent Set.

Definition 1.4 Independence Number: The Cardinality of a maximum In- dependent Set is called the Independence Number of the graph G and it is denoted as $\beta_0(G)$.

2. Independence Bondage Number of Graph

2.1 Definitions for Independence Bondage Number of Graph

Definition 2.1.1 Bondage Number[5]: The Smallest number of edges whoseremoval from a graph G causes the modified graph to have a larger Dominationnumber than G is called Bondage Number.

Definition 2.1.2 Independence Bondage Number: The Smallest number of edges whose removal from a graph G causes the modified graph to have a larger Independence number than G is called Independence Bondage Number.It is denoted by $I_b(G)$.

2.2 Results on Independence Bondage Number of Graph

Lemma 2.2.1 If K edges can be removed from a graph G to yield a subgraph H with $I_b(H) = r$ then $I_b(G) \leq K + r$.

Proof. Let G be a Graph. If we remove K edges from G then Suppose Re- sultant graph be H. Now $I_b(H) = r$. So , let H_1 be the graph obtained by removing r edges from H . Now we can see that $\gamma_1(H_1) > \gamma_1(H) \geq \gamma_1(G)$. So, $I_b(G) \leq K + r$.

Theorem 2.2.2 If G is a Graph then $I_b(G) \leq deg(u) + deg(v) - 1$ for every pair u and v of vertices which are adjacent.

Proof. Let G be a graph and u and v are adjacent vertices in graph G.Consider the graph $H = G - (E_v \cup E_v - uv)$ (where E_x denotes set of edges incident with vertex x). Let S be Maximum Independent set of H. It is obvious thatonly $u \in S$ or $v \in S$ as S is a Maximum Independent set. Now let H_1 be the Graph obtained by removing edge uv from graph H then $\gamma_1(H_1) > \gamma_1(H)$. So, $I_b(H) = 1$. So by lemma 3.2.1 $I_b(G) \leq [deg(u) - 1] + [deg(v) - 1] + 1 = deg(u) + deg(v) - 1$.

Theorem 2.2.3 If G is a Graph then $I_b(G) \leq deg(u) + deg(v) + deg(w) - 2$ for every pair u and v of vertices with $d(u, v) = 2$ and $w \in [N(u) \cap N(v)]$.

Proof. Let G be a Graph with $d(u, v) = 2$ and $w \in [N(u) \cap N(v)]$.

Consider the Graph $H = G - (E_u \cup E_v \cup E_w) - uw - vw$.

Let S be Maximum Independent set of graph H. Then it is obvious that$u \in S, v \in S$ and w does not belong to S as S is Maximum Independent Set. Now if we remove edge uw and vw from H and Let H_1 be resultant graphthen $\gamma_1(H_1) > \gamma_1(H)$. So $I_b(H) = 2$

So by lemma 3.2.1, $I_b(G) \leq [deg(u) - 1] + [deg(v) - 1] + [deg(w) - 2] + 2 =$
$deg(u) + deg(v) + deg(w) - 2$.

Notation. It is useful to partition the Vertex set V of G into two sets accord- ing to how their removal affects $\beta_0(G)$.

$V_I^- = [v \in V(G)/\beta_0(G - v) < \beta_0(G)]$.

$V_I^0 = [v \in V(G)/\beta_0(G - v) = \beta_0(G)]$.

Theorem 2.2.4 If G is a Graph then $I_b(G) \leq min[deg(v)/v \in V_I^0]$.

Proof. If $v \in V_I^0$ then $\beta_0(G - v) = \beta_0(G)$ and therefore there is a maximum Independent set S of graph G such that v does not belong to S (reader canrefer) [6]

So if we remove all edges incident with v then $S \cup v$ is an independent set in the graph obtained by removing these edges.

Thus Independence bondage number $I_b(G) \leq deg(v)$ and hence $I_b(G) \leq min[deg(v)/v \in V_I^0]$.

Note. It may be noted that in any graph G with at least one edge there is a vertex v such that $\beta_0(G - v) = \beta_0(G)$ or equivalently there is a vertex v which do not belong to some maximum Independent set of graph G. Therefore we have the following corollary.

Corollary 2.2.5 If G is a Graph with at least one edge then $I_b(G) \leq deg(v)$. where v does not belong to S for some Maximum Independent set S of graph G.

Theorem 2.2.6 If G is a Graph then

$I_b(G) \leq min[\beta_0(G), min(deg(v)/v \in V^0\ \ )\ ].$ $_I$

Proof. If $v \in V^0$ then $\beta_0(G - v) = \beta_0(G)$ and therefore there is a maximum Independent set S of graph G such that v does not belong to S

So if we remove $\beta_0(G)$ edges incident to v or $deg(v)$ edges incident with v whichever is minimum then $S \cup v$ is an independent set in the resultant graph.

Thus Independent bondage number

$I_b(G) \leq min[\beta_0(G), min(deg(v)/v \in V^0\ \ )].$ $_I$

Illustrations on Independence Bondage Number of Graph

Illustration 2.3.1 Here we give an illustration for which $I_b(G) = deg(u) + deg(v) - 1$ for some adjacent vertices u and v. which represents sharp upperbound.

Figure 2.3.1

So $I_b(P_5) = deg(u) + deg(v) - 1 = 1 + 2 - 1 = 1.$

Moreover, $I_b(G) \leq deg(v)$, v does not belong to S which also represents sharpupper bound.

Illustration 2.3.2 Here we give an illustration of a graph which representssharp upper bound according to theorem 3.2.3.

Figure 2.3.2

So $I_b(G) = deg(u) + deg(v) + deg(w) - 2 = 1 + 1 + 3 - 2 = 3.$

Illustration 2.3.3 Here we give an illustration of Peterson graph.

In peterson graph for every vertex v, $\beta_0(G - v) = \beta_0(G)$. And therefore

$I_b(G) \leq 3$.

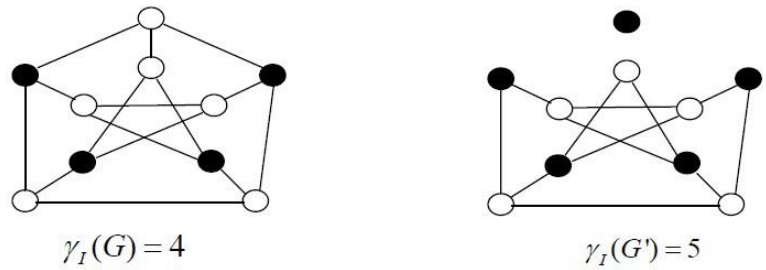

Figure 2.3.3

Illustration 2.3.4 Here we give an illustration of Complete graph for which $I_b(G) = 1$ as $\beta_0(G) = 1$.

Figure 2.3.4

Illustration 2.3.5 Here we give exact values for Independence Number, Inde- pendence Bondage Number and Independece Reinforcement Number of selected families of graphs.

Graph G	Independence Number ($\gamma_I(G)$)	Independence Bondage Number ($I_b(G)$)	Independence Reinforcement Number ($I_R(G)$)
P_n (path)	$\left\lceil \frac{n}{2} \right\rceil$	2, n=odd 1, n=even	1, n=odd 3, n=even $n \geq 3$
C_n (cycle)	$\left\lfloor \frac{n}{2} \right\rfloor$	1, n=odd 2, n=even	2, n=even 6, n=odd 5, n=5
S_n (star graph)	n-1	n-1	1
K_n (complete graph)	1	1	Not possible

Table 2.3.5

Illustration 2.3.6 Here we give exact values for Independence Number, Independence Bondage Number and Independece Reinforcement Number of Com-plete Grid graphs.

Grid graphs	Independence Number ($\gamma_I(G)$)	Independence Bondage Number ($I_b(G)$)	Independence Reinforcement Number ($I_R(G)$)
$P_2 \times P_n$	n	2	2
$P_3 \times P_n$	$\frac{3n+1}{2}, n = odd$ $\frac{3n}{2}, n = even$	2, n=even 3, n=0dd	2
$P_4 \times P_n$	2n	2	2
$P_5 \times P_n$	$\frac{5n+1}{2}, n = odd$ $\frac{5n}{2}, n = even$	2	2

Table 2.3.6

References

[1] J. F. Fink, M.S. Jacobson, L.F.Kinch and J.Roberts,(1990): *The bondage number of graph*, Discrete Math. 86 47-57..

[2] J. Huang and J.-M. Xu (2007): *The total domination and bondage numbers of extendedde bruijn and Kautz digraphs*,Computer and Mathematics with Applications, 53(8) , 1206-1213.

[3] J. Huang and J.-M. Xu(2007) : *The bondage numbers and efficient dominations of vertex- transitive graphs*, Discrete Math., 308(4) , 571-582.

[4] M. D. Elias, V. S. A. Subramanian, (2007): *Total bondage number of a grap* AKCE Int. J. Graphs Combin. 4 (2), 203-209.

[5] Teresa W. Haynes,Stephern T. Hedetniemi, Peter J. Slater (1998): *Changing and Un- changing*. Fundamental of Domination, Marshel Decar , 135-154.

[6] D.K.Thakkar and J.C.Bosamiya (2011): *Graph critical with respect to varients of domi- nation,* ,Ph.D thesis,Saurashtra university september - 2011 .

[7] N. Sridharan, M.D. Elias, V.S.A. Subramanian(2007): *upper bound for total bondagenumber of graphs* AKCE J. Graphs. Combin., 4, No. 2 , pp. 203-209.