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Softly ii-normal spaces

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Abstract: In this paper, we establish and study a new class of softly normal called softly ii-normal spaces. The relationships among normal, ii-normal and Int-normal spaces are investigated. Moreover, we established some functions related with softly ii-normal spaces and obtain some preservation theorems of softly ii-normal spaces.

Keywords: ii-open**,** softly ii-normal, normal spaces.

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1. Introduction

In 1937, Stone **[4]** established the concept of regular-open sets. In 2019, Mohammed and Abdullah. **[1]** established the concepts of ii-open, i-open sets and obtained their properties. In 2020, Hamant Kumar **[3]** established the concepts of ii-normal spaces and its properties.

2. Preliminaries

2.1 Definition. A subset A of a topological space X is said to be

1. **regular open [4]** if A = int(cl(A)).

2. **INT-open [1]** if

(a) $G \neq \phi$, X.

(b) INT(A) = G

3. **ii-open [1]** set if there exist an open set G belongs to τ, such that

(a) $G \neq \phi$. X

(b) A $\subset$ cl(A ∩ G)

(c) INT(A) = G.

4. The complement of regular open (resp. **ii-open**) set is said to be **regular closed** (resp. **ii-closed**). The intersection of all ii-closed sets containing A is called the **ii-closure of A** and denoted **ii-cl**(**A**).The union of all ii-open subsets of X which are contained in A is called the **ii-interior of A** and denoted by **ii-int**(**A**).

The family of all regular open (resp. regular closed, ii-open, ii-closed, int-open, int-closed) sets of a space X is denoted by R-O(X) (resp. R-C(X), ii-O(X), ii-C(X), int-O(X), int-C(X))).

We have the following implications for the properties of subsets :

open $\Rightarrow$ ii-open $\Rightarrow$ int-open

Where none of the implications is reversible as can be seen from the following examples.

2.2. Example. Let X = {p, q, r, s} and $\mathfrak{Z}$ = {X, ϕ, {q, s}, {p, q, s}, {q, r, s}}. Then

1) closed sets in X are X, ϕ, {p}, {r}, {p, r}.

2) int-open sets in X are ϕ, X, {p}, {q}, {p, q}, {p, q, r} {p, q, s}.

2.3. Example. Let X = {p, q, r, s} and $\mathfrak{Z}$ = {ϕ, {p}, {r}, {p, q}, {p, r}, {p, q, r}, X}. Then

1) closed sets in X are X, ϕ, {s}, {q, s}, {r, s}, {p, r, s}, {q, r, s}.

2) Int-closed sets in X are X, ϕ, {p}, {q}, {r}, {s}, {p, q}, {p, r}, {p, s}, {q, r},{q, s}, {p, q, s}, {q, r, s}.

2.4. Example. Let X = {p, q, r} and $\mathfrak{Z}$ = {X, ϕ, {p}, {q}, {p, q}}. Then

1) closed sets in X are X, ϕ, {r}, {p, r}, {q, r}.

2) ii-open sets in X are ϕ, X, {p}, {q}, {p, q}, {p, r} {q, s}.

3. Softly ii-Normal Spaces

3.1. Definition. A topological space X is said to be **softly ii-normal** if for every pair of disjoint closed subsets H and K, one of which is π-closed and the other is regular closed, there exist disjoint ii-open sets U, V of X such that H $\subset$ U and K $\subset$ V.

3.2. Definition. A topological space X is said to be **int-normal [2]** if for every pair of disjoint closed subsets H, K of X, there exist disjoint INT-open sets U, V of X such that H $\subset$ U and K $\subset$ V.

3.3. Definition. A topological space X is said to be **ii-normal [3]** if for every pair of disjoint closed subsets H, K of X, there exist disjoint ii-open sets U, V of X such that $H \subset U$ and $K \subset V$.

3.4. Definition. A topological space X is said to be **softly int-normal** if for every pair of disjoint closed subsets H and K, one of which is π-closed and the other is regular closed, there exist disjoint int-open sets U, V of X such that $H \subset U$ and $K \subset V$.

3.4. Remark.

By the definitions and examples stated above, we have the following diagram:

softly normal ⇒ softly ii-normal ⇒ softly int-normal

3.5. Example. Let X = {p, q, r, s} and $\mathfrak{Z}$ = {τ, {p}, {q}, {r}, {p, q}, {p, r}, { q, r}, {p, q, r}, {p, q, s}, X }. The pair of disjoint closed subsets of X are A = {r} and B = {s}. Also U = {q, r} and V = {p, s} are disjoint int-open sets such that $A \subset U$ and $B \subset V$. Hence X is softly int-normal but not normal.

3.6. Example. Let X = {p, q, r, s} and $\mathfrak{Z}$ = {ϕ, {r}, {s}, {q, r}, {r, s}, {p, r, s}, {q, r, s}, X}. The pair of disjoint closed subsets of X are A = {p} and B = {q}. Also U = {p, r} and V = {q, s} are disjoint int- open sets such that $A \subset U$ and $B \subset V$. Hence X is softly int-normal but neither softly normal nor normal.

3.7. Example. Let X = {p, q, r, s} and $\mathfrak{Z}$ = {ϕ, {q}, {s}, {q, s}, {p, q, s}, {q, r, s}, X}. The pair of disjoint closed subsets of X are A = {p} and B = {r}. Also U = {p, s} and V = {q, r} are disjoint int-open sets such that $A \subset U$ and $B \subset V$. Hence X is int-normal but not normal. Since U and V are not open.

3.8. Example. Let X = {p, q, r, s} and $\mathfrak{Z}$ = {ϕ, {p}, {q, r, s}, X}. The pair of disjoint closed subsets of X are A = {p} and B = {q, r, s}. Also U = {p} and V = {q, r, s} are disjoint ii-open sets such that $A \subset U$ and $B \subset V$. Hence X is softly normal as well as softly ii-normal. Since U and V both are open sets.

3.8. Theorem. For a topological space X, the following are equivalent:

(1) X is softly ii-normal.

(2) For each π-closed set G and each regularly open set H with $G \subset H$,

(3) There exist an ii-open set U s. t. $G \subset U \subset$ ii-cl(U) $\subset H$. For each regularly closed set G and each π-open set H with $G \subset H$, there exist an int-open set U s. t. $G \subset U \subset$ ii-cl(U) $\subset H$.

(4) For any pair possessing of disjoint closed set G and H, one of which is π-closed and the other is regularly closed, there exist ii-open sets U and V s. t. $G \subset U$ and $H \subset V$ and also int-cl(U) $\cap$ ii-cl(V) = ϕ.

Proof: (1) $\Rightarrow$ (2).

Suppose (1). Let G and H are any pair of disjoint closed sets one of which is π-closed and the other is regularly open s. t. $G \subset H$. Then $G \cap (X - H) = \phi$ where $(X - H)$ is regularly closed. Then there exist disjoint ii-open sets U, V s. t. $G \subset U$ and $(X - H) \subset V$. Because $U \cap V = \phi$, then ii-cl(U) $\cap V = \phi$. So ii-cl(U) $\subset (X - V) \subset (X - (X - H)) = H$. Hence $G \subset U \subset$ ii-cl(U) $\subset$ H.

(2) $\Rightarrow$ (3).

Suppose (2).

Consider G is any regularly closed set and H is any π-open set s. t. $G \subset H$. Then $(X - H) \subset (X - G)$, where $(X - H)$ is π-closed and $(X - G)$ is regularly open. Therefore by (2), there exist an ii-open set M s. t. $(X - H) \subset M \subset$ ii-cl(M) $\subset (X - G)$. So $G \subset$ (X - ii-cl(M)) $\subset (X - M) \subset H$. Therefore, we suppose U = (X – ii-cl(M)), which is int-open and since $M \subset$ ii-cl(M), then (x – ii-cl(M)) $\subset (X - M)$. Therefore $U \subset (X - M)$, thus ii-cl(U) $\subset$ ii-cl(X – M) = $(X - M) \subset H$.

(3) $\Rightarrow$ (4).

Suppose (3).

Consider G and H are any regular closed and π-closed set with $G \cap H = \phi$. Then $G \subset (X - H)$ where $(X - H)$ is π-open. By (3), there exists an ii-open set U s. t. $A \subset U \subset$ ii-cl(U) $\subset (X - H)$. But ii-cl(U) is ii-closed, using (3), again we have an int-open set M such that $G \subset U \subset$ ii-cl(U) $\subset M \subset$ ii-cl(M) $\subset (X - H)$. Suppose (X – ii-cl(M)), then V is ii-open set and $H \subset V$. We get (X – ii-cl(M)) $\subset (X - M)$, so $V \subset (X - M)$, therefore ii-cl(V) $\subset$ ii-cl(X – M) = $(X - M)$. Then we get ii-cl(U) $\subset M$ and ii-cl(V) $\subset (X - M)$. So ii-cl(U) $\cap$ ii-cl(V) = ϕ.

(4) $\Rightarrow$ (1) is obvious.

3.9. Theorem. For a topological space X, the following are equivalent :

(1) X is softly ii-normal.

(2) For any pair of open sets A and B, one of which is π-open and the other is regular open their union is X, there exist an ii-closed sets U, V such that $A \subset U$ and $B \subset V$ and U union V = X.

(3) For each π-closed set G and each regular open set H consisting G, there is an ii-open set V such that $G \subset V \subset$ ii-cl(V) $\subset H$.

Proof. (1) ⇒ (2), (2) ⇒ (3), (3) ⇒ (1).

(1) ⇒ (2). Suppose A, B are π-open and regular open sets respectively in a softly ii-normal space X such that A union B = X. Then X – A and X – B are π- closed and regular closed sets of X with $(X – A) \cap (X – B) = \phi$. By softly ii-normal of X, there exist disjoint ii-open sets A_1, B_1 such that $X – A \subset A_1$ and $X – B \subset B_1$. Suppose $U = X – A_1$ and $V = X – B_1$. Then U and V are ii-closed sets such that $U \subset A$, $V \subset B$ and U union V = X.

(2) ⇒ (3) and (3) ⇒ (1) are clear. By using previous theorem (3.8), this is easy to produce the following theorem, which is a Urysohns lemma version to softly ii-normal. A proof can be introduced by a similar manner of the normal case.

3.10. Theorem. A topological space X is soft ii-normality iff for each pair of disjoint closed sets G and H of which is π-closed and the other is regularly closed, there exist a continuous function g on X into [a, b], with its usual topology, s. t. g(G) = {a} and g(H) = {b}. As above this can be see that the inverse image of a π-closed set under an open continuous function is π-closed and the inverse image of a regularly closed set under an open continuous function is regularly closed.

3.11. Theorem. Suppose X is a soft ii-normality and $g : X \rightarrow Y$ is an open continuous one-one function. Then g(X) is a soft ii-normality.

Proof: Suppose G be any π-closed subset in g(X) and consider H be any regularly closed subset in g(X) s. t. $G \cap H = \phi$. Then g^{-1} (G) is a π-closed set in X, which is disjoint from the regularly closed set g^{-1} (H). Because X is softly ii-normal, there are two disjoint open sets S and T s. t. $g^{-1}(G) \subset S$ and $g^{-1}(H) \subset T$. Because g is injective and open. Hence the result follows. **Corollary:** Softly ii-normal space is a topological property.

3.12. Lemma: Suppose N be a closed domain subspace of a topological space X. If G be an ii-open set in X, then $G \cap N$ is ii-open set in N.

3.13. Theorem. A closed domain subspace of a soft ii-normality is soft ii-normality.

Proof: Suppose N be a closed domain subspace of a soft ii-normality X. Suppose G, H be any disjoint closed sets in N s. t. G and H are regularly closed and π-closed respectively. Then G and H are disjoint closed sets in X such that G is regularly closed and H is π-closed in X. By softly ii-normal space of X, there exist disjoint ii-open sets S and T of X such that $G \subset S$ and $H \subset T$. By using the **Lemma (3.12),** we get $S \cap N$, $T \cap N$ are disjoint ii-open sets in N such that $G \subset S \cap N$ and $H \subset T \cap N$. Therefore, N is softly ii-normal subspace. Because each closed and open (clopen) subset is a closed domain, then we get the following corollary.

Corollary: Softly ii-normal space is a hereditary under the clopen subspaces.

REFERENCES

1.Amir Mohammed and Beyda Abdullah, ii-open sets in topological spaces, International Mathematical Forum, Vol. 14, (2019), 41-48.

2. Bhopal Singh Sharma and Anuj Kumar, Int-normal spaces in topological spaces, International Journal of Mathematics Trends and Technology (IJMTT), Vol. 66, (2020), 67-70.

3. Hamant Kumar, ii-normal spaces in topological spaces, *Journal of Emerging Technologies and Innovative Research (JETIR)*, **7**(2020), no.8, 1510-1520.

4. M. H. Stone, Applications of the theory of Boolean rings to general topology. Trans. Amer. Math. Soc. **41**(1937), 375-381.