JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

CONTINUED FRACTION FOR GENERALIZED THIRD AND EIGHTH ORDER MOCK THETA FUNCTIONS

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ABSTRACT : A continued fraction representation has been obtained for Fine's function using certain linear q-difference equation. This in turn procedure the continued fraction for generalized eight order mock theta functions and further a continued fraction representation for generalized third order mock theta functions, has been given by using Resonance formula.

Keywords : Basic hypergeometric series**,** Generalized mock theta functions, Continued fractions.

I. INTRODUCTION

S. Ramanujan in his last letter to G.H. Hardy [9, pp 354-355] introduced seventeen functions whom he called mock theta functions, as they were not theta functions. He stated two conditions for a function to be a mock theta function:

(0) For every root of unity ζ, there is a θ-function $\theta_\zeta(q)$ such that the difference f $(q) - \theta_\zeta(q)$ is bounded as $q\rightarrow \zeta$ radially.

(1) There is no single θ-function which works for all ζ i.e., for every θ-function (q) there is some root of unity ζ for which difference f $(q) - \theta(q)$ is unbounded as $q \rightarrow \zeta$ radially.

Of the seventeen mock theta functions, four were of third order, ten of fifth order in two groups with five functions in each group and three of seventh order. Ramanujan did not specify what he meant by the order of a mock theta function. Later Watson [15] added three more third order mock theta functions, making the four third order mock theta functions to seven. G.E. Andrews [13] while visiting Trinity College Cambridge University discovered some notebooks of Ramanujan, and called it the "Lost" Notebook. In the Notebook Andrews found seven more mock theta functions and some identities and Andrews and Hickerson [14] called them of sixth order.

The third order mock theta functions of Ramanujan's are

$$f(q) = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(-q)_n^2} ,$$

$$\varphi(q) = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(-q^2; q^2)_n} ,$$

$$\psi(q) = \sum_{n=1}^{\infty} \frac{q^{n^2}}{(q ;q^2)_n},$$

$$\chi(q) = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(1 - q + q^2) \dots \dots (1 - q^n + q^{2n})} ,$$

$$\omega(q) = \sum_{n=0}^{\infty} \frac{q^{2n(n+1)}}{(q;q^2)_{n+1}^2}$$

$$\vartheta(q) = \sum_{n=0}^{\infty} \frac{q^{n(n+1)}}{(-q; q^2)_{n+1}}$$

The eighth order mock theta functions are

$$S_0(q) = \sum_{n=0}^{\infty} \frac{q^{n^2}(-q;q^2)_n}{(-q^2;q^2)_n} ,$$

$$S_1(q) = \sum_{n=0}^{\infty} \frac{q^{n^2+2n}(-q;q^2)_n}{(-q^2;q^2)_n},$$

$$T_0(q) = \sum_{n=0}^{\infty} \frac{q^{(n+1)(n+2)}(-q^2;q^2)_n}{(-q;q^2)_{n+1}},$$

$$T_1(q) = \sum_{n=0}^{\infty} \frac{q^{n^2+n}(-q^2;q^2)_n}{(-q;q^2)_{n+1}},$$

$$U_0(q) = \sum_{n=0}^{\infty} \frac{q^{n^2}(-q;q^2)_n}{(-q^4;q^4)_n} ,$$

$$U_1(q) = \sum_{n=0}^{\infty} \frac{q^{(n+1)^2}(-q;q^2)_n}{(-q^2;q^4)_{n+1}} ,$$

$$V_0(q) = -1 + 2\sum_{n=0}^{\infty} \frac{q^{n^2}(-q; q^2)_n}{(q; q^2)_n}$$

$$= -1 + 2\sum_{n=0}^{\infty} \frac{q^{2n^2}(-q^2;q^4)_n}{(q;q^2)_{2n+1}}$$

$$V_1(q) = \sum_{n=0}^{\infty} \frac{q^{(n+1)^2}(-q; q^2)_n}{(q; q^2)_{n+1}}$$

$$= \sum_{n=0}^{\infty} \frac{q^{2n^2+2n+1}(-q^4; q^4)_n}{(q; q^2)_{2n+2}}$$

The generalized third order mock theta functions are [2];

$$f(t,\alpha,\beta,z;q)=\frac{1}{(t)_\infty}\sum_{n=0}^{\infty}\frac{(t)_n q^{n^2-4n+n\beta}\alpha^n z^{2n}}{(-z;q)_n(-\alpha z/q;q)_n}$$

$$\phi(t,\alpha,\beta,z;q)=\frac{1}{(t)_\infty}\sum_{n=0}^{\infty}\frac{(t)_n q^{n^2-3n+n\beta}z^{2n}}{(-\alpha z^2/q\,;q^2)_n}$$

$$\psi(t,\alpha,\beta,z;q)=\frac{1}{(t)_\infty}\sum_{n=0}^{\infty}\frac{(t)_n q^{n^2-n+n\beta}z^{2n+1}}{(\alpha z^2/q^2;q^2)_{n+1}},$$

$$\nu(t,\alpha,\beta,z;q)=\frac{1}{(t)_\infty}\sum_{n=0}^{\infty}\frac{(t)_n q^{n^2-2n+n\beta}z^{2n}}{(-\alpha^2 z^2/q^3;q^2)_{n+1}},$$

$$\omega(t,\alpha,\beta,z;q)=\frac{1}{(t)_\infty}\sum_{n=0}^{\infty}\frac{(t)_n q^{2n^2-5n-4+n\beta}\alpha^{2n}z^{4(n+1)}}{(z^2/q;q^2)_{n+1}(\alpha^2 z^2/q^3;q^2)_{n+1}}$$

$$\chi(t,\beta,z;q)=\frac{1}{(t)_\infty}\sum_{n=0}^{\infty}\frac{(t)_n q^{n^2-3n+n\beta}z^{2n}}{(\nu z;q)_n(-\nu^2 z;q)_n}$$

and

$$\rho(t,\beta,z;q)=\frac{z^4}{q^4}\frac{1}{(t)_\infty}\sum_{n=0}^{\infty}\frac{(t)_n q^{2n^2-3n+n\beta}z^{4n}}{(\nu^2 z^2/q;q^2)_{n+1}(\nu^{-2}z^2/q;q^2)_{n+1}}$$

Where $\nu=e^{\frac{\pi i}{3}}$.

For $\beta=1$ and $z=q$ we have generalized five third order mock theta functions namely f, ϕ, ψ, ν, ω of Andrews [13]. For $t=0$, $\beta=1$, $\alpha=q$ and $z=q$ the generalized functions f, ϕ, ψ and χ reduce to the third order mock theta functions of Ramanujan and ω, ν and ρ to the third order mock theta functions of Watson[15].

The generalized eighth order mock theta functions [17]:

$$S_0(t,\alpha,z;q)=\frac{1}{(t)_\infty}\sum_{n=0}^{\infty}\frac{(t)_n q^{n^2-2n+n\alpha}z^n(-z^2/q;q^2)_n}{(-z^2;q^2)_n}$$

$$S_1(t,\alpha,z;q)=\frac{1}{(t)_\infty}\sum_{n=0}^{\infty}\frac{(t)_n q^{n^2+n\alpha}z^n(-z^2/q;q^2)_n}{(-z^2;q^2)_n},$$

$$T_0(t,\alpha,z;q)=\frac{1}{(t)_\infty}\sum_{n=0}^{\infty}\frac{(t)_n q^{n^2+n+n\alpha}z^{n+2}(-z^2;q^2)_n}{(-z^2/q\,;q^2)_{n+1}},$$

$$T_1(t,\alpha,z;q)=\frac{1}{(t)_\infty}\sum_{n=0}^{\infty}\frac{(t)_n q^{n^2-n+n\alpha}z^n(-z^2;q^2)_n}{(-z^2/q;q^2)_{n+1}},$$

$$U_0(t,\alpha,z;q)=\frac{1}{(t)_\infty}\sum_{n=0}^{\infty}\frac{(t)_n q^{n^2-3n+n\alpha}z^{2n}(-z^2/q\,;q^2)_n}{(-z^2q^2;q^4)_n},$$

$$U_1(t,\alpha,z;q)=\frac{1}{(t)_\infty}\sum_{n=0}^{\infty}\frac{(t)_n q^{n^2-n+n\alpha}z^{2n+1}(-z^2/q\,;q^2)_n}{(-z^2;q^4)_{n+1}},$$

$$V_0(t,\alpha,z;q)=-1+\frac{2}{(t)_\infty}\sum_{n=0}^{\infty}\frac{(t)_n q^{n^2-2n+n\alpha}z^n(-z;q^2)_n}{(z^2/q\,;q^2)_n},$$

$$=-1+\frac{2}{(t)_\infty}\sum_{n=0}^{\infty}\frac{(t)_n q^{2n^2-3n+n\alpha}z^{2n}(-z^2;q^4)_n}{(z^2/q\,;q^2)_{2n+1}}$$

and

$$V_1(t,\alpha,z;q) = \frac{1}{(t)_\infty}\sum_{n=0}^{\infty}\frac{(t)_n q^{n^2+n\alpha} z^{n+1}(-z;q^2)_n}{(z^2/q\,;q^2)_{n+1}}$$

$$= \frac{1}{(t)_\infty}\sum_{n=0}^{\infty}\frac{(t)_n q^{2n^2-n+n\alpha} z^{2n+1}\left(-z^2q^2;q^4\right)_n}{(z^2/q;q^2)_{2n+2}} .$$

For $t = 0, \alpha = 1$ and $z = q$ these generalized functions reduce to eighth order mock theta functions of Gordon and McIntosh [16].

II. NOTATIONS

We shall use the following usual basic hypergeometric notations:

For $|q^k| < 1$,

$$(a;q^k)_n = \prod_{r=1}^{n}\left(1 - aq^{k(r-1)}\right), \quad n \geq 1$$

$$(a;q^k)_0 = 1,$$

$$(a;q^k)_\infty = \prod_{j=1}^{\infty}\left(1 - aq^{kj}\right)$$

$${}_A\phi_{A-1}\left[\begin{matrix} a_1, a_2, \ldots\ldots, a_A\,; \\ b_1, b_2, \ldots\ldots, b_{A-1}; q_1, \ z\end{matrix}\right]$$

$$= \sum_{n=0}^{\infty}\frac{(a_1;q_1)_n \ldots\ldots (a_A;q_1)_n \ z^n}{(b_1;q_1)_n \ldots\ldots (b_{A-1};q_1)_n \ (q_1;q_1)_n}, |z| < 1$$

III. CONTINUED FRACTION REPRESENTATION FOR GENERALIZED THIRD ORDER MOCK THETA FUNCTIONS

We shall give the continued fraction representation for generalized third order mock theta functions by using Resonance formula of Ramanujan [6]

$$\frac{\sum_{n=0}^{\infty}\dfrac{\left(1-\lambda q^{2n}\right)\left(\frac{-\lambda}{a}\right)_n (b)_n \left(\frac{-\lambda}{c}\right)_n (\lambda)_n \left(\frac{ac}{b}\right)^n q^{n^2+n}}{(q)_n \ (-cq)_n \left(\frac{\lambda q}{b}\right)_n \ (-aq)_n}}{(1-\lambda)\left(1+\frac{aq}{b}\right)\sum_{n=0}^{\infty}\dfrac{\left(1-\lambda q^{2n+1}\right)\left(\frac{-\lambda}{a}\right)_n (b)_n \left(\frac{-\lambda q}{c}\right)_n (\lambda q)^n \left(\frac{ac}{b}\right)^n q^{n^2+2n}}{(q)_n \ (-cq)_n \left(\frac{\lambda q}{b}\right)_{n+1} \ (-aq)_{n+1}}}$$

$$= \quad 1 + \cfrac{\left(1-\frac{1}{b}\right)(a+\lambda)q}{\left(1+\frac{aq}{b}\right) + \cfrac{\lambda q^2+cq}{1+\cfrac{\left(1-\frac{1}{bq}\right)\left(aq^2+\lambda q^3\right)}{\left(1+\frac{aq}{b}\right)+\frac{\lambda q^4+cq^2}{1}\ldots\ldots\ldots\ldots}}} \qquad \ldots \ldots\ldots (3.1)$$

(i) Representation of $f(t,\alpha,\beta,z;q)$ as continued fraction:

Letting $t=0\,,\ \lambda=0\,,a=\frac{z}{q}\,,b=q\,,c=\frac{\alpha z}{q^2}\,,\beta=1\,,\alpha=1$ in equation (3.1) we get the continued fraction for $f(t,\alpha,\beta,z;q)$ as follows:

$$\frac{\sum_{n=0}^{\infty}\frac{q^{n^2-3n}\,z^{2n}}{(-z;q)_n\left(\frac{-z}{q};q\right)_n}}{\sum_{n=0}^{\infty}\frac{q^{n^2-2n}\,z^{2n}}{\left(\frac{-z}{q};q\right)_n\ (-zq;q)_n}}=1+\cfrac{z\left(1-\frac{1}{q}\right)}{\left(1+\frac{z}{q}\right)+\cfrac{\frac{z}{q}}{1+\cfrac{zq\left(1-\frac{1}{q^2}\right)}{\left(1+\frac{z}{q}\right)+1\ldots\ldots}}}$$

$$\frac{f(0,1,1,z;q)}{\frac{1}{1+z}\left(1+\frac{z}{q}\right)\sum_{n=0}^{\infty}\frac{q^{n^2-3n}\,z^{2n}}{(-z;q)_n\left(\frac{-z}{q};q\right)_n}}=1+\cfrac{z\left(1-\frac{1}{q}\right)}{\left(1+\frac{z}{q}\right)+\cfrac{\frac{z}{q}}{1+\cfrac{zq\left(1-\frac{1}{q^2}\right)}{\left(1+\frac{z}{q}\right)+1\ldots\ldots}}}$$

(ii) Representation of $\phi(t,\alpha,\beta,z;q)$ as continued fraction:

Letting $t=0\,,\lambda=0\,,a=\frac{i\alpha^{\frac{1}{2}}z}{q^{\frac{3}{2}}}\,,b=q\,,c=\frac{-i\alpha^{\frac{1}{2}}z}{q^{\frac{3}{2}}}\,,\alpha=1\,,\beta=0$ in equation (3.1) we get, the continued fraction for $\emptyset(t,\alpha,\beta,z;q)$ as follows :-

$$\frac{\sum_{n=0}^{\infty}\frac{q^{n^2-3n}\,z^{2n}}{\left(\frac{-z^2}{q};q^2\right)_n}}{\left(1+\frac{i\alpha^{1/2}z}{q^{3/2}}\right)\sum_{n=0}^{\infty}\frac{q^{n^2-2n}\ z^{2n}}{\left(\frac{i\alpha^{1/2}z}{q^{3/2}}\right)_n\left(\frac{-i\alpha^{1/2}z}{q^{1/2}};q\right)_n}}=1+\cfrac{\left(1-\frac{1}{q}\right)\left(\frac{iz}{q^{1/2}}\right)}{\left(1+\frac{iz}{q^{3/2}}\right)-\cfrac{\frac{iz}{q^{3/2}}}{1+\cfrac{\left(1-\frac{1}{q^2}\right)\left(\frac{iz}{q^{-1/2}}\right)}{\left(1+\frac{iz}{q^{3/2}}\right)+\cdots}}}$$

$$\frac{\phi(0,1,0,z\,;q)}{\left(1+\frac{i\alpha^{1/2}z}{q^{3/2}}\right)\sum_{n=0}^{\infty}\frac{q^{n^2-2n}\ z^{2n}}{\left(\frac{i\alpha^{1/2}z}{q^{3/2}}\right)_n\left(\frac{-i\alpha^{1/2}z}{q^{1/2}};q\right)_n}}=1+\cfrac{\left(1-\frac{1}{q}\right)\left(\frac{iz}{q^{1/2}}\right)}{\left(1+\frac{iz}{q^{3/2}}\right)-\cfrac{\frac{iz}{q^{3/2}}}{1+\cfrac{\left(1-\frac{1}{q^2}\right)\left(\frac{iz}{q^{-1/2}}\right)}{\left(1+\frac{iz}{q^{3/2}}\right)+\cdots}}}$$

(iii) Representation of $\psi\,(t,\alpha,\beta,z;q)$ as continued fraction:

Letting $t=0,\lambda=0,a=\frac{\alpha^{1/2}z}{q}\,,c=\frac{-\alpha^{1/2}z}{q}\,,b=q\,,\lambda=0,\alpha=-1,\beta=-1$ in equation (3.1) we get, the continued fraction for $\psi(t,\alpha,\beta,z;q)$ as follows:-

$$\frac{\frac{z}{\left(1+\frac{z^2}{q^2}\right)}\sum_{n=0}^{\infty}\frac{q^{n^2-2n}\,z^{2n}}{(-z^2\,;q^2)_n}}{\left(1+\frac{iz}{q}\right)\left(\frac{1}{1+iz}\right)\sum_{n=0}^{\infty}\frac{q^{n^2-n}\ z^{2n}}{(iz\,;q)_n\,(-iz\,;q)_n}}=1+\cfrac{\left(1-\frac{1}{q}\right)(iz)}{\left(1+\frac{iz}{q}\right)-\cfrac{\frac{iz}{q}}{1+\cfrac{\left(1-\frac{1}{q^2}\right)(izq)}{\left(1+\frac{iz}{q}\right)-\ izq}}}$$

$$\frac{\psi(0,-1,-1,z\,;q)}{\left(1+\frac{iz}{q}\right)\left(\frac{1}{1+iz}\right)\sum_{n=0}^{\infty}\frac{q^{n^2-n}\ z^{2n}}{(iz\,;q)_n\,(-iz\,;q)_n}} = 1+\cfrac{\left(1-\frac{1}{q}\right)(iz)}{\left(1+\frac{iz}{q}\right)-\cfrac{\frac{iz}{q}}{1+\cfrac{\left(1-\frac{1}{q^2}\right)(izq)}{\left(1+\frac{iz}{q}\right)-\ izq}}}$$

(iv) Representation of $\boldsymbol{\nu}\,(\boldsymbol{t},\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{z};\boldsymbol{q})$ as continued fraction:

Letting $t=0\,,\lambda=0\,,a=\frac{-i\alpha z}{q^{3/2}}\,,\ c=\frac{i\alpha\,z}{q^{3/2}}\,,\ b=q\,,\ \lambda=0\,,\ \alpha=1\,,\beta=-1$ in equation (3.1) we get, the continued fraction for $\nu(t,\alpha,\beta,z;q)$ as follows:-

$$\frac{\frac{z}{\left(1+\frac{z^2}{q^3}\right)}\sum_{n=0}^{\infty}\frac{q^{n^2-3n}\ z^{2n}}{\left(\frac{-\alpha^2z^2}{q};q^2\right)_n}}{\left(1+\frac{iz}{q^{3/2}}\right)\left(\frac{1}{1-\frac{iz}{q^{1/2}}}\right)\sum_{n=0}^{\infty}\frac{q^{n^2-2n}\ z^{2n}}{\left(\frac{z^2}{q};q^2\right)_n}} = 1+\cfrac{\left(1-\frac{1}{q}\right)(iz)}{\left(1-\frac{z}{q^{3/2}}\right)+\cfrac{\frac{iz}{q^{1/2}}}{1+\cfrac{\left(1-\frac{1}{q^2}\right)\left(\frac{-iz}{q^{-1/2}}\right)}{\left(1-\frac{iz}{q^{3/2}}\right)+\left(\frac{iz}{q^{1/2}}\right)+\cdots\ldots\ldots}}}$$

$$\frac{\frac{z}{\left(1+\frac{z^2}{q^3}\right)}\upsilon(0,1,-1,z;q)}{\left(1+\frac{iz}{q^{3/2}}\right)\left(\frac{1}{1-\frac{iz}{q^{1/2}}}\right)\sum_{n=0}^{\infty}\frac{q^{n^2-2n}\ z^{2n}}{\left(\frac{z^2}{q};q^2\right)_n}} = 1+\cfrac{\left(1-\frac{1}{q}\right)(iz)}{\left(1-\frac{z}{q^{3/2}}\right)+\cfrac{\frac{iz}{q^{1/2}}}{1+\cfrac{\left(1-\frac{1}{q^2}\right)\left(\frac{-iz}{q^{-1/2}}\right)}{\left(1-\frac{iz}{q^{3/2}}\right)+\left(\frac{iz}{q^{1/2}}\right)+\cdots\ldots\ldots}}}$$

(v) Representation of $\boldsymbol{\omega}\,(\boldsymbol{t},\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{z};\boldsymbol{q})$ as continued fraction:

Letting $q\to q^2\,,t=0\,,\lambda=0\,,a=\frac{-\alpha^2z^2}{q^3}\,,\ c=\frac{-z^2}{q}\,,b=q^2\,,\ \lambda=0\,,\ \alpha=1\,,\beta=1$ in equation (3.1) we get, the continued fraction for $\omega(t,\alpha,\beta,z;q)$ as follows:-

$$\frac{\sum_{n=0}^{\infty}\frac{q^{2n^2-4n}\ z^{4n}}{\left(\frac{z^2}{q};q^2\right)_n\ (z^2q;q^2)_n}}{\frac{\left(1-\frac{\alpha^2z^2}{q^3}\right)}{\left(1-\frac{\alpha^2z^2}{q}\right)}\sum_{n=0}^{\infty}\frac{q^{2n^2-2n}\ \alpha^{2n}\ z^{4n}}{(z^2q;q^2)_n\ \ (\alpha^2z^2q;q^2)_n}} = 1-\cfrac{\frac{z}{q}\left(1-\frac{1}{q^2}\right)}{\left(1-\frac{z^2}{q^3}\right)-\cfrac{z^2q}{1-\cfrac{\left(1-\frac{1}{q^4}\right)\left(\frac{z^2}{q}\right)}{\left(1-\frac{z^2}{q^3}\right)-(z^2q)\ldots\ldots\ldots}}}$$

$$\frac{\omega(0,1,1,z;q)}{\frac{\left(1-\frac{\alpha^2z^2}{q^3}\right)}{\left(1-\frac{\alpha^2z^2}{q}\right)}\sum_{n=0}^{\infty}\frac{q^{2n^2-2n}\ \alpha^{2n}\ z^{4n}}{(z^2q;q^2)_n\ \ (\alpha^2z^2q;q^2)_n}} = 1-\cfrac{\frac{z}{q}\left(1-\frac{1}{q^2}\right)}{\left(1-\frac{z^2}{q^3}\right)-\cfrac{z^2q}{1-\cfrac{\left(1-\frac{1}{q^4}\right)\left(\frac{z^2}{q}\right)}{\left(1-\frac{z^2}{q^3}\right)-(z^2q)\ldots\ldots\ldots}}}$$

IV. CONTINUED FRACTION REPRESENTATION FOR GENERALIZED EIGHTH ORDER MOCK THETA FUNCTIONS

The continued fraction representation for Fine's function is given in [3] as:

$$(1-b)^{-1}\,(1-t)\,F(t,\alpha;tq) = \cfrac{1}{\frac{-(b-atq)}{(1-t)}+A_0+\cfrac{B_0}{A_1+\cfrac{B_1}{A_2+\cdots\ldots\ldots}}}$$

$$A_i = \frac{1+b-(1+a)tq^{i+1}}{1-tq^i} \quad , \quad B_i = \frac{atq^{i+2}-b}{1-tq^i} \quad \ldots\ldots \quad (4.1)$$

We shall give the continued fraction for generalized eighth order mock theta function by using (4.1)

(i) Representation of $S_0(t,\alpha,z;q)$ as continued fraction

Taking $q \to q^2$ $a = \frac{-z^2}{q^3}$, $b = \frac{-z^2}{q^2}$, $t \to zq^{n+\alpha-4}$ in equation (4.1) we get the continued fraction for $S_0(t,\alpha,z;q)$ as:

$$(1+\frac{z^2}{q^3})^{-1}(1-zq^{n+\alpha-4})\,S_0(t,\alpha,z;q) = \cfrac{1}{\frac{-\left(\frac{z^2}{q^2}-z^3q^{n+\alpha-5}\right)}{(1-zq^{n+\alpha-4})} + P_0 + \cfrac{Q_0}{P_1 + \cfrac{Q_1}{P_2+\cdots\ldots}}}$$

$$P_i = \frac{1+b-(1+a)tq^{i+1}}{1-tq^i} \quad , \quad Q_i = \frac{atq^{2i}-b}{1-tq^{2+i}}$$

(ii) Representation of $S_1(t,\alpha,z;q)$ as continued fraction :

Taking $q \to q^2$, $a = \frac{-z^2}{q^3}$, $b = \frac{-z^2}{q^2}$, $t \to zq^{n+\alpha-2}$ in equation (4.1) we get the continued fraction for $S_1(t,\alpha,z;q)$ as:

$$\left(1+\frac{z^2}{q^3}\right)^{-1}(1\text{-}zq^{n+\alpha-2})\,S_1(t,\alpha,z;q) = \cfrac{1}{\left(\frac{\frac{z^2}{q^2}-z^3q^{n+\alpha-3}}{1-z^3q^{n+\alpha-3}}\right) + X_0 + \cfrac{Y_0}{X_1 + \cfrac{Y_1}{X_2+\cdots\ldots}}}$$

$$X_i = \frac{1+b-(1+a)tq^{i+1}}{1-tq^{i+2}} \quad , \quad Y_i = \frac{atq^{2i}-b}{1-tq^{2+i}}$$

(iii) Representation of $T_0(t,\alpha,z;q)$ as continued fraction :

Taking $q \to q^2, a = \frac{-z^2}{q^2}$, $b = \frac{-z^2}{q}$, $t \to zq^{n+\alpha+1}$ in equation (4.1) we get the continued fraction for $T_0(t,\alpha,z;q)$ as:

$$\left(1+\frac{z^2}{q}\right)^{-1}(1\text{-}zq^{n+\alpha+1})\,T_0(t,\alpha,z;q) = \cfrac{1}{\left(\frac{\frac{z^2}{q}-z^3q^{n+\alpha+1}}{1-z^3q^{n+\alpha+1}}\right) + R_0 + \cfrac{S_0}{R_1 + \cfrac{S_1}{R_2+\cdots\ldots}}}$$

$$R_i = \frac{(1+b)-(1+a)\,tq^{i+1}}{(1-tq^i)} \quad , \quad S_i = \frac{(atq^{2i}-b)}{(1-tq^i)}$$

(iv) Representation of $T_1(t,\alpha,z;q)$ as continued fraction :

Taking $q \to q^2, a = \frac{-z^2}{q^2}$, $b = \frac{-z^2}{q}$, $t \to zq^{n+\alpha-1}$ in equation (4.1) we get the continued fraction for $T_1(t,\alpha,z;q)$ as:

$$(1+\frac{z^2}{q})^{-1}\,(1-zq^{n+\alpha-1})\,T_1(t,\alpha,z;q) = \cfrac{1}{\left(\frac{\frac{z^2}{q}-z^3q^{n+\alpha-1}}{1-zq^{n+\alpha-1}}\right)+U_0+\cfrac{V_0}{U_1+\cfrac{V_1}{U_2+\cdots\ldots}}}$$

$$U_i = \frac{(1+b)-(1+a)\,tq^{i+1}}{(1-tq^i)} \quad , \quad V_i = \frac{(atq^{2i}-b)}{(1-tq^i)}$$

V. CONCLUSION

Mock theta functions are mysterious function. These investigations will be helpful in understanding more about these functions. I would like to point out (3.1) and (4.1) can be used to get continued fraction of third and eight order generalized mock theta function. In (3.1) continued fraction representations for third order generalized mock theta function by resonance formula of Ramanujan's and (4.1) continued fraction representations for eight order generalized mock theta function by Fine's function.

VI . ACKNOWLEDGEMENT

I am thankful to Dr. Sameena Saba for her help and guidance. My MCN number is IU/R&D/2022-MCN0001555.

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