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The Production D-Impaction Model

Characterization of the Nature of Production, Area and Yield on the Basis of Food Grains and Non-food Grains for the Effective Period

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Abstract: This article presents a model approach to characterize the nature of production, area and yield for the principal crops based on food grains and non-food grains. For time discretization, the D-Impaction model and the population D-Impaction model will be used to establish the presented model. Further, the article demonstrates the effective production, area and yield levels corresponding to the effective year between two consecutive sessions in the sense of index number for the principal crops of India since 2010 till 2020 elaborately which theoretically as well as experimentally ensures the validity of the model. The computational analysis depict that the nature of the effective production level and the effective production both alter with the corresponding variation in the production for the effective period. Moreover, the doubling and the half period for the effective production level and the effective production; to make model utility redundantly, applications in demographics, environment, economics, industry, science etc. are provided by allowing some appropriate instances.

Index Terms: **Effective Production, Effective Area, Effective Yield, Effective Principal, Effective Population.**

I. INTRODUCTION

An index number is used to express relative change, mainly designed to measure changes in variable or a group of related variables like price, quantity or value related to base period, more generally over two different situations. An index number is a statistical measure defined by a single ratio (or a percentage) [11, 16-17].

In the presented article, we are intended to construct a model which characterize the nature of entities like ‘production’, ‘area’ and ‘yield’ under the principal crops easily based on the effective change in the given entity. In this direction, our prime aim is to obtain the effective level first. The effective level can be obtained by following the D-Impaction model [4], by which the effective moral standard of a human is given due to Sanskars and Kusanskars below as:

$$M_S(t) = D_{sk}\, e^{(R_s - R_k)\, t}, \tag{1}$$

where,

Sanskar means to make a person or a thing suitable by the process of purification,

R_s = The rate of flow of Sanskars per unit time,

R_k = The rate of flow of Kusanskars per unit time,

R_s, R_k are in %, such that

$$R_s + R_k = 100\%; \tag{2}$$

and D_{sk} is called the D-Factor defined as

$$D_{sk} = \left|\frac{R_s - R_k}{R_s + R_k}\right|, \tag{3}$$

such that

$$0 \leq D_{sk} < 1 \tag{4}$$

Also, $|R_s - R_k|$ is called the effective difference and D_{sk}, the effective part of both R_s and R_k.

Finally, the effective production will be given by the population D-Impaction model [3]. If P_0 be the population, given for the base year, of any given species which changes exponentially for a given region, then the effective population is given as follows:

$$P(t) = P_0 \times P_E(t) \tag{5}$$

where

$P_E(t)$ is the effective population level given as:

$$P_E(t) = D_{bd}.\, e^{(R_b - R_d)\, t}; \tag{6}$$

R_b and R_d are birth rate and death rate respectively; and

$$D_{bd} = \left|\frac{R_b - R_d}{R_b + R_d}\right|, \tag{7}$$

such that

$$0 \leq D_{bd} < 1, \tag{8}$$

Also, from [3], the half period and the doubling period $T_{E(1/2)}$, T_E for effective population level and $T_{(1/2)}$, T for effective population are estimated by the following formulae:

$$T_E = \frac{\ln 2}{R_b - R_d}, R_b > R_d \tag{9}$$

$$T_{E(1/2)} = \frac{-\ln 2}{R_b - R_d}, R_b < R_d \tag{10}$$

$$T = \frac{\ln (2/D_{bd})}{R_b - R_d} = \frac{\ln 2 - \ln D_{bd}}{R_b - R_d}, R_b > R_d \tag{11}$$

$$T_{(1/2)} = \frac{\ln (1/2D_{bd})}{R_b - R_d} = \frac{-(\ln 2 + \ln D_{bd})}{R_b - R_d}, R_b < R_d \tag{12}$$

The statistical data for agricultural production is collected from [5, 8], area under the principal crops from [6, 9] and yield under the principal crops from [7, 10] in view of index numbers. Principal crops are those which have significant index numbers, that is, which perform for the preparation of index numbers [1-2, 18-19]. Index numbers of agricultural production, area and yield for given commodities are calculated by the following formulae [1, 2]:

For individual crop indices (for kth year)

$$\text{Index number of production} = \frac{P_{jk}}{P_{j0}} \times 100 = I_P\ ;$$

$$\text{Index number of area} = \frac{A_{jk}}{A_{j0}} \times 100 = I_A\ ;$$

$$\text{Index number of yield} = \frac{\text{Index number of production}}{\text{Index number of area}} = \frac{I_P}{I_A} \times 100.$$

For any subgroup G of commodities with weight W_j,

$$\text{Index number of production} = \frac{\sum_{j\in G} W_j I_P}{\sum_{j\in G} W_j} \times 100 = I_{PW}\ ;$$

$$\text{Index number of area} = \frac{\sum_{j\in G} W_j I_A}{\sum_{j\in G} W_j} \times 100 = I_{AW}\ ;$$

$$\text{Index number of yield} = \frac{\text{Index number of production with weight}}{\text{Index number of area with weight}} = \frac{I_{PW}}{I_{AW}} \times 100,$$

where

A_{jk} = The area under jth crop in the kth year

A_{j0} = The area under jth crop in the base year period

P_{jk} = The production of jth crop in the kth year

P_{j0} = The production of jth crop in the base year period

W_j = Weight of jth crop

Let $y = f(x)$ be a monotonic function which is defined on a domain D and differentiable in (a, b); $[a, b] \subset D$. Then, y is said to be increasing monotonically (strictly) at $c \in (a, b)$ according as $f'(c) \geq 0$ (> 0); where prime $(\,'\,)$ represents the derivative of $f(x)$ with respect to x. Also, y will attain its local minimum at c when $f''(c) > 0$ and if y is differentiable in D, then $c \in D$ is said to be the point of global minimum if $f(c) < f(k); \forall\, k \in D$ such that $f'(k) = 0, f''(k) > 0;\ \forall\, k \in D$ [12-15].

II. METHODOLOGY

To construct the desired model, our prime intension is towards to formulate first by assuming the following that:

$Y_{i\sim i+1}$ = The effective period between the sessions Y_i and Y_{i+1}
$D_{F_{i\sim i+1}}$ = Rate of the production of food grains per effective period $Y_{i\sim i+1}$ (in %)
$D_{N_{i\sim i+1}}$ = Rate of the production of non-food grains per effective period $Y_{i\sim i+1}$ (in %)
$D_{FN_{i\sim i+1}}$ = The effective factor for the effective period $Y_{i\sim i+1}$
$W_{i\sim i+1}(t)$ = The effective production level or the effective productivity for the effective period $Y_{i\sim i+1}$
where $i = 1, 2, \ldots, 9$.

Notice that if the data is given in years, then $Y_{i\sim i+1}$ is the effective period between the years Y_i and Y_{i+1}.

For two consecutive sessions Y_i and Y_{i+1}, Let

$$F_{i,i+1} = F_{P_{i+1}} - F_{P_i}\,, \tag{13}$$

and

$$N_{i,i+1} = N_{P_{i+1}} - N_{P_i}\,, \tag{14}$$

where

F_{P_i}= The production of food grains in the session Y_i,
N_{P_i}= The production of non-food grains in the session Y_i.

Define

$$D_{F_{i\sim i+1}} = \frac{F_{i,i+1}}{F_{i,i+1} + N_{i,i+1}} \times 100\%\,, \tag{15}$$

$$D_{N_{i\sim i+1}} = \frac{N_{i,i+1}}{F_{i,i+1} + N_{i,i+1}} \times 100\%\,, \tag{16}$$

By using from Eq. 2-4 and Eq. 15-16, we have

$$D_{F_{i\sim i+1}} + D_{N_{i\sim i+1}} = 100\%\,, \tag{17}$$

$$D_{FN_{i\sim i+1}} = \left|\frac{\left|D_{F_{i\sim i+1}}\right| - \left|D_{N_{i\sim i+1}}\right|}{\left|D_{F_{i\sim i+1}}\right| + \left|D_{N_{i\sim i+1}}\right|}\right|; 0 \leq D_{FN_{i\sim i+1}} < 1 \tag{18}$$

where $|.|$ represents the modulus or absolute value.
Hence, from Eq. 17-18 and Eq. 1, we have
The effective production level

$$W_{i\sim i+1}(t) = D_{FN_{i\sim i+1}}\, e^{(D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}})\,t}\,, \tag{19}$$

Means are defined as follows:
First kind mean, by taking means of $F_{i,i+1}$; $N_{i,i+1}$ and $D_{FN_{i\sim i+1}}$,

$$F_M = \frac{\sum_{i=1}^{9} F_{i,i+1}}{C_N}\,, \tag{20}$$

$$N_M = \frac{\sum_{i=1}^{9} N_{i,i+1}}{C_N}\,, \tag{21}$$

$$D_{F_M} = \frac{F_M}{F_M + N_M} \times 100\,, \tag{22}$$

$$D_{N_M} = \frac{N_M}{F_M + N_M} \times 100\,, \tag{23}$$

Second kind mean, by taking means of $D_{F_{i\sim i+1}}$; $D_{N_{i\sim i+1}}$ and $D_{FN_{i\sim i+1}}$,

$$D_{MF} = \frac{\sum_{i=1}^{9} D_{F_{i\sim i+1}}}{C_N}\,, \tag{24}$$

$$D_{MN} = \frac{\sum_{i=1}^{9} D_{N_{i\sim i+1}}}{C_N}, \qquad (25)$$

$$D_{MFN} = \frac{\sum_{i=1}^{9} D_{FN_{i\sim i+1}}}{C_N}, \qquad (26)$$

where

C_N = Number of the consecutive effective years (= 9, here).

Using from Eq. 19-26, we obtain

The effective mean production level (first kind)

$$W_{M1}(t) = D_{MFN}\, e^{(D_{F_M} - D_{N_M})\,t}, \qquad (27)$$

The effective mean production level (second kind)

$$W_{M2}(t) = D_{MFN}\, e^{(D_{MF} - D_{MN})\,t}, \qquad (28)$$

Now, Using from Eq. 5-6, we get

The effective production

$$W(t)_{i\sim i+1} = W(0)_{i\sim i+1} \times W_{i\sim i+1}(t) = W(0)_{i\sim i+1} \times D_{FN_{i\sim i+1}}\, e^{(D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}})\,t}, \qquad (29)$$

where

$$W(0)_{i\sim i+1} = W(0)_{i+1} - W(0)_i\,; \qquad (30)$$

such that

$$W(0) = \text{Food grains} + \text{Non-food grains} = \text{Total Production (Initially)} \qquad (31)$$

and the effective mean production

$$W_1(t) = W_{M1}(0) \times W_{M1}(t) = W_{M1}(0) \times D_{MFN}\, e^{(D_{F_M} - D_{N_M})\,t}, \qquad (32)$$

and

$$W_2(t) = W_{M2}(0) \times W_{M2}(t) = W_{M2}(0) \times D_{MFN}\, e^{(D_{MF} - D_{MN})\,t}, \qquad (33)$$

where

$W(0)_{i\sim i+1}$ is the initial agricultural production or availability (stock) as the base entity in the effective period $Y_{i\sim i+1}$.

Analogously, for area and yield under principal crops, we can construct the mathematical structure by using from Eq. 27-33 as follows:

The effective areal level

$$A_{i\sim i+1}(t) = D_{FN_{i\sim i+1}}\, e^{(D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}})\,t}, \qquad (34)$$

The effective yield level

$$E_{i\sim i+1}(t) = D_{FN_{i\sim i+1}}\, e^{(D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}})\,t}, \qquad (35)$$

The effective area

$$A(t)_{i\sim i+1} = A(0)_{i\sim i+1} \times A_{i\sim i+1}(t) = A(0)_{i\sim i+1} \times D_{FN_{i\sim i+1}}\, e^{(D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}})\,t}, \qquad (36)$$

The effective yield

$$E(t)_{i\sim i+1} = E(0)_{i\sim i+1} \times E_{i\sim i+1}(t) = E(0)_{i\sim i+1} \times D_{FN_{i\sim i+1}}\, e^{(D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}})\,t}, \qquad (37)$$

The effective mean area

$$A_1(t) = A_{M1}(0) \times A_{M1}(t) = A_{M1}(0) \times D_{MFN}\, e^{(D_{F_M} - D_{N_M})\,t}, \qquad (38)$$

and

$$A_2(t) = A_{M2}(0) \times A_{M2}(t) = A_{M2}(0) \times D_{MFN}\, e^{(D_{MF} - D_{MN})\,t}, \qquad (39)$$

The effective mean yield

$$E_1(t) = E_{M1}(0) \times E_{M1}(t) = E_{M1}(0) \times D_{MFN}\, e^{(D_{F_M} - D_{N_M})\,t}, \qquad (40)$$

and

$$E_2(t) = E_{M2}(0) \times E_{M2}(t) = E_{M2}(0) \times D_{MFN}\, e^{(D_{MF} - D_{MN})\,t}, \qquad (41)$$

By applying the above formulations, the required calculations may be done as given below:

For $Y_{1\sim 2}$,

$$F_{1\sim 2} = F_{P_2} - F_{P_1} = 119.5 - 114.3 = 5.2\,,$$

and

$$N_{1\sim 2} = N_{P_2} - N_{P_1} = 129.6 - 128.0 = 1.6\,,$$

$$D_{F_{1\sim 2}} = \frac{5.2}{5.2 + 1.6} \times 100 = 76.47\%\,,$$

$$D_{N_{1\sim2}} = \frac{1.6}{5.2 + 1.6} \times 100 = 23.53\% \,,$$

and

$$D_{FN_{1\sim2}} = \left|\frac{76.47-23.53}{76.47+23.53}\right| = 0.5294 \,,$$

For $Y_{2\sim3}$,

$$F_{2\sim3} = F_{P_3} - F_{P_2} = 119.4 - 119.5 = -0.1 \,,$$

and

$$N_{2\sim3} = N_{P_3} - N_{P_2} = 129.0 - 129.6 = -0.6 \,,$$

$$D_{F_{2\sim3}} = \frac{(-0.1)}{(-0.1)+(-0.6)} \times 100 = 14.29\% \,,$$

$$D_{N_{2\sim3}} = \frac{(-0.6)}{(-0.1)+(-0.6)} \times 100 = 85.71\% \,,$$

and

$$D_{FN_{2\sim3}} = \left|\frac{14.29-85.71}{14.29+85.71}\right| = 0.7142 \,,$$

and so on. Now,
To construct effective π-diagram initially, we define

$$Z_i = \frac{W_{i\sim i+1}(0)}{\sum_{i=1}^{9} W_{i\sim i+1}(0)} \times 100\% \qquad (i = 1, 2, \dots, 9)\,; \tag{42}$$

where

$W_{i\sim i+1}(0)$ = The effective agricultural productivity for the effective period $Y_{i\sim i+1}$ at t = 0.

Also, we have

$$\sum_{i=1}^{9} W_{i\sim i+1}(0) = 4.3642, \sum_{i=1}^{9} A_{i\sim i+1}(0) = 3.2106, \sum_{i=1}^{9} E_{i\sim i+1}(0) = 4.9540.$$

Table 1.1 Index Numbers of Agricultural Production

(Base: Triennium Ending 2007-2008=100)

Production	Session→	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	2015-2016	2016-2017	2017-2018	2018-2019	2019-2020
	Weight ↓	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}
Food grains	50.7	114.3	119.5	119.4	123.3	115.9	115.7	131.1	136.8	134.4	139.8
Non-food grains	49.3	128.0	129.6	129.0	136.4	132.2	126.1	134.7	142.1	142.0	151.3
All Commodities	100.0	121.1	124.5	124.2	129.8	124.0	120.8	132.8	139.4	138.1	145.5

Table 1.2 The Effective Agricultural Production Level

$Y_{i\sim i+1}$	$F_{i,i+1}$	$N_{i,i+1}$	$D_{F_{i\sim i+1}}$ (in%)	$D_{N_{i\sim i+1}}$ (in%)	$D_{FN_{i\sim i+1}}$	$W_{i\sim i+1}(t) = D_{FN_{i\sim i+1}} e^{(D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}})t}$
$Y_{1\sim2}$	5.2	1.6	76.47	23.53	0.5294	$0.5294\, e^{0.5294\, t}$
$Y_{2\sim3}$	- 0.1	- 0.6	14.29	85.71	0.7142	$0.7142\, e^{-0.7142\, t}$
$Y_{3\sim4}$	3.9	7.4	34.51	65.49	0.3098	$0.3098\, e^{-0.3098\, t}$
$Y_{4\sim5}$	- 7.4	- 4.2	63.79	36.21	0.2758	$0.2758\, e^{0.2758\, t}$
$Y_{5\sim6}$	- 0.2	- 6.1	03.17	96.83	0.9366	$0.9366\, e^{-0.9366\, t}$
$Y_{6\sim7}$	15.4	8.6	64.17	35.83	0.2834	$0.2834\, e^{0.2834\, t}$
$Y_{7\sim8}$	5.7	7.4	43.51	56.49	0.1298	$0.1298\, e^{-0.1298\, t}$
$Y_{8\sim9}$	- 2.4	- 0.1	96.00	04.00	0.9200	$0.9200\, e^{0.9200\, t}$
$Y_{9\sim10}$	5.4	9.3	36.73	63.27	0.2654	$0.2654\, e^{-0.2654\, t}$

Table 1.3 The Effective Agricultural Mean Production Level

F_M	F_N	D_{F_M}	D_{F_N}	D_{MF}	D_{MN}	D_{MFN}	$W_{M1}(t)$	$W_{M2}(t)$
2.83	2.59	52.21	47.79	48.07	51.93	0.4849	$0.4849\, e^{0.0442\, t}$	$0.4849\, e^{-0.0386\, t}$

Table 1.4 The Effective Initial Agricultural Production Level

Effective Period	$Y_{1\sim2}$	$Y_{2\sim3}$	$Y_{3\sim4}$	$Y_{4\sim5}$	$Y_{5\sim6}$	$Y_{6\sim7}$	$Y_{7\sim8}$	$Y_{8\sim9}$	$Y_{9\sim10}$
$W_{i\sim i+1}(0)$	0.5294	0.7142	0.3098	0.2758	0.9366	0.2834	0.1298	0.9200	0.2654
Z_i(in %)	12.13	16.37	07.10	06.32	21.46	06.49	02.97	21.08	06.08

Table 2.1 Index Numbers of Area Under Principal Crops

(Base: Triennium Ending 2007-2008=100)

Production	Session→	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	2015-2016	2016-2017	2017-2018	2018-2019	2019-2020
	Weight ↓	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}
Food grains	50.7	104.4	104.1	102.0	105.6	105.4	103.9	109.0	107.4	105.6	107.9
Non-food grains	49.3	113.9	116.0	116.0	119.2	118.5	119.5	116.2	118.2	121.7	124.0
All Commodities	100.0	109.1	110.0	108.9	112.3	111.9	111.6	112.6	112.7	113.5	115.9

Table 2.2 The Effective Areal Level Under Principal Crops

$Y_{i\sim i+1}$	$F_{i,i+1}$	$N_{i,i+1}$	$D_{F_{i\sim i+1}}$ (in%)	$D_{N_{i\sim i+1}}$ (in%)	$D_{FN_{i\sim i+1}}$	$A_{i\sim i+1}(t) = D_{FN_{i\sim i+1}}\, e^{(D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}})\,t}$
$Y_{1\sim 2}$	-0.3	2.1	- 16.67	116.67	0.7500	$0.7500\, e^{-1.3334\,t}$
$Y_{2\sim 3}$	-2.1	0.0	100.00	0.00	1.0000	$1.0000\, e^{1.0000\,t}$
$Y_{3\sim 4}$	3.6	3.2	52.94	47.06	0.0588	$0.0588\, e^{0.0588\,t}$
$Y_{4\sim 5}$	-0.2	-0.7	22.22	77.78	0.5556	$0.5556\, e^{-0.5556\,t}$
$Y_{5\sim 6}$	-1.5	1.0	300.00	- 200.00	0.2000	$0.2000\, e^{5.0000\,t}$
$Y_{6\sim 7}$	5.1	-3.3	283.33	- 183.33	0.2143	$0.2143\, e^{4.6666\,t}$
$Y_{7\sim 8}$	-1.6	2.0	- 400.00	500.00	0.1111	$0.1111\, e^{-9.0000\,t}$
$Y_{8\sim 9}$	-1.8	3.5	- 105.88	205.88	0.3208	$0.3208\, e^{-3.1176\,t}$
$Y_{9\sim 10}$	2.3	2.3	50.00	50.00	0.0000	$0.0000\, e^{0.0000\,t}$

Table 2.3 The Effective Mean Areal Level Under Principal Crops

F_M	F_N	D_{F_M}	D_{F_N}	D_{MF}	D_{MN}	D_{MFN}	$A_{M1}(t)$	$A_{M2}(t)$
0.39	1.12	25.83	74.17	31.77	68.23	0.3567	$0.3567\, e^{-0.4834\,t}$	$0.3567\, e^{-0.3646\,t}$

Table 2.4 The Effective Initial Areal Level Under Principal Crops

Effective Period	$Y_{1\sim 2}$	$Y_{2\sim 3}$	$Y_{3\sim 4}$	$Y_{4\sim 5}$	$Y_{5\sim 6}$	$Y_{6\sim 7}$	$Y_{7\sim 8}$	$Y_{8\sim 9}$	$Y_{9\sim 10}$
$A_{i\sim i+1}(0)$	0.7500	1.0000	0.0588	0.5556	0.2000	0.2143	0.1111	0.3208	0.0000
Z_i(in %)	23.36	31.15	01.83	17.31	06.23	06.67	03.46	09.99	0.0000

Table 3.1 Index Numbers of Yield of Principal Crops

(Base: Triennium Ending 2007-2008=100)

Production	Session→	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	2015-2016	2016-2017	2017-2018	2018-2019	2019-2020
	Weight ↓	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}
Food grains	50.7	109.5	114.9	117.1	116.8	110.0	111.3	120.3	127.4	127.3	129.5
Non-food grains	49.3	112.4	111.7	111.2	114.4	111.6	105.5	115.8	120.2	116.7	122.0
All Commodities	100.0	111.0	113.5	114.0	115.5	110.8	108.3	118.0	123.7	121.7	125.6

Table 3.2 The Effective Yield Level of Principal Crops

$Y_{i\sim i+1}$	$F_{i,i+1}$	$N_{i,i+1}$	$D_{F_{i\sim i+1}}$ (in%)	$D_{N_{i\sim i+1}}$ (in%)	$D_{FN_{i\sim i+1}}$	$E_{i\sim i+1}(t) = D_{FN_{i\sim i+1}}\, e^{(D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}})\,t}$
$Y_{1\sim 2}$	5.4	-0.7	114.89	- 14.89	0.7705	$0.7705\, e^{1.2978\,t}$
$Y_{2\sim 3}$	2.2	-0.5	129.41	- 29.41	0.6296	$0.6296\, e^{1.5882\,t}$
$Y_{3\sim 4}$	-0.3	3.2	- 10.34	110.34	0.8286	$0.8286\, e^{-1.2068\,t}$
$Y_{4\sim 5}$	-6.8	-2.8	70.83	29.17	0.4166	$0.4166\, e^{0.4166\,t}$
$Y_{5\sim 6}$	1.3	-6.1	- 27.08	127.08	0.6487	$0.6487\, e^{-1.5416\,t}$
$Y_{6\sim 7}$	9.0	10.3	46.63	53.37	0.0674	$0.0674\, e^{-0.0674\,t}$
$Y_{7\sim 8}$	7.1	4.4	61.74	38.26	0.2348	$0.2348\, e^{0.2348\,t}$
$Y_{8\sim 9}$	-0.1	-3.5	2.78	97.22	0.9444	$0.9444\, e^{-0.9444\,t}$
$Y_{9\sim 10}$	2.2	5.3	29.33	70.67	0.4134	$0.4134\, e^{-0.4134\,t}$

Table 3.3 The Effective Mean Yield Level of Principal Crops

F_M	F_N	D_{F_M}	D_{F_N}	D_{MF}	D_{MN}	D_{MFN}	$E_{M1}(t)$	$E_{M2}(t)$
2.22	1.07	67.48	32.52	46.46	53.54	0.5504	$0.5504\, e^{0.3456\,t}$	$0.5504\, e^{-0,0708\,t}$

Table 3.4 The Effective Initial Level of Principal Crops

Effective Period	$Y_{1\sim2}$	$Y_{2\sim3}$	$Y_{3\sim4}$	$Y_{4\sim5}$	$Y_{5\sim6}$	$Y_{6\sim7}$	$Y_{7\sim8}$	$Y_{8\sim9}$	$Y_{9\sim10}$
$E_{i\sim i+1}(0)$	0.7705	0.6296	0.8286	0.4166	0.6487	0.0674	0.2348	0.9444	0.4134
Z_i(in %)	15.55	12.71	16.73	08.41	13.09	01.36	04.74	19.06	08.35

III. Applications

Here, we are serving some suitable applications to make model useful in various disciplines as given below:

3.1 The doubling and half period (for effective period $Y_{i\sim i+1}$)

To find the doubling and half period, we have from Eq. 9-12 as follows:
For effective production level,
Doubling period

$$T_E = \frac{ln\ 2}{D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}}}, D_{F_{i\sim i+1}} > D_{N_{i\sim i+1}}; \quad (43)$$

and half period

$$T_{E(1/2)} = \frac{-\ ln\ 2}{D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}}}, D_{F_{i\sim i+1}} < D_{N_{i\sim i+1}} \quad (44)$$

For effective production,
Doubling period

$$T = \frac{ln\left(2/D_{FN_{i\sim i+1}}\right)}{D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}}} = \frac{ln\ 2 - ln\left(D_{FN_{i\sim i+1}}\right)}{D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}}}, D_{F_{i\sim i+1}} > D_{N_{i\sim i+1}}; \quad (45)$$

and half period

$$T_{(1/2)} = \frac{ln\left(1/2D_{FN_{i\sim i+1}}\right)}{D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}}} = \frac{-\left[ln\ 2 + ln\left(D_{FN_{i\sim i+1}}\right)\right]}{D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}}}, D_{F_{i\sim i+1}} < D_{N_{i\sim i+1}} \quad (46)$$

Similarly, we can define the doubling and half period for the area and yield.
Now, we put some related illustrations by using from Eq. 43-46 as discussed below:

Example 1 For effective production level, consider

For the effective year $Y_{1\sim2}$: $T_E = \frac{ln\ 2}{52.94\%} = 1.31$

For the effective year $Y_{6\sim7}$: $T_E = \frac{ln\ 2}{28.34\%} = 2.45$

For the effective year $Y_{8\sim9}$: $T_E = \frac{ln\ 2}{92\%} = 0.75$

Example 2 For effective production level, consider

For the effective year $Y_{2\sim3}$: $T_{E(1/2)} = \frac{-ln\ 2}{-71.42\%} = 0.97$

For the effective year $Y_{7\sim8}$: $T_{E(1/2)} = \frac{-ln\ 2}{-12.98\%} = 5.34$

For the effective year $Y_{9\sim10}$: $T_{E(1/2)} = \frac{-ln\ 2}{-26.54\%} = 2.61$

Example 3 For effective production, consider
For the effective year $Y_{1\sim2}$:

$W(0)_{1\sim2} = (119.5 + 129.6) - (114.3 + 128.0) = 6.8$ and $T = \frac{ln\ 2 - \ln\ (0.5294)}{52.94\%} = 2.51$

For the effective year $Y_{1\sim2}$:

$W(0)_{7\sim8} = (136.8 + 142.1) - (131.1 + 134.7) = 13.1$ and $T = \frac{-\ [\ln 2 + \ln\ (0.1298)]}{-12.98\%} = -10.39$

Minus sign implies that the effective production would be half of its initial before the time of 10.39.
Similar calculations may be done for area and yield.

3.2 Consequences

The following analogous approaches for the given effective year $Y_{i\sim i+1}$ may be served in view of Eq. 29 as follows (symbols have their own meaning as discussed in population D-Impaction model differ from only indices $i\sim i+1$):
For the effective population (presented for birth-death),

$$P(t)_{i\sim i+1} = P(0)_{i\sim i+1} \times D_{BD_{i\sim i+1}}\, e^{\left(D_{B_{i\sim i+1}} - D_{D_{i\sim i+1}}\right)t}\ ;\ D_{B_{i\sim i+1}} + D_{D_{i\sim i+1}} = 100\% \tag{47}$$

For the effective population (presented for growth-decay),

$$P(t)_{i\sim i+1} = P(0)_{i\sim i+1} \times D_{GD_{i\sim i+1}}\, e^{\left(D_{G_{i\sim i+1}} - D_{D_{i\sim i+1}}\right)t}\ ;\ D_{G_{i\sim i+1}} + D_{D_{i\sim i+1}} = 100\% \tag{48}$$

For the effective population (presented for sex-ratio, that is, male-female),

$$P(t)_{i\sim i+1} = P(0)_{i\sim i+1} \times D_{MF_{i\sim i+1}}\, e^{\left(D_{M_{i\sim i+1}} - D_{F_{i\sim i+1}}\right)t}\ ;\ D_{M_{i\sim i+1}} + D_{F_{i\sim i+1}} = 100\% \tag{49}$$

For the effective population (presented for area enclosed by male-female population),

$$A(t)_{i\sim i+1} = A(0)_{i\sim i+1} \times D_{EMF_{i\sim i+1}}\, e^{\left(D_{EM_{i\sim i+1}} - D_{EF_{i\sim i+1}}\right)t}\ ;\ D_{EM_{i\sim i+1}} + D_{EF_{i\sim i+1}} = 100\% \tag{50}$$

For the effective population (presented for vegetarian & non-vegetarian),

$$P(t)_{i\sim i+1} = P(0)_{i\sim i+1} \times D_{VN_{i\sim i+1}}\, e^{\left(D_{V_{i\sim i+1}} - D_{N_{i\sim i+1}}\right)t}\ ;\ D_{V_{i\sim i+1}} + D_{N_{i\sim i+1}} = 100\% \tag{51}$$

For the effective principal (presented for profit-loss),

$$S(t)_{i\sim i+1} = S(0)_{i\sim i+1} \times D_{PL_{i\sim i+1}}\, e^{\left(D_{P_{i\sim i+1}} - D_{L_{i\sim i+1}}\right)t}\ ;\ D_{P_{i\sim i+1}} + D_{L_{i\sim i+1}} = 100\% \tag{52}$$

For the effective food consumption (presented for food grains & non-food grains),

$$F(t)_{i\sim i+1} = F(0)_{i\sim i+1} \times D_{fN_{i\sim i+1}}\, e^{\left(D_{f_{i\sim i+1}} - D_{N_{i\sim i+1}}\right)t}\ ;\ D_{f_{i\sim i+1}} + D_{N_{i\sim i+1}} = 100\% \tag{53}$$

The effective temperature or weather measure (presented for heat-cold),

$$M(t)_{i\sim i+1} = M(0)_{i\sim i+1} \times D_{HC_{i\sim i+1}}\, e^{\left(D_{H_{i\sim i+1}} - D_{C_{i\sim i+1}}\right)t}\ ;\ D_{H_{i\sim i+1}} + D_{C_{i\sim i+1}} = 100\% \tag{54}$$

The effective production (presented for production-wastage),

$$R(t)_{i\sim i+1} = R(0)_{i\sim i+1} \times D_{PW_{i\sim i+1}}\, e^{\left(D_{P_{i\sim i+1}} - D_{W_{i\sim i+1}}\right)t}\ ;\ D_{P_{i\sim i+1}} + D_{W_{i\sim i+1}} = 100\% \tag{55}$$

IV. Results And Conclusion

The model is applicable there, where change is taken exponentially. The model is sufficient to characterize the nature of production, area, yield and such structural data like population, weather measure, principal, food consumption, etc. for the effective period. More generally, in D-Impaction and population D-Impaction model we are able to characterize the nature of the given entity for the given time while the presented model is primly defined for the effective period between two consecutive times (say, year, month, day, hour, session, decade, century etc.). In absence of non-food grains, the model is purely in favor of food grains; and in absence of food grains, the model is purely in favor of non-food grains. Thus, both are said to be ideal model here. The model is in favor of food grains if rate of food grains is always greater than that of non-food grains; and in favor of non-food grains if rate of food grains is always less than that of non-food grains. As like population D-Impaction model, the production D-Impaction model provide us a statistical approach to characterize the nature of production and this also leads us to facilitate to characterize the such structural data (as discussed in from Eq. 47-55). Due to same adoption of differential equations by production (by following population growth model), effective production level and effective production for the effective period we conclude that all these have same in nature.

Also, Eq. 19 and from Eq. 27-41 provide the mathematical structure for the effective production, area, yield levels and the effective production, area, yield for the effective period. Eq. 42 gives us the formula to construct π-diagram.

Table 1.1, 2.1 and 3.1 provide us required statistical data while from Table 1.2-1.4, Table 2.2-2.4 and Table 3.2-3.4 give us the computational data and structure to sketch appropriate graphs and figures.

Now, we shall represent given data graphically and analyze its nature. For this, Fig. 1, Fig. 2 and Fig. 3 are generated by using MATLAB TOOL while Fig. 4 to Fig. 9 by using M.S. EXCEL. Figure 1 represents the nature of the effective production level $W_{i\sim i+1}(t)$ for the effective year $Y_{i\sim i+1}$ where the graphs occurring upwards, are in favor of food grains and downwards, in favor of non-food grains. Also, $W_{8\sim 9}(t)$ adopts the fastest growth while $W_{5\sim 6}(t)$, the fastest decay in favor of food grains. The first mean $W_{M1}(t)$ is in favor of food grains while the second mean $W_{M2}(t)$, in favor of non-food grains. Figure 2 shows the nature of the effective areal level $A_{i\sim i+1}(t)$ for the effective year $Y_{i\sim i+1}$. Also, $A_{5\sim 6}(t)$ obtains the fastest growth while $A_{7\sim 8}(t)$, the fastest decay in favor of food grains except $A_{9\sim 10}(t)$. The first mean $A_{M1}(t)$ and the second mean $A_{M2}(t)$ both are in favor of non-food grains such that $A_{M1}(t)$ decays rapidly than $A_{M2}(t)$. Figure 3 depicts the nature of the effective yield level $E_{i\sim i+1}(t)$ for the effective year $Y_{i\sim i+1}$. Also, $E_{2\sim 3}(t)$ adopts the fastest growth while $E_{5\sim 6}(t)$, the fastest decay in favor of food grains. The first mean $E_{M1}(t)$ is in favor of food grains while the second mean $E_{M2}(t)$, in favor of non-food grains. Figure 4 exhibits the nature of the effective production level $W_{i\sim i+1}(t)$ initially which are interpolated by the polynomial of degree 6 as

$$y = -0.0031x^6 + 0.0907x^5 - 1.0456x^4 + 5.9273x^3 - 17.083x^2 + 23.066x - 10.43$$

with R-squared value on the chart as $R^2 = 0.8493$ while Fig. 5 provides the corresponding π-chart for $W_{i\sim i+1}(0)$. The global maximum is attained by this polynomial for $Y_{5\sim 6}$ and global minimum for $Y_{7\sim 8}$. Figure 6 demonstrates the nature of the effective areal level $A_{i\sim i+1}(t)$ initially which are interpolated by the polynomial of degree 6 as

$$y = -0.0018x^6 + 0.0531x^5 - 0.631x^4 + 3.7059x^3 - 11.113x^2 + 15.478x - 6.7291$$

with R-squared value on the chart as $R^2 = 0.8047$ while Fig. 7 provides the corresponding π-chart for $A_{i\sim i+1}(0)$. The global maximum is attained by this polynomial for $Y_{2\sim 3}$ and global minimum for $Y_{9\sim 10}$. Figure 8 exhibits the nature of the effective yield level $E_{i\sim i+1}(t)$ initially which are interpolated by the polynomial of degree 6 as

$$y = -0.001x^6 + 0.0275x^5 - 0.2814x^4 + 1.3859x^3 - 3.434x^2 + 3.9631x - 0.8991$$

with R-squared value on the chart as $R^2 = 0.787$ while Fig. 9 provides the corresponding π-chart for $E_{i\sim i+1}(0)$. The global maximum is attained by this polynomial for $Y_{8\sim 9}$ and global minimum for $Y_{6\sim 7}$.

Also, the discussed model can be written analogous to the population D-Impaction model defined for given time only, not for effective period, as follows:

$$W(t) = W_0 \times P_E(t) = W_0 \times D_{FN}.e^{(D_F - D_N)\,t};\ D_F + D_N = 100\%,$$

where

$$D_F = \frac{\text{Food grains}}{(\text{Food grains}) + (\text{Non-food grains})};\ D_N = \frac{\text{Non-food grains}}{(\text{Food grains}) + (\text{Non-food grains})}$$

and for the given base time

$$W_0 = \text{Food grains} + \text{Non-food grains} = \text{Total Production (Initially)}$$

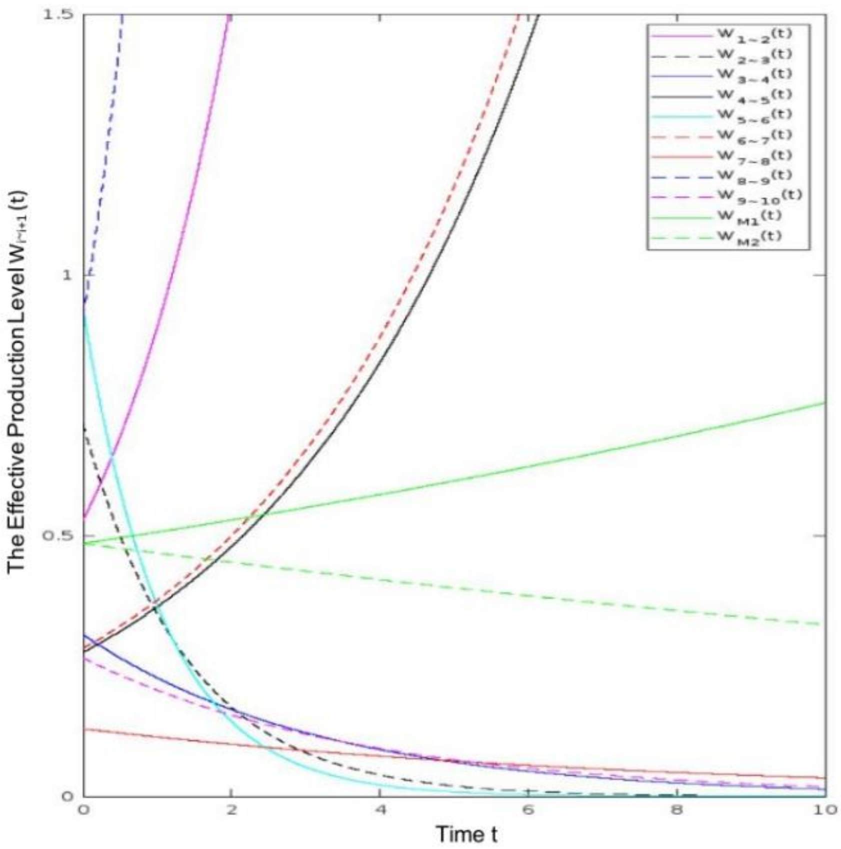


Fig. 1 The Profile Picture of the Effective Production Level $W_{i\sim i+1}(t)$ for the Effective Year $Y_{i\sim i+1}$

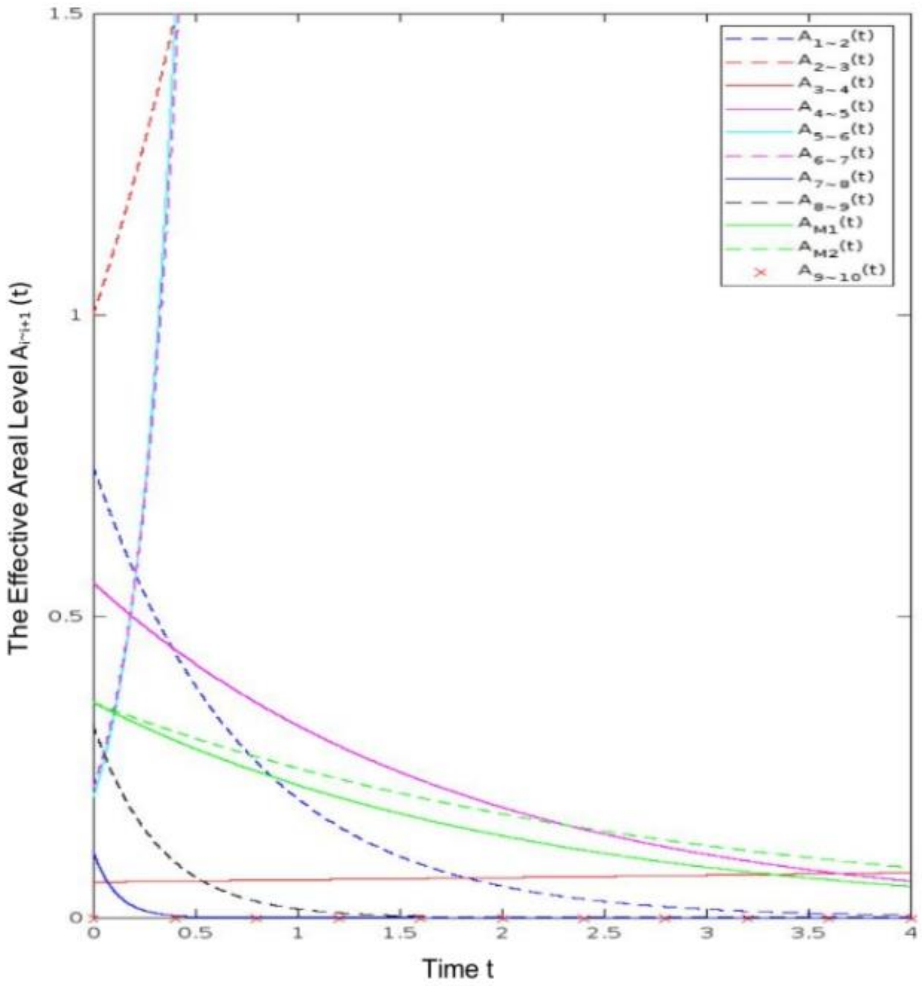


Fig. 2 The Profile Picture of the Effective Areal Level $A_{i\sim i+1}$ (t) for the Effective Year $Y_{i\sim i+1}$

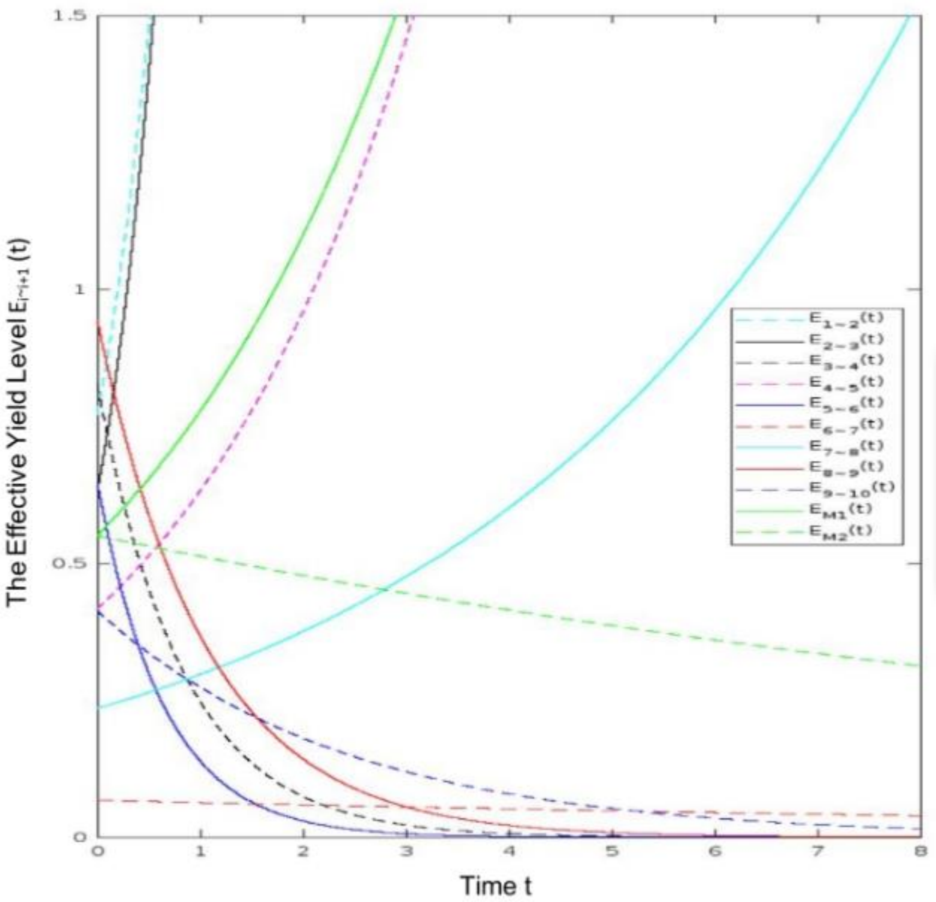


Fig. 3 The Profile Picture of the Effective Yield Level $E_{i\sim i+1}$ (t) for the Effective Year $Y_{i\sim i+1}$

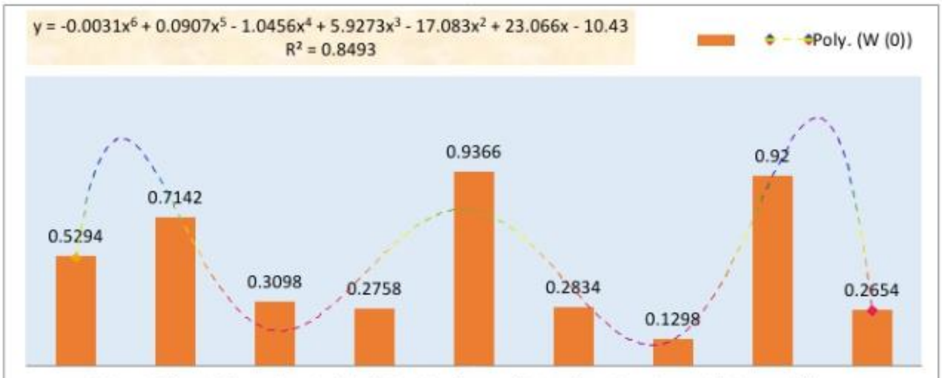
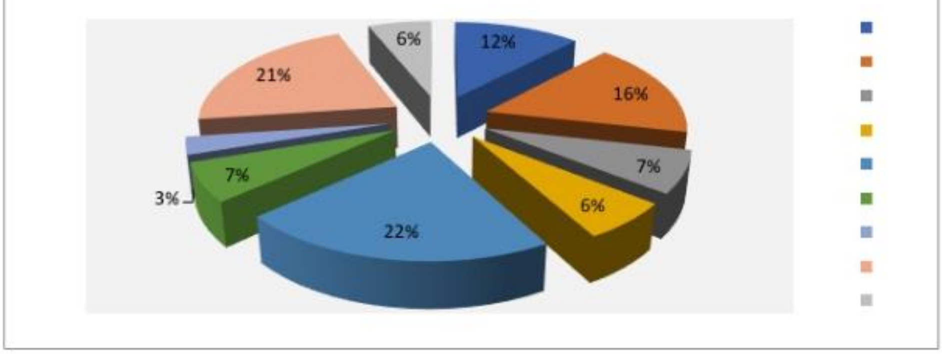


Fig. 4 The Effective Initial Agricultural Production Level $W_{i\sim i+1}$ (0) for the Effective Year $Y_{i\sim i+1}$

Fig. 5 Pie-chart of the Effective Initial Agricultural Production Level $W_{i\sim i+1}$ (0) for the Effective Year $Y_{i\sim i+1}$



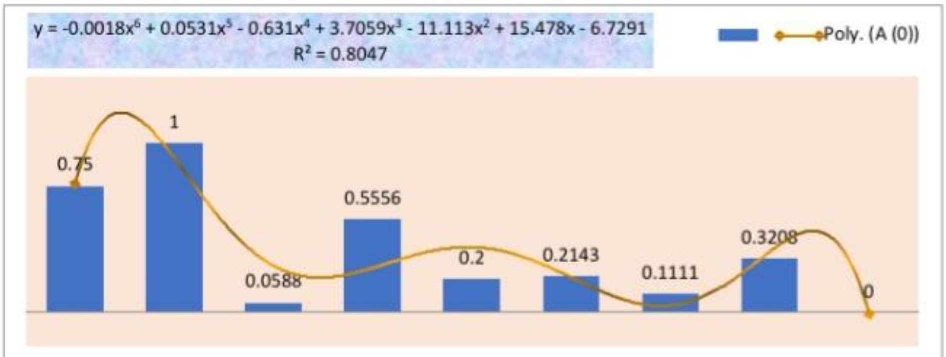
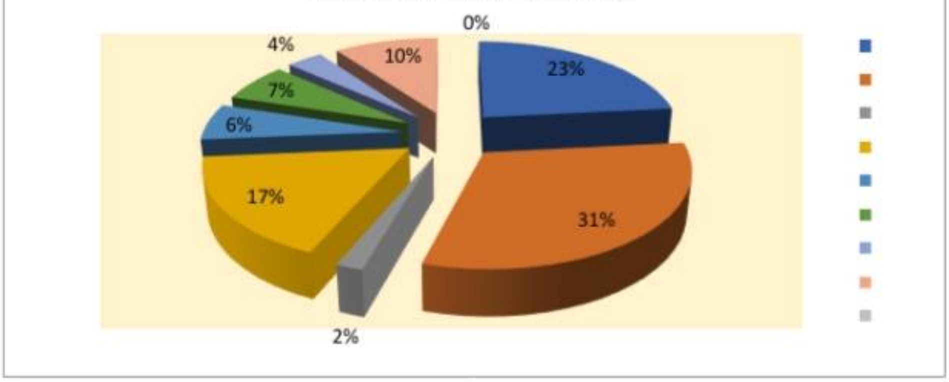


Fig. 6 The Effective Initial Areal Level $A_{i\sim i+1}$ (0) for the Effective Year $Y_{i\sim i+1}$

Fig. 7 Pie-chart of the Effective Initial Areal Level $A_{i\sim i+1}$ (0) for the Effective Year $Y_{i\sim i+1}$



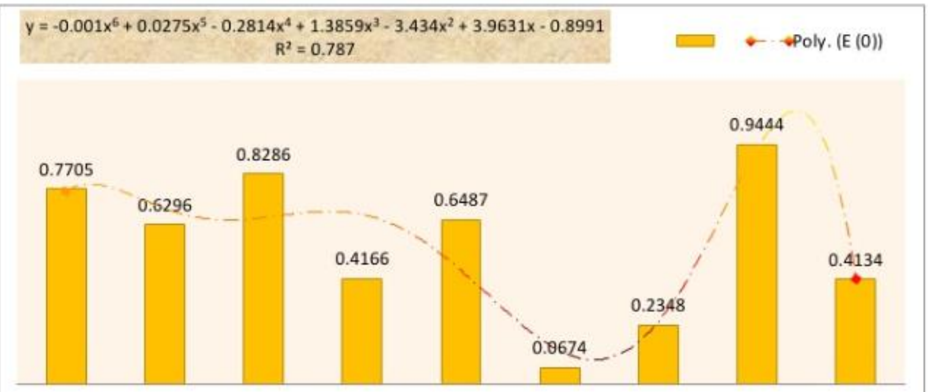
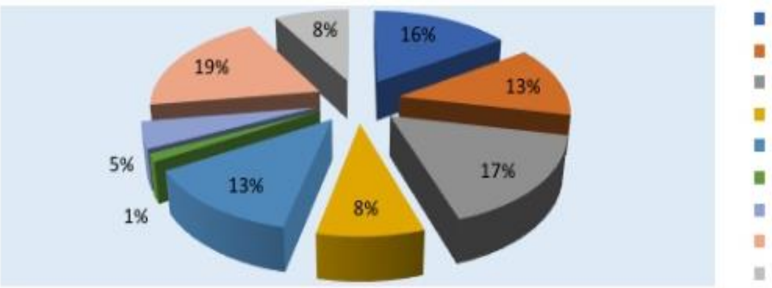


Fig. 8 The Effective Initial Yield Level $E_{i\sim i+1}$ (0) for the Effective Year $Y_{i\sim i+1}$

Fig. 9 Pie-chart of the Effective Initial Yield Level $E_{i\sim i+1}$ (0) for the Effective Year $Y_{i\sim i+1}$



Now, by D-Impaction model, from Eq. 19, $W_{i\sim i+1}(t)$ has the global minimum as $W_{i\sim i+1}(0) = D_{FN_{i\sim i+1}}$, when $D_{F_{i\sim i+1}} > D_{N_{i\sim i+1}}$ and has the global maximum as $W_{i\sim i+1}(0) = D_{FN_{i\sim i+1}}$, when $D_{F_{i\sim i+1}} < D_{N_{i\sim i+1}}$. From Fig. 1 to Fig. 3, it may be seen that both assertions are graphically true. Do same for $A_{i\sim i+1}(t)$, $E_{i\sim i+1}(t)$, first and second mean. Also, from Eq. 29, we have

$$\frac{dW(t)_{i\sim i+1}}{dt} = W(0)_{i\sim i+1} \times D_{FN_{i\sim i+1}}\left(D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}}\right).e^{\left(D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}}\right)t} \Rightarrow \frac{dW(t)_{i\sim i+1}}{dt} = \begin{cases} > 0; D_{F_{i\sim i+1}} > D_{N_{i\sim i+1}} \\ < 0; D_{F_{i\sim i+1}} < D_{N_{i\sim i+1}} \end{cases}, \quad (56)$$

and

$$\frac{d^2W(t)_{i\sim i+1}}{dt^2} = W(0)_{i\sim i+1} \times D_{FN_{i\sim i+1}}\left(D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}}\right)^2.e^{\left(D_{F_{i\sim i+1}} - D_{N_{i\sim i+1}}\right)t} \Rightarrow \frac{d^2W(t)_{i\sim i+1}}{dt^2} > 0; t > 0, \quad (57)$$

Obviously, from Eq. 56, when $D_{F_{i\sim i+1}} > D_{N_{i\sim i+1}}$, then $W(t)_{i\sim i+1}$ is an increasing function which satisfy the condition of minima, from Eq. 57 and has the global minimum at $t = 0$ as $W(0)_{i\sim i+1} \times D_{FN_{i\sim i+1}}$. From Eq. 56, when $D_{F_{i\sim i+1}} < D_{N_{i\sim i+1}}$, then $W(t)_{i\sim i+1}$ is a decreasing function which satisfy the condition of maxima, from Eq. 57 and has the global maximum at $t = 0$ as $W(0)_{i\sim i+1} \times D_{FN_{i\sim i+1}}$ also. Similar task may be done for $A(t)_{i\sim i+1}$, $E(t)_{i\sim i+1}$, first and second mean.

V. REFERENCES

[1] Agricultural Statistics at a Glance 2018, available on agricoop.gov.in https://agricoop.gov.in › filesPDF Agricultural Statistics at a Glance 2018, pp. 452-454 (19 January 2022)

[2] Agricultural Statistics at a Glance 2020 available on dacnet.nic.in https://eands.dacnet.nic.in › ...PDF Agricultural Statistics at a Glance 2020, pp. 298-299 (30 May 2022)

[3] Arya, D. V., Sharma, P. K, & Gupta, S. (2022). The Population D-Impaction Model: Characterization of the Nature of Population by the Concept of Effective Change. International Journal of Current Science, 12(2), 897-907.

[4] Arya, D. V., Sharma, P. K., Sharma, M. B., & Gupta, S. (2022). Vedic Sanskar (D-Impaction) Model: The Characterization of Moral Standard of Human Beings on the Basis of Sanskars and Kusanskars. International Journal of Creative Research Thoughts, 10(1), b241-b249.

[5] Economic Survey 2019-20: Statistical Appendix, Table 1.12 Index Numbers of Agricultural Production available on thehindubusinessline.com https://www.thehindubusinessline.com › ...PDF Economic Survey 2021-22 Statistical Appendix – The Hindu Business Line, Volume 2, pp. 31 (31 May 2022)

[6] Economic Survey 2019-20: Statistical Appendix, Table 1.13 Index Numbers of Area under Principal Crops available on thehindubusinessline.com https://www.thehindubusinessline.com › ...PDF Economic Survey 2021-22 Statistical Appendix – The Hindu Business Line, Volume 2, pp. 32 (31 May 2022)

[7] Economic Survey 2019-20: Statistical Appendix, Table 1.14 Index Numbers of Yield of Principal Crops available on thehindubusinessline.com https://www.thehindubusinessline.com › ...PDF Economic Survey 2021-22 Statistical Appendix – The Hindu Business Line, Volume 2, pp. 33 (31 May 2022)

[8] Economic Survey 2021-22: Statistical Appendix, Table 1.12 Index Numbers of Agricultural Production available on indiabudget.gov.in https://www.indiabudget.gov.in › ...PDF Economic Survey 2021-22 Statistical Appendix - Union Budget, pp. 34 (19 February 2022)

[9] Economic Survey 2021-22: Statistical Appendix, Table 1.13 Index Numbers of Area under Principal Crops available on indiabudget.gov.in https://www.indiabudget.gov.in › ...PDF Economic Survey 2021-22 Statistical Appendix - Union Budget, pp. 35 (19 February 2022)

[10] Economic Survey 2021-22: Statistical Appendix, Table 1.14 Index Numbers of Yield of Principal Crops available on indiabudget.gov.in https://www.indiabudget.gov.in › ...PDF Economic Survey 2021-22 Statistical Appendix - Union Budget, pp. 36 (19 February 2022)

[11] Foundation Programme: Business Economics Chapter 3 available on icsi.edu https://www.icsi.edu › websitePDF BUSINESS ECONOMICS – ICSI, pp. 345-346

[12] https://en.wikipedia.org/wiki/Derivative_test (20 April 2022)

[13] https://mathworld.wolfram.com/Extremum Test.html (20 April 2022)

[14] https://www.google.com/url?sa=t&source=web&rct=j&url=https://math.stackexchange.com/questions/1591520/what-is-difference-between-maxima-or-minima-and-global-maxima-or-minima&ved=2ahUKEwiyuszm8J33AhVmRmwGHStaDfYQFnoECAkQBQ&usg=AOvVaw2oHMM9vfV3UVfdRNAZWy3b (18 April 2022)

[15] https://www.google.com/url?sa=t&source=web&rct=j&url=https://www.mathworks.com/help/optim/ug/local-vs-global-optima.html&ved=2ahUKEwiyuszm8J33AhVmRmwGHStaDfYQFnoECAwQBQ&usg=AOvVaw3_hkBppNIPxE0Bw9dEHWxi (18 April 2022)

[16] Index Numbers available on ncert.nic.in https://ncert.nic.in › kest108PDF CHAPTER- VIII- Index Number Ch.8 (Ver-12).pmd – NCERT, pp. 108 (28 may 2022)

[17] Index Numbers available on nios.ac.in https://nios.ac.in › documentsPDF 11 INDEX NUMBERS – NIOS, pp. 204-205 (19 January 2022)

[18] Pocket Book of Agricultural Statistics 2017, available on agricoop.nic.in https://agricoop.nic.in › filesPDF Pocket Book of AGRICULTURAL STATISTICS, pp. 109 (19 January 2022)

[19] Pocket Book of Agricultural Statistics 2020, available on gov.in https://desagri.gov.in › ... Pocket book of Agricultural Statistics | Official website of Directorate of ..., pp. 116 (30 May 2022)