JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

Fuzzy Queuing Model Using DSW Algorithm With Dodecagonal Fuzzy Number

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ABSTRACT

In this paper we study fuzzy single server queuing model in dodecagonal fuzzy numbers using α- cut rule. The arrival and the service rate are fuzzy natures and also analyzed the performance measures of this model in dodecagonal fuzzy numbers. In the numerical example we are deduce that the efficiency of this model.

Key Words - Queuing theory, α-cut, Membership function, Dodecagonal fuzzy number, DSW algorithm, interval analysis.

Mathematical Subject classification; 93B22,60K25,68M20

1. INTRODUCTION

Queuing theory is the describe of waiting liens.it is great extent applications in service institutions as well as manufacturing units. Queue is quite common in many region for example, in Bank, in telephone exchange , in Supermarket , at petrol station ,at railway station, at computer system etc. Normally [7] a queue is created on a production channel when the customer waits for service when the numeral of customers exceeds the number of service facilities or the service features do not work properly, customers take longer to receive service because this server Takes more time to serve. it helps to determine the balance between cost of offering the service and cost transacted due to delay in offering service.

Queuing model was introduced by A.K.Erlang in 1909 [3].Queuing models are aids to determine the compose number of counters so as to satisfy the customers keeping the total cost minimum. It helps the customers should get service with minimum time. It tries to answer the questions like expected number of customer in the system and queue, the mean waiting time and system response time.

Fuzzy sets have been introduced by Lofti.A.Zadeh in 1965 [13], A fuzzy set is a class with no sharp boundary between membership function. Li.R.J and Lee.E.S.[4],fuzzy set theory and fuzzy linear programing introduced by Zimmermann.H.J.[14], multi-server queuing models using dsw algorithm with hexagonal fuzzy number. Narayanmoorthy.S and Ramya.L[6] many -server queuing models using DSW algorithm with fuzzy range. Shanmugasundaram.S and Venkadesh [7],[8],[9], define new membership function and new ranking function onheptagonal fuzzy number. Durai.K and Karpagam.A [2],discussed on comparison with heptagonal and octagonal fuzzy number.Shanmugasundaram.S and Thalmilselvi.K [15]. The approximate method dsw algorithm used to determine a membership function of performance measures in single server fuzzy queueing model with dodecagonal fuzzy number. The paper organization is to six part 1.be introduction, 2.bassic concepts,3.algorithm,4.numerical problam,5.results,6. discussion and conclusion and last 7.references.

2. BASIC CONCEPTS

Given a fuzzy set A, any α ∈ [0,1], for which A_F* = {(x ∈ U | μ_F (x)) =α} is not empty, is called a level of A.

2.1. Interval Calculation

A fuzzy range may be a dodecagonal fuzzy range denoted by $\mu_{A_{dodeca}}(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12})$ wherever $\mu_{A_{dodeca}}$ are real set and its membership perform $\mu_{A_{dodeca}}$ is given below.

$$\mu_{A_{dodeca}}(x) = \begin{cases} 0, & for\ a_0 \le x \le a_1 \\ k_1\left(\frac{x-a_1}{a_2-a_1}\right), & for\ a_1 \le x \le a_2 \\ k_1, & for\ a_2 \le x \le a_3 \\ k_1 + (1-k_1)\left(\frac{x-a_3}{a_4-a_3}\right), & for\ a_3 \le x \le a_4 \\ k_2, & for\ a_4 \le x \le a_5 \\ k_2 + (1-k_2)\left(\frac{x-a_5}{a_6-a_5}\right), & for\ a_5 \le x \le a_6 \\ 1, & for\ a_6 \le x \le a_7 \\ k_2 + (1-k_2)\left(\frac{x-a_7}{a_8-a_7}\right), & for\ a_7 \le x \le a_8 \\ k_2, & for\ a_8 \le x \le a_9 \\ k_1 + (1-k_1)\left(\frac{x-a_9}{a_{10}-a_9}\right), & for\ a_9 \le x \le a_{10} \\ k_1, & for\ a_{10} \le x \le a_{11} \\ k_1\left(\frac{x-a_{11}}{a_{12}-a_{11}}\right), & for\ a_{11} \le x \le a_{12} \\ 0, & for\ a_{12} \le x \\ where\ k & 0 \le k_1 \le k_2 \le 1 \end{cases}$$

Let C and D be two interval numeral described by controlled couple of actual digit with lowest data and highest data.

$C = \{C_1, C_2, C_3, C_4\}$ and $D = \{D_1, D_2, D_3, D_4\}$

Delineate a calculation rule with the image * Wherever ={+,−,·,÷} notional the operation.

I. $C + D = \{C_1 + D_1, C_2 + D_2, C_3 + D_3, C_4 + D_4\}$
II. $C - D = \{C_1 + D_4, C_2 + D_3, C_3 + D_2, C_4 + D_1\}$
III. $C \times D = \{C_1D_1,\ C_2D_2,\ C_3D_3, C_4D_4\}$
IV. $C \div D = \{\frac{C_1}{D_4}, \frac{C_2}{D_3}, \frac{C_3}{D_2}, \frac{C_4}{D_1}\}$
V. $\alpha \times C = \{\alpha C_1, \alpha C_2, \alpha C_3, \alpha C_4\} = \begin{cases} [\alpha C_1, \alpha C_2, \alpha C_3, \alpha C_4]\ for\ \alpha > 0 \\ [\alpha C_4, \alpha C_3, \alpha C_2, \alpha C_1]\ for\ \alpha < 0 \end{cases}$

3. DSW ALGORITHM

This is the most straight forward technique for continuous membership perform is described by a nonstop curve of α-level in term from α= zero to α = one. Yet we've one input mapping given by Ɏ = f (X) that's to be enlarged for fuzzy sets Ƀ = f(X) and that we wish to disintegrate A into the list of α-level interval say I_α. It uses total α numbers in a very interval analysis and arithmetic's the DSW algorithmic rule consists of the subsequent postulates:-

$\mathbf{P_1}$. choose α wherever, zero ≤ α ≤ one.

$\mathbf{P_2}$. find the intervals within the input membership functions that correspond to the current α.

$\mathbf{P_3}$. calculate the interval for the output membership perform and select α level.

$\mathbf{P_4}$. If reuse the process steps 1-3 for different values of α has been to find out.

3.1. $M^F/M^F/1$;∞/FCFS Model

$M^F/M^F/1$ model assumptions:

The arrivals follow Poisson dissemination, with a mean arrival rate λ.
The service time has exponential dissemination, average service rate μ.
Arrivals are infinite range of population.
The first-come-first-served rule is to get serve customers.
The server range is one queue model.

possibility of no customers in the system is $P_0 = \frac{1}{1+\sum_{n=1}^{\infty}\prod_{i=0}^{n-1}\frac{\lambda_i}{\mu_{i+1}}} = 1-\rho$

The possibility of exactly one client in the system is $P_1 = \rho P_0$.

Similarly the probability of having two customers in the system would be

$P_3 = \rho P_1 = \rho 2\ P_0$

And the possibility of having exactly n customers in the system is

$\sum_{n=0}^{\infty} P_n = P_0 + P_0 \sum_{n=1}^{\infty}\prod_{i=1}^{n-1}\frac{\lambda_i}{\mu_{i+1}} = 1,$

Average expected range of customer within the system

$$(\mathrm{AEL_s}) = \sum_{n=0}^{\infty} nP_n$$

$$= \sum_{n=0}^{\infty} n.(1-\rho)\rho^n = (1-\rho).\rho \sum_{n=1}^{\infty} n.\rho^{n-1}$$

$$= \rho(1-\rho).[1+2\rho+3\rho+\cdots]$$

$$\mathrm{AEL_s} = \frac{\rho.(1-\rho)}{(1-\rho)^2} = \frac{\rho}{1-\rho} = \frac{x}{y-x}$$

To find average expected numeral of customer within the queue

$$(\mathrm{AEL_q}) = \sum_{n=1}^{\infty}(n-1)P_n$$

Since there are (n-1) units in the queue excluding one in the service

$$=\sum_{n=1}^{\infty} n\,P_n - \sum_{n=1}^{\infty} P_n$$

$$= \mathrm{EL_s} - \left[\sum_{n=1}^{\infty} P_n - P_0\right]$$

$$\mathrm{AEL_q} = \frac{\rho}{1-\rho} - 1 + (1-\rho) = \frac{\rho^2}{1-\rho} = \frac{x^2}{y(y-x)}$$

The probability density function of waiting time distribution.

At any time, let there be n units in the system, (n-1) waiting and one in service. If one unit arrive at this time, then it has to wait for time between ω and $\omega + d\omega$ only when (n-1) waiting units are served in time ω and one unit in service is served in time $d\omega$.

Since servicing distribution is also poisson. Probability [(n-1) units waiting are served in time ω]

$$=\frac{\mu\omega^{n-1}.e^{-\mu\omega}}{(n-1)!}$$

And probability of [one unit in service is served in time ω] $=\mu d\omega$

Probability that a new arrival is taken into service after a time lying between ω and $\omega+d\omega = \psi_n(\omega)\,d\omega$

Probability [(n-1) units waiting are served in time ω] × Probability [one unit in service is served in time $d\omega$]

$$=\frac{\mu\omega^{n-1}.e^{-\mu\omega}}{(n-1)!}.\mu d\omega$$

Let w be the waiting time of a unit who has to wait s.t. $\omega \le W \le \omega + d\omega$.

Since the queue length can vary between 1 and ∞, therefore, the probability of the density function to the waiting time is given by

$$\psi_n(\omega)\,d\omega = \sum_{n=1}^{\infty} P_n.\psi_n(\omega)\,d\omega = \sum_{n=1}^{\infty}(1-\rho)\rho^n.\frac{\mu\omega^{n-1}.e^{-\mu\omega}}{(n-1)!}.\mu d\omega$$

$$=\mu\rho(1-\rho).e^{-\mu\omega}\sum_{n=1}^{\infty}\frac{(\rho\mu\omega)^{n-1}}{(n-1)}\,d\omega$$

$$\psi(\omega)d\omega = \rho(\mu-\lambda)e^{-(\mu-\lambda)\omega}\,d\omega \qquad \omega > 0$$

The average expected waiting time or excluded service time within the queue $(\mathrm{AEW}_q) = \int_0^{\infty}\omega\,\psi(\omega)\,d\omega$

$$=\int_0^{\infty}\omega\,\mu\rho(1-\rho)e^{-(1-\rho)\mu\omega}\,d\omega = \mu\rho(1-\rho).\int_0^{\infty}\omega e^{-(1-\rho)\mu\omega}\,d\omega$$

$$AEW_q = \frac{\rho}{\mu(1-\rho)} = \frac{1}{y(y-x)}$$

Average expected waiting period and including service time also within the system $(\mathrm{AEW}_s) = \int_0^{\infty} v.\mu(1-\rho)e^{-(1-\rho)\mu v}\,dv$

$$AEW_s = \mu\rho(1-\rho)\int_0^{v} v.e^{-(1-\rho)\mu v}\,dv = \frac{1}{\mu(1-\rho)} = \frac{1}{y-x}$$

4. NUMERICAL EXAMPLES

Consider a FM/FM/1 queue, where the both the arrival rate and service rate are dodecagonal fuzzy numbers represented by Ƀ=[1,2,3,4,5,6,7,8,9,10,11,12] and Ɏ =[21,22,23,24,25,26,27,28,29,30,31,32].

The confidence interval at the probability level α as [1+α, 12−α] and [21+α, 32−α].

Where, x=[1+α, 12−α] and y = [21+α, 32−α].

$\mathrm{AEL_s} = \frac{x}{y-x}$, $\mathrm{AEL_q} = \frac{x^2}{y(y-x)}$, $\mathrm{AEW_s} = \frac{1}{y-x}$, $\mathrm{AEW_q} = \frac{1}{y(y-x)}$

Table - The α cuts of AEL_s AEL_q AEW_s AEW_q at **α** values

α	AEL_S	Customer	AEL_q	Customer	AEW_s	Time	AEW_q	Time
0	**0.0500,**	**0.6000**	**0.95238,**	**90.0000**	**0.05**	**0.05**	**0.9523,**	**7.5000**
0.1	0.0645,	0.5855	1.56326,	86.4863	0.05	0.05	1.2118,	7.3856
0.2	0.0880,	0.5620	2.84705,	80.8819	0.05	0.05	1.6176,	7.1959
0.3	0.1205,	0.5295	5.18349,	73.3233	0.05	0.05	2.1508,	6.9238
0.4	0.1620,	0.4880	9.03407,	64.0172	0.05	0.05	2.7882,	6.5591
0.5	0.2125,	0.4375	14.8969,	53.2608	0.05	0.05	3.5051,	6.0869
0.6	0.2720,	0.3780	23.2654,	41.4757	0.05	0.05	4.2767,	5.4862
0.7	0.3405,	0.3095	34.5961,	29.2601	0.05	0.05	5.0801,	4.7269
0.8	0.4180,	0.2320	49.2874,	17.4753	0.05	0.05	5.8956,	3.7662
0.9	0.5045,	0.1455	67.6690,	7.39249	0.05	0.05	6.7065,	2.5403
1	**0.6000,**	**0.0500**	**90.0000,**	**0.95238**	**0.05**	**0.05**	**7.5000,**	**0.9523**

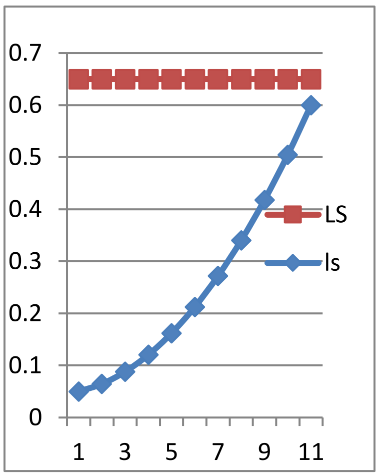

Fig:1 AEL_s

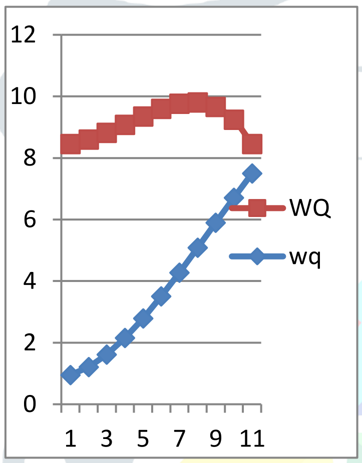

Fig: 2 AEW_q

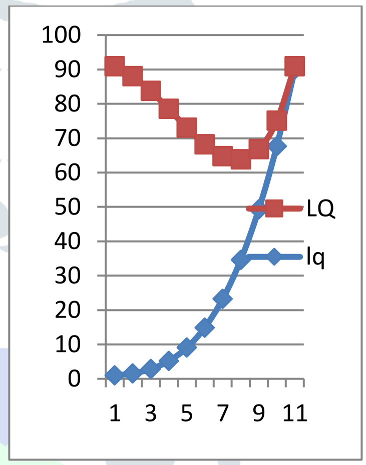

Fig: 3 AEL_q

We performed by using MATLAB of fuzzy arrival rate and fuzzy service rate and α cuts intervals also derived in table.

5. RESULTS

The performance measures from the table such as

1. Expected numeral of customers within the system (AEL_s) [0.5000, 0.6000]
2. Expected length of queue(AEL_q) [0.95238, 90.0000]
3. Average range expected waiting time in the system (AEW_s) [0.05, 0.05]
4. Average waiting time or excluded service time of a customer within the queue (AEW_q) [0.9523, 7.5000]

6. DISCUSSION AND CONCLUSION

Let's try to understand the result of single queue model. The result is based on the α level technique. The model gives the result using the Fuzzy dodecagonal number. The model has two intervals. One is the maximum and the other is the minimum. There is a contradiction in the results of both intervals. It follows that the minimum level of α results in the minimum interval. The same number occurs at the maximum interval of the maximum level of α. The time to service the queue in the model is very short. This explains the utility of the model here. Usability is hundred percent. In this model we look at the queue. There is no queue at the beginning of the model. It is inspired by first come first served. The queue starts increasing continuously along with the α level. Further experiments can be done in this model. Increased breakdown of customer behavior is an exception to uncertainty.

In the above discussion we study the performance measures of dodecagonal fuzzy number using α-cut the arrival and service times are fuzzy in nature. The numerical example shows its coincidence.

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