



MHD FLOW OF DUSTY VISCOUS FLUID THROUGH A POROUS MEDIUM BOUNDED BY AN OSCILLATING POROUS PLATE IN SLIP FLOW REGIME

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ABSTRACT:

A theoretical analysis for fluid velocity, dust particle velocity and skin-friction of the flow of dusty viscous an incompressible fluid of small electrical conductivity in porous medium. Near and oscillating infinite porous flat plate in slip flow regime under influence of transverse magnetic field of uniform strength. Fixed relative to the fluid has been carried out. The velocity of fluid and dust particle decreases with the increase in density of dust particle. But the skin- friction decreases with increase in density of dust particle.

INTRODUCTION:

Due to importance of dusty viscous flows in petroleum industry in the purification of crude oil, in physiological flows and in other technological fields, various studies have appeared in the literature. The dispersion and fall out of pollutants in air or in water have necessitated the study of the flow of dusty fluids. Saffman (1962) has formulated the basic

equations for the flow dusty fluid. Since then many researchers have discussed the problems of dusty fluid. Korchevskie and Marchochunik(1965), Michael and Miller (1966), Micheal and Norey(1968), Agrawal and Varshney(1986). The study of fluctuating flow is important in the paper industry and many other technological fields. Due to this reason many research works²⁻¹³ have paid their attention

towards the fluctuating flows of viscous, incompressible fluid parallel infinite plate. Stokes¹¹ and Reyleigh⁸ studied the flow of a viscous and incompressible fluid about an infinite flat wall which executes linear harmonic oscillations parallel to itself. Stuarts¹³ investigated the response of skin- friction and temperature of an infinite plate thermometer, to fluctuation in the stream with suction at the plate. Ong, R.S. and Nicholls⁶ extended the method to obtain the flow in a magnetic field near an infinite wall which oscillate in its own plan. Gupta and Babu⁴ studied the flow of a viscous incompressible fluid through a porous medium near an oscillating infinite porous flat plate in slip flow regime. This work is the extension of the paper of Gupta and Sharma (1990) with dusty viscous fluid.

Nomenclature

V velocity of suction or injection

K_0 permeability constant

H_0 strength of uniform transverse
magnetic field

N_0 density of dust particle

ρ density of fluid

d radius of spherical particle

m mass of the dust particle

M magnetic field parameter

R refraction parameter

μ Coefficient of kinematic viscosity

FORMULATION:

Let us consider an unsteady flow of an incompressible dusty viscous liquid u and v ; be the component of fluid velocity in x and y direction respectively v be the velocity of dust particle along x direction taken along and perpendicular to the porous plate. As the plate is infinite in length and uniform suction is imposed over it, the physical variables depend only on y and t . A uniform magnetic field H_0 is acting along the y axis. For problem in aeronautical engineering, the magnetic Reynold number is usually small. Under such conditions the induced magnetic field due to the flow may be neglected with respect to the applied field-the pressure P in the fluid is assumed constant, if V represents the velocity of suction or injection at the plate.

The equation of continuity

$$\frac{\partial u}{\partial x} = 0 \quad \dots$$

(1) With the condition $y = 0, v' = V$ leads to the result $v' = V$ every-where. The boundary layer equation describing the flow of a viscous, incompressible fluid through a porous medium (assumed highly porous) of permeability K in slip flow regime under the influence of a uniform

transverse magnetic field of strength H_0 , the equation in the form:

$$y^* = \frac{U_0 y}{\nu}, u^* = \frac{u}{U_0}, V^* = \frac{V}{U_0}, t^* = \frac{U_0^2 t}{\nu}, n^* = \frac{\nu n}{U_0^2}, K^* = \frac{U_0^2 K}{\nu^2}$$

$$\frac{\partial u}{\partial t} + V \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\nu}{K} u - \frac{\sigma \mu_e^2 H_0^2}{\rho} u + \frac{K_0 N_0}{\rho} (v - u)$$

From equation (2) and (3)

(2)

$$\frac{\partial u^*}{\partial t^*} + V \frac{\partial u^*}{\partial y^*} = \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\nu}{U_0^2} \frac{u^*}{K^*} - \frac{\sigma \mu_e^2 H_0^2 \nu u^*}{\rho U_0^2} + \frac{\nu K_0 N_0}{\rho U_0^2} (v^* - u^*)$$

And

$$m \left[\frac{\partial v}{\partial t} + (v \cdot \nabla) v \right] = K_0 (u - v) \quad \dots \quad (6)$$

(3)

$$\text{And } \frac{\partial v^*}{\partial t^*} = \frac{K_0}{m U_0^2} (u^* - v^*) \quad \dots$$

Where $\nu = \frac{\mu}{\rho}$, μ and ρ are the coefficient of

(7)

kinematic viscosity and corresponding density respectively K be the permeability, K_0 be the stocks resistance coefficient (for spherical particles of radius d , it is $\sigma \pi m_0 d$) N_0 be the density of dust particles and m be mass of dust particle. First order velocity slip boundary condition, when the plate execute linear harmonic oscillation in its own plane, is given by (Street¹²) –

$$\text{Let } \frac{K_0}{m U_0^2} = \frac{1}{w} \text{ and } \frac{m N_0 \nu}{\rho} = l \quad \dots$$

(8)

After dropping the stars

$$\frac{\partial u}{\partial t} + V \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \frac{u}{K} - M^2 u + \frac{l}{w} (v - u) \quad \dots$$

(9)

And

$$u = U_0 \cos(nt) + L_1 \frac{\partial u}{\partial y} \text{ at } y=0 \quad \dots (4)$$

$$\frac{\partial v}{\partial t} = \frac{1}{w} (u - v) \quad \dots$$

(10)

$$\text{And } u \rightarrow 0 \text{ as } y \rightarrow \infty \quad \dots (5)$$

Where $L_1 = (2 - m_1) \frac{L}{m_1}$, $L = \left[\frac{\pi}{2 p \rho} \right]^{1/2}$ is the

With boundary conditions

mean free path and m_1 the Maxwells reflection coefficient.

$$v = u = \cos(nt) + R \frac{\partial u}{\partial y} \text{ at } y=0 \quad \dots (11)$$

Introducing the following non dimensional quantities,

$$v = u = 0 \text{ as } y \rightarrow \infty \quad \dots (12)$$

Where $R = L_1 \frac{U_0}{v}$ and $\frac{\mu_e H_0 \sqrt{v\sigma}}{U_0 \rho} = M$ are the refraction and magnetic field parameters respectively.

SOLUTION OF THE PROBLEM:

Let the solution be assumed in the form:-

$$u = F(y) \cos(nt - Ay) \quad \dots (13)$$

$$v = G(y) \sin(nt - Ay) \quad \dots (14)$$

Where A is a Constant, it will be determined later on putting for u in (9) and for v in (10) and equating the coefficients of sine and cosine terms on both sides, we get,

$$2AF' - F|VA - n| + \frac{l}{w} g = 0 \quad \dots(15)$$

$$F'' - VF' - F\left(A^2 + \frac{1}{K} + M^2 + \frac{l}{w}\right) = 0 \quad \dots(16)$$

And $gn = \frac{F}{w}$

$$g = \frac{F}{nw} \quad \dots (17)$$

Equation (15) becomes-

$$2AF' - F\left(VA - n - \frac{l}{nw^2}\right) = 0 \quad \dots(18)$$

With boundary conditions-

$$|1 - AR \tan(nt)| F(y) = 1 + RF'(y) \quad \text{at } y = 0$$

$$F(y) \rightarrow 0 \quad \text{as} \quad F(y) \rightarrow \infty$$

...(19) In equation (16) and (18) dashes denote the differentiation with respect to y, the solution of equation (18).

$$F(y) = \frac{e^{-S\left|(n+\frac{l}{nw^2})A^{-1}-V\right|y}}{1 + .5R\left(\left(n+\frac{l}{nw^2}\right)A^{-1}-V\right) - AR \tan(nt)}$$

From equation (20) the value of F(y) in equation (16)

$$4A^4 + SA^2 - \left(n + \frac{l}{w^2n}\right) = 0$$

Where $S = \left(V^2 + 4\left(\frac{1}{K} + M^2 + \frac{l}{w}\right)\right)$
 ...(21)

Since A^2 is remain positive, hence

$$A^2 = \frac{1}{8} \left[-S + \sqrt{S^2 + 16\left(n + \frac{l}{nw^2}\right)^2} \right]$$

Therefore,

$$A = \frac{1}{2\sqrt{2}} \left[-S + \sqrt{S^2 + 16\left(n + \frac{l}{nw^2}\right)^2} \right]^{\frac{1}{2}} \quad \dots(22)$$

Neglecting negative value of A. To obtain u, eliminating F(y) from (13 and (20) we get,

$$u = \frac{e^{-S\left|(n+\frac{l}{nw^2})A^{-1}-V\right|y} \cos(nt - y)}{1 + .5R\left(\left(n+\frac{l}{nw^2}\right)A^{-1}-V\right) - AR \tan(nt)} \quad \dots(23)$$

And to obtain v eliminating g(y) from (14) by the help of (17) and (20)

$$v = \frac{1}{nw} \frac{e^{-.5\left(n+\frac{l}{nw^2}\right)A^{-1}-V}y \sin(nt-y)}{1+.5R\left(n+\frac{l}{nw^2}\right)A^{-1}-V}-AR \tan(nt)$$

...(24)

Where A is given in (22) the non dimensional skin-friction at the plate is

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{A \sin nt - .5\left(n+\frac{l}{nw^2}\right)A^{-1}-V \cos(nt)}{1+.5R\left(n+\frac{l}{nw^2}\right)A^{-1}-V}-AR \tan(nt)$$

...(25)

y	v l=0.2	v l=0.2	v l=0.2
0.000	0.132	0.131	0.126
0.225	0.060	0.057	0.049
0.450	0.027	0.025	0.019
0.675	0.012	0.011	0.007
0.900	0.006	0.005	0.003

Table 2

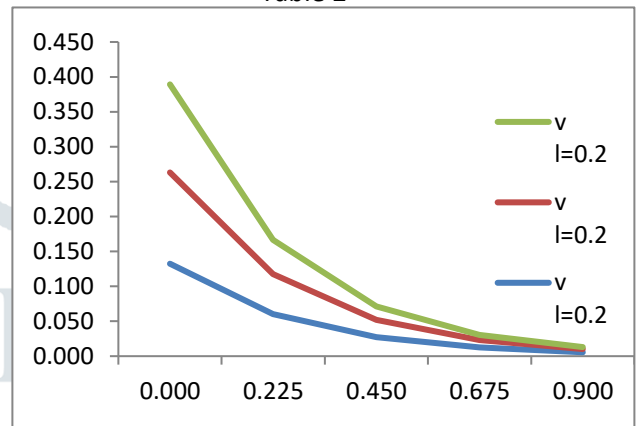


Figure 2

(Velocity profiles for the dust particle with different values of the density of dust particle)

y	u l=0.2	u l=0.3	u l=0.6
0.000	0.758	0.747	0.722
0.225	0.366	0.346	0.298
0.450	0.177	0.160	0.123
0.675	0.085	0.074	0.051
0.900	0.041	0.034	0.021

Table 1

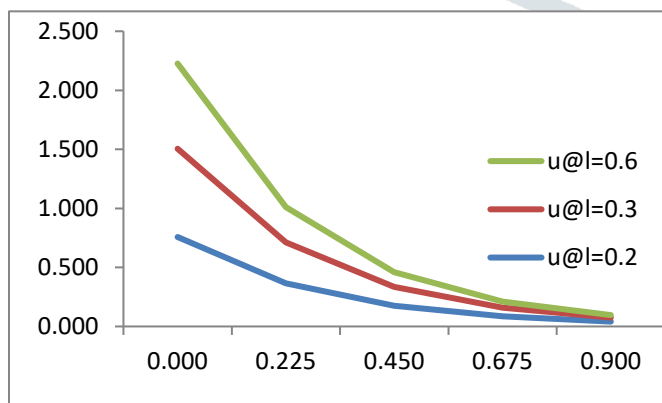


Figure 1

(Velocity profiles with different values of density of dust particle)

l	τ
0.2	-0.19208
0.3	-0.19445
0.4	-0.19705
0.5	-0.19972
0.6	-0.20238

Table 3

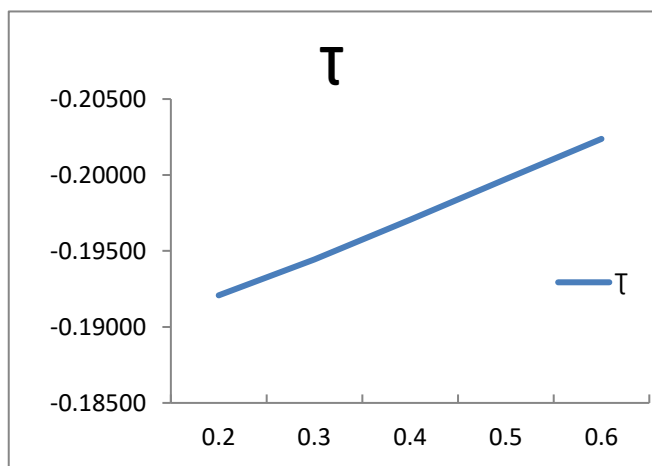


Figure3

(Skin-friction with different value of density of dust particles)

RESULT AND DISCUSSION:-

Figure (1) shows velocity profiles with different values of density of dust particle at constant $n=2$, $V=-2$, $t=2$, $w=0.2$, $M=1$, $R=0.1$ and $l=0.2, 0.3, 0.6$, we find that fluid velocity decreases with the increase of the density of dust particle.

Figure (2) shows velocity profiles for the dust particle with different values of the density of dust particle at constants $n=2$, $V=-2$, $w=0.2$, $M=1$, $R=0.1$, $K=1$ and $l=0.2, 0.3, 0.6$, we find that velocity of the dust particle also decreases with the increase in density of dust particles.

Figure (3) shows the skin-friction with different value of density of dust particles at constants $n=2$, $V=-2$, $w=0.2$, $M=1$, $K=1$, $R=0.1$ and $l=0.2, 0.3, 0.6$, we find that skin friction decreases with the increase of density of dust particles it is interesting to note that the velocity of dust particle is less than the velocity of the fluid.

Note: If we put $N_0=0$ we get the same results as Gupta and Sharma.

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