



Performance Analysis of Multi-Hop Relaying System Over $\alpha - \eta - \kappa - \mu$ Channel

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Abstract: Wireless Communication is an active area of research in communication technology, and its applications seem limitless. More effort is being devoted to the problem of channel modelling and shadowing in the channel with various propagation models and channel parameters. One of the possible scenarios is the multi-hop propagation system (which is an extension of two-hop system) used to connect source to sink via multiple intermediate relays. It is well known that shadowing and multi path fading are two effects that degrade communication signal during propagation. In this context, the recently developed α - η - κ - μ channel model can be used to model the behaviour of mm wave for the multi-hop communication system in aforesaid fading environment. The fading distribution α - η - κ - μ which is in fact a most general form of α - κ - μ and α - η - μ fading distributions, explores the linearity as well as nonlinearity of the wireless propagation medium. This paper investigates channel behaviour under multi-hop condition with the help of performance matrix Average Bit Error Rate using α - η - κ - μ fading model for multi-hop communication system in closed-form formulas.

Keywords: Multi-hop Communication, Signal to Noise Ratio (SNR), Amount of Fading (AoF), Bit Error Rate (BER), Average Bit Error Rate (ABER), Source to Sink Average Bit Error Rate, Average Channel Capacity (ACC) per Unit Bandwidth, α - η - κ - μ Fading Channel.

1. INTRODUCTION:

An accurate description of a signal is essential to correctly characterize the system with which it interacts. Wireless communication, in particular, are affected by a number of phenomena, such as interference, path loss and fading and the true depiction of the signal in this scenario is paramount for a proper system design. Short term fading describes the phenomenon that raises a great deal of interest and to which a considerable attention has been given. Various models are proposed to describe short term fading such as Rayleigh [1], Hoyt [2] and Rice [3]. These are most traditional and well established and their statistics has been described jointly in terms of envelope and phase, arising directly from the definition of their physical models. Bearing in mind, the distinguished importance of environmental characteristics, a number of fading models has been proposed that find wide acceptance in wireless communication. The more realistic models of fading environment are Nakagami-m [4] and Weibull [5], followed by α - μ [6], κ - μ [7] and η - μ [7]. Nakagami-m ranges from positive semi-Gaussian to Rayleigh and from Rayleigh to positive-Gaussian, depending upon the Nakagami parameter. Weibull provides a similar flexibility ranging from an impulse at the origin to negative exponential to Rayleigh to near positive Gaussian, depending upon the Weibull parameter.

The other fading models, namely α - μ , κ - μ and η - μ are more generalised physical models, first comprising Weibull and Nakagami-m, the second Rice and Nakagami-m and third Hoyt and Nakagami-m. Further developments have led to even more general envelop based fading models, namely α - κ - μ [8] and α - η - μ [8]. The first encompassing α - μ and κ - μ and the second comprising α - μ and η - μ .

Communication systems and techniques has been evolving dramatically moving from the personal-centred towards devices/things-centred systems and in a wireless fashion. Applications and services abound and the demand for new ones seem limitless. The wireless channel, previously restricted to outdoor environments, soon move to indoor ones, then to confined places, further to body area networks, to vehicle-to-vehicle to relay [9], and to all sorts of new scenarios.

It is obvious that any given fading model that applies to a given situation does not so to some other. Thus, it is desirable that most general models such as α - η - κ - μ [10] which comprise and explain all above said models has been proposed, with the parameters correctly describing the propagation phenomenon and physical environmental conditions.

In recent years, a lot of attention has been attracted by ad-hoc wireless networks, because of multiple potential applications, both civilian and military. Various approaches have been appeared in the literature for the study of this type of networks. Most of the studies focus on single-hop network but the real-life scenarios are closer to multi-hop network system. For example, in a smart-dust type sensor network [11], where nodes may be thrown over the terrain, it is very likely that the final distribution of the nodes will be irregular. The irregularity significantly affects the connectivity and bandwidth requirement of the network.

Along with this, in coming years, the multi-hop technique will play an important role in the wireless communication systems, as it can not only supply higher power density to coverage area but can also construct flexible service networks. However, the device-to-device (D2D) oriented communications power and bandwidth resources are scarce and expensive in multi-hop communication system. As a result, it is crucial to make effort to enhance the utilization efficiency of power and bandwidth resource.

In view of the above discussions, this paper derives expressions for S2S-ABER for multi-hop communication system. The expressions have been derived for most generalised cases of multi-hop communication system considering links with H-hops with independent and identical distribution [12]. Here H represents intermediate nodes, which may vary from 2 to 10, depending upon application and environmental conditions. Finally, the analytical results have been verified with the simulation results.

The remainder of this paper is organised as follows: Section 2 presents channel model and expressions for PDF of the channel have been derived. Section 3 describes the system models for single and multi-hop wireless communication system. In section 4, Source-to-Sink, Average Bit Error Rate (ABER) performance matrix for multi-hop wireless communication system has been obtained. In section 5, numerical and simulation results are discussed.

2. CHANNEL MODEL:

Considering the channel as α - η - κ - μ fading distribution, it can be represented by the Probability Density Function (PDF) as [06]

$$f_R(r) = \int_{-\frac{2\pi}{\alpha}}^{\frac{2\pi}{\alpha}} f_{R,\theta}(r, \theta) d\theta \quad (1)$$

where $f_{R,\theta}$ is the joint envelope and phase distribution of α - η - κ - μ channel and given as [06, Eq. 8]

Again, substituting $\theta' = \frac{2\pi}{\alpha} \theta$ in Eq. (1), we can rewrite it as

$$f_R(r) = \frac{2\pi}{\alpha} \int_{-1}^1 f_{R,\theta}\left(r, \frac{2\pi}{\alpha} \theta'\right) d\theta \quad (2)$$

The above Eq. 2 can also be represented as

$$f_R(r) = \frac{2\pi}{\alpha} \sum_{a=1}^{N_a} \omega_a f_{R,\theta}\left(r, \frac{2\pi}{\alpha} \theta_a\right) \quad (3)$$

The above equation is obtained using Legendre-Gauss Quadrature Integral approximation [13], where θ_a 's and ω_a 's are the nodes and weights taken as variables. There are various other methods available for

approximation, but we have used Legendre-Gauss Quadrature Integral approximation since it limits the computational error within 2%. Na is number of terms equal to 25, which is sufficient to neglect computational error.

Considering γ as instantaneous SNR and $\bar{\gamma}$ as average SNR, the two can be related using Eq.

$$\gamma = \bar{\gamma} \left(\frac{r}{\bar{r}} \right)^2 \quad (4)$$

Using Eq. (4) we obtain PDF of instantaneous SNR through random variable transformation given as

$$f_{\gamma}(\gamma) = \frac{2\pi}{\alpha} \sum_{a=1}^{N_a} \omega_a f_{\gamma, \theta} \left(\gamma, \frac{2\pi}{\alpha} \theta_a \right) \quad (5)$$

where $f_{\gamma, \theta} \left(\gamma, \frac{2\pi}{\alpha} \theta_a \right)$ is given as,

$$\begin{aligned} f_{\gamma, \theta} \left(\gamma, \frac{2\pi}{\alpha} \theta_a \right) = & A \frac{\gamma^{\frac{\alpha}{4}(\mu+2)-1} \left| \sin \left(\frac{2\pi}{\alpha} \theta_a \right) \right|^{\frac{\mu}{p+1}} \left| \cos \left(\frac{2\pi}{\alpha} \theta_a \right) \right|^{\frac{\mu p}{p+1}}}{2 \bar{\gamma}^{\frac{\alpha}{4}(\mu+2)}} \\ & \times \exp \left(-B \left(\eta \sin^2 \left(\frac{2\pi}{\alpha} \theta_a \right) + p \cos^2 \left(\frac{2\pi}{\alpha} \theta_a \right) \right) \left(\frac{\gamma}{\bar{\gamma}} \right)^{\frac{\alpha}{2}} \right) \\ & \times \exp \left(C \cos \left(\frac{2\pi}{\alpha} \theta_a - \phi \right) \left(\frac{\gamma}{\bar{\gamma}} \right)^{\frac{\alpha}{4}} \right) \\ & \times \frac{I_{\frac{\mu}{p+1}-1} \left(D \left| \sin \left(\frac{2\pi}{\alpha} \theta_a \right) \right| \left(\frac{\gamma}{\bar{\gamma}} \right)^{\alpha/4} \right)}{\cosh \left(D \left| \sin \left(\frac{2\pi}{\alpha} \theta_a \right) \right| \left(\frac{\gamma}{\bar{\gamma}} \right)^{\alpha/4} \right)} \times \frac{I_{\frac{\mu p}{p+1}-1} \left(E \left| \cos \left(\frac{2\pi}{\alpha} \theta_a \right) \right| \left(\frac{\gamma}{\bar{\gamma}} \right)^{\alpha/4} \right)}{\cosh \left(E \left| \cos \left(\frac{2\pi}{\alpha} \theta_a \right) \right| \left(\frac{\gamma}{\bar{\gamma}} \right)^{\alpha/4} \right)} \end{aligned} \quad (6)$$

The Envelope PDF for α - η - κ - μ fading environment represented by Eq. (5) is discussed and plotted in [14] for various values of α which is taken here for performance measurement.

3. SYSTEM MODEL:

Communication system model used in this paper is shown in Fig.1. Message bits x_0 are QPSK modulated and transmitted to the communication channel. Communication channel has been modelled by α - η - κ - μ distribution. At receiver side AWGN noise \mathbf{u} is added to the signals giving \widehat{x}_1 which after demodulation gives x_1 . Finally, Average BER has been calculated through message and received bits. x_0 , \mathbf{u} and \widehat{x}_1 are defined as

$$x_0 = [x_i : x_i \in \{0, 1\}, \forall i \in I^+] \quad (7)$$

$$\mathbf{u} = [u_i : u \sim N(0, \sigma^2)] \quad (8)$$

$$\widehat{x}_1 = h\widetilde{x}_0 + \mathbf{u} \quad (9)$$

where \widetilde{x}_0 is the mapped version of x_0 and I is the diagonal matrix with diagonal elements h_{ii} 's, which are random variables with α - η - κ - μ distribution. The performance of the system can be measured through Average BER (Λ).

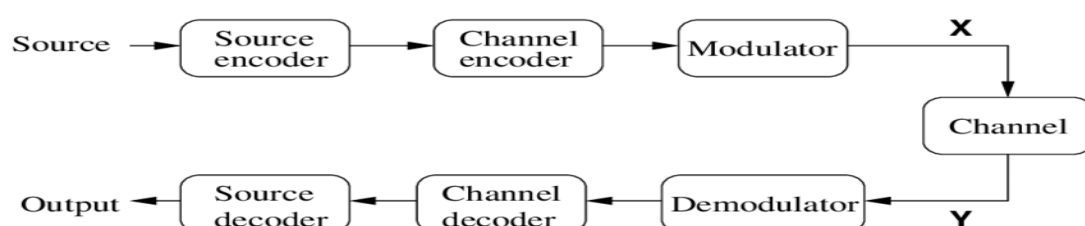


Fig 1. Basic Communication System Model

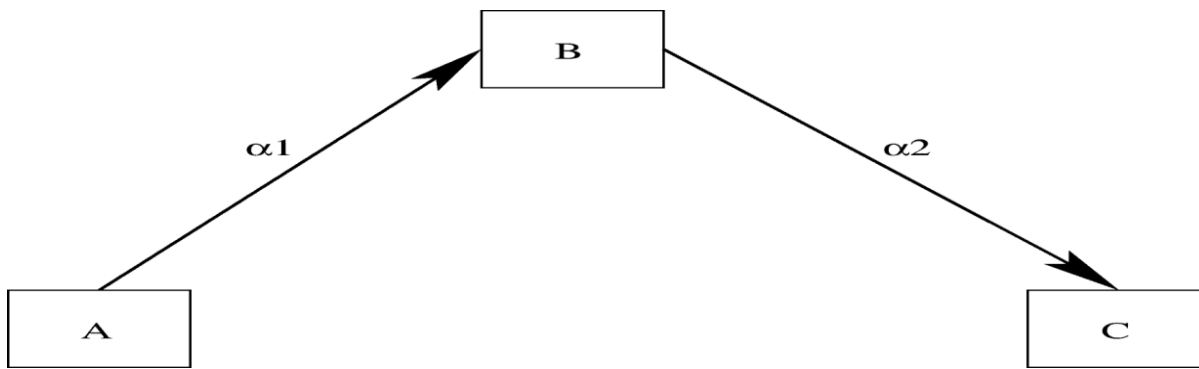


Fig 2. Multi-hop Communication Scenario

Fig.2 shows a typical multi-hop scenario with $H + 1$ nodes. The message passes from source to sink node in multiple hops using decode and forward technique. Each intermediate link is established as discussed in Fig. 1 and modelled as α - η - κ - μ fading channel. Hence, the received bits x_r at any intermediate node can be calculated after demodulation of \widehat{x}_r defined as,

$$\widehat{x}_r = h\widehat{x}_{r-1} + u \quad (10)$$

where \widehat{x}_{r-1} has same definition as of \widehat{x}_0 but for any intermediate node n_{r-1} . Λ_{Di} is the Average BER for i^{th} hop and $\Theta(H)$ is the S2S-ABER from source to sink node having H hops.

4. PERFORMANCE MEASURES:

Here we will discuss the aforesaid system model considering α - η - κ - μ fading condition and multi-hop network with H intermediate relays. The performance analysis is required so that we can analyse and accordingly optimise the system resources for efficient designing of wireless communication system. The three performance matrices namely AoF, ACC and ABER for two-hop network is already discussed in [14], and S2S-ABER is being extended for multi-hop wireless communication system.

4.1. S2S-Average Bit Error Rate (ABER)

Source to Sink Bit Error Rate with H intermediate relays is defined as cumulative average of BER of each individual link. Average Symbol Error Rate for coherent detection schemes for an arbitrary fading channel can be obtained as

$$\Lambda(\bar{\gamma}) = 2A_c \int_0^\infty Q(\sqrt{2B_c\bar{\gamma}}) f_{\bar{\gamma}}(\gamma) d\gamma \quad (11)$$

where A_c and B_c are constants for various modulation schemes.

From Eq. (6) and Eq. (11) ABER for QPSK modulation scheme over α - η - κ - μ channel is given as,

$$\Lambda(\bar{\gamma}) = F^0 \sum_{a=1}^{N_a} G_a^0 \sum_{b=1}^{N_b} \omega_b f_a(0, x_b) \int_0^{\pi/2} e^{\frac{-H_a x_b^2 / \alpha}{\sin^2(\delta)}} d\delta \quad (12)$$

The above equation can be solved through numerical methods using any mathematical software like MATLAB.

In multi-hop communication, data travel from source to sink through multiple hops. To judge the overall performance of multi-hop system, End-to-End or Source-to-Sink Average Bit Error Rate (S2S-ABER) has been calculated. In multi-hop scenario with H hops all having links with independent and identical distribution (iid) and same average SNR can be obtained as [15]

$$\Theta_{iid}(\bar{\gamma}) = \frac{1}{2} (1 - (1 - 2\Lambda(\bar{\gamma}))^H) \quad (13)$$

5. RESULTS AND DISCUSSION:

For the discussion of above performance matrix, three parameters set for various values of α , η , κ , μ , p and q are considered. Graph for S2S-ABER have been plotted for multi-hop communication system and compared with two-hop system. Moreover, the analytical result has been verified with the simulation result.

Table 1: α - η - κ - μ Distribution Parameters for Simulation

Parameter set	α	η	κ	μ	p	q
Set-1	2	3	4	3	2	1
Set-2	3	3	5	2	1	1
Set-3	4	2	3	3	1	1

As N_b increases, the results become more and more accurate. However, higher value of N_b increases the computation burden. Therefore, choice of N_b depends on the requirement of accuracy as per the application and available computational resources. Number of terms for other parameter sets can also be obtained in similar manner.

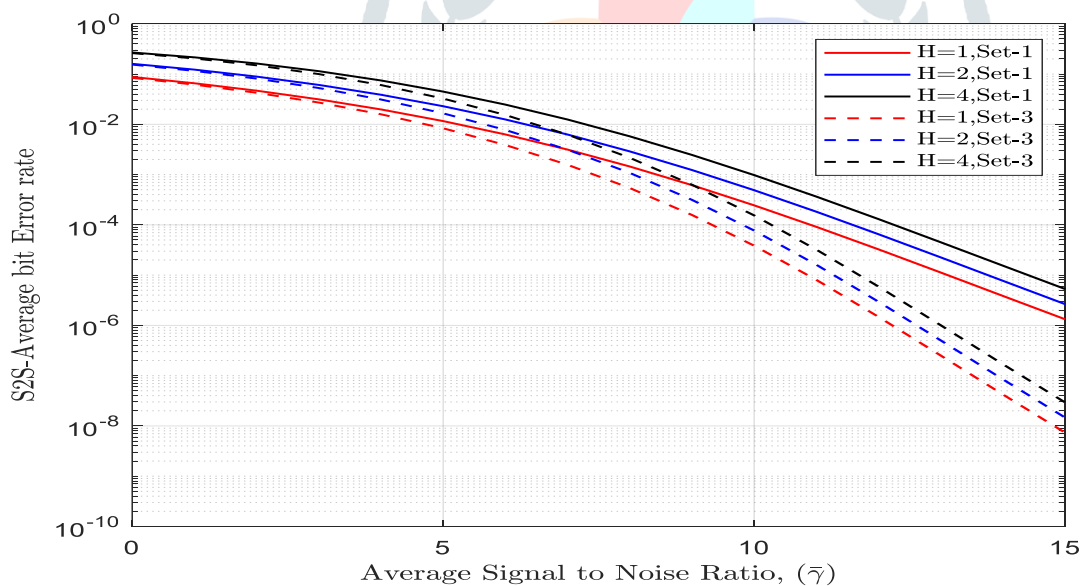


Fig.3 S2S-Average Bit Error rate w.r.t. Average SNR for various parameter set of α - η - κ - μ Channel

For multi-hop scenario, S2S-ABER results have been plotted for independent and identical links (iid). Following assumptions have been considered to simulate the multi-hop scenario:

- 1- Packets received at each node are decoded and forwarded to next hop.
- 2- Average SNR values of the packets received at each hop is assumed to be same.

Fig. 3 depicts variation in S2S-ABER for multi-hop scenario with iid links. It is evident that S2S-ABER value increases with increase in number of hops. For 15 dB average SNR, S2S-ABER increases from approximately 10^{-6} to 10^{-4} with variation in number of hops from 2 to 30. This result also discusses about the improvement in the power density distribution. Increasing the number of hops will improve the coverage area for given wireless communication system.

The above result of multi-hop system may be compared with two-hop system which is already plotted in [14] and the graph is shown below in Fig. 4.

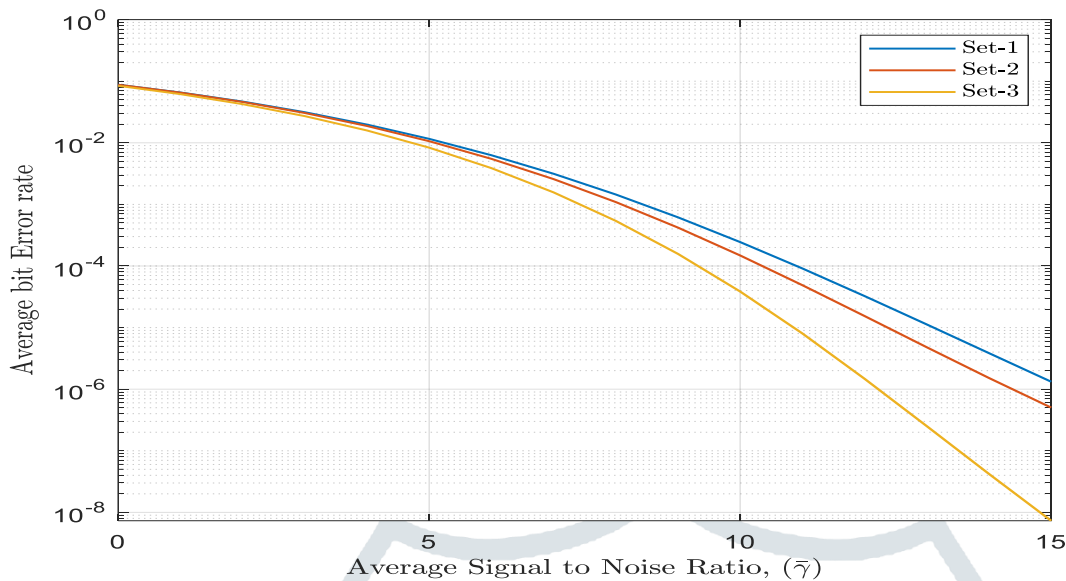


Fig 4. Average Bit Error Rate w.r.t. Average SNR for α - η - κ - μ Channel

When compared with the two-hop system, the power density and the coverage area for given territory is significantly improved. This is happening because of improvement in Bit Error Rate performance for multi-hop network. Two-hop system does not allow flexibility in variation in channel length corresponding to fading environment while multi-hop provide degree of freedom to change or vary channel length between hops with respect to the environmental conditions or availability of intermediate relays.

6. CONCLUSION

This paper derives the expression of instantaneous SNR for α - η - κ - μ fading channel through random variable transformation. The derived expressions have further been used to obtain S2S ABER for multi-hop wireless communication system and compared with two-hop result. The comparison shows better performance of multi-hop over two-hop. Here the multi-hop communication system is considered only for one scenario i.e., independent and identical links (iid) while the other scenario which is independent and non-identical links (inidd) is a special case of iid. Derived analytical expressions have been validated through simulations. This analysis can also be carried out through closed form expressions. However, the computational results obtained using closed form expressions are very close to the result obtained in this paper.

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