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A NEW METHOD FOR Φ_n USING GRILLS IN NANO TOPOLOGICAL SPACE.

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Abstract : In this paper, we introduce the concept of Φ_n in nano topological space $(U, \mathcal{N}, \mu)$ through the grill. We establish the relationship between open sets in nano topology and Φ_n sets in nano topological space. We launch some μ_Φ-function like μ_Φ-α, μ_Φ-pre, μ_Φ-$semi$, μ_Φ-β, μ_Φ-b. Few properties are deals with μ_Φ-function. Finally, we discussed some of the new sets: μ- $\star$ -closed set, μ- $\star$ - dense in itself, μ- $\star$ -perfect, and μ-co-dense.

Keywords: Φ_n, μ_Φ-α, μ_Φ-pre, μ_Φ-$semi$, μ_Φ-β, μ_Φ-b, μ- $\star$ - dense in itself, μ- $\star$ -perfect and μ-co-dense.

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1. Introduction

The functions Φ_n [1, 6] are the primary structure of the grill were introduced. Lellis Thivagar and Carmel Richard [3] proposed a nano topological space with respect to a subsets of X of an universe U which is defined in terms of lower approximation, upper approximation and boundary region of X. The members of nano topological space are called nano open sets. Some of the properties are portrayed using Kuratowski's [2] closure operator. In this paper, We introduce the combination of grill in nano topological space as μ_Φ-function .

2. Preliminaries

We recall the following definition in the sequel:

Definition 2.1 [2] *A non-empty sub-collection $\mathcal{G}$ of a space $\mathcal{X}$ which carries topology τ named grill on this space if the following conditions are true:*

1. $\phi \notin \mathcal{G}$.
2. $A \in \mathcal{G}$ and $A \subseteq B \subseteq \mathcal{X} \Rightarrow B \in \mathcal{G}$.
3. if $A \cup B \in \mathcal{G}$ for $A, B \subseteq \mathcal{X}$, then $A \in \mathcal{G}$ or $B \in \mathcal{G}$.

Definition 2.2 [6] *Let $(X, \tau, \mathcal{G})$ be a grill topological space. An operator $\Phi: \mathcal{P}(\mathcal{X}) \to \mathcal{P}(\mathcal{X})$ is defined as follows* : $\Phi(A) = \Phi_{\mathcal{G}}(A, \tau) = \{x \in \mathcal{X}: A \cap U \in \mathcal{G}$ for every open set U containing $x\}$ for each $A \in \mathcal{P}(\mathcal{X})$. The mapping Φ is called the operator associated with the grill $\mathcal{G}$ and the topology τ.

Definition 2.3 [3] *Let $\mathcal{U}$ be a nonempty finite set of members called the universe and $\mathcal{R}$ be an equivalence relation on $\mathcal{U}$ called as the indiscernibility relation. Objects belonging to the same equivalence class are said to be indiscernible with each other. The pair $(\mathcal{U}, \mathcal{R})$ is said to be the approximation space. Let $\mathcal{X} \subset \mathcal{U}$. Then,*

1. The lower approximation of $\mathcal{X}$ with respect to $\mathcal{R}$ is the set of all members, which can be for certain classified as X with respect to $\mathcal{R}$ and it is denoted by $\mathcal{L}_{\mathcal{R}}(\mathcal{X})$.That is,

$$\mathcal{L}_{\mathcal{R}}(\mathcal{X}) = \cup_{x \in \mathcal{U}} \{\mathcal{R}(\mathcal{X}): \mathcal{R}(\mathcal{X}) \subseteq \mathcal{X}\},$$

where $\mathcal{R}(\mathcal{X})$ denoted the equivalence class determined by $\mathcal{X}$.

2. The upper approximation of $\mathcal{X}$ with respect to $\mathcal{R}$ is the set of all members, which can be possibly classified as $\mathcal{X}$ with respect to $\mathcal{R}$ and it is denoted by $\mathcal{U}_{\mathcal{R}}(\mathcal{X})$. That is,

$$\mathcal{U}_{\mathcal{R}}(\mathcal{X}) = \cup_{x \in \mathcal{U}} \{\mathcal{R}(\mathcal{X}): \mathcal{R}(\mathcal{X}) \cap \mathcal{X} \neq \phi\},$$

3. The boundary region of $\mathcal{X}$ with respect to $\mathcal{R}$ is the set of all members, which can be neither in nor as not-$\mathcal{X}$ with respect to $\mathcal{R}$ and it is denoted by $\mathcal{B}_{\mathcal{R}}(\mathcal{X})$. That is,

$$\mathcal{B}_{\mathcal{R}}(\mathcal{X}) = \mathcal{U}_{\mathcal{R}}(\mathcal{X}) - \mathcal{L}_{\mathcal{R}}(\mathcal{X}).$$

Definition 2.4 [3] *If $(\mathcal{U},\mathcal{R})$ is an approximation space and $\mathcal{X},\mathcal{Y} \subseteq \mathcal{U}$, then*

1. $\mathcal{L}_{\mathcal{R}}(\mathcal{X}) \subseteq \mathcal{X} \subseteq \mathcal{U}_{\mathcal{R}}(\mathcal{X})$.
2. $\mathcal{L}_{\mathcal{R}}(\phi) = \mathcal{U}_{\mathcal{R}}(\phi) = \phi$ and $\mathcal{L}_{\mathcal{R}}(\mathcal{U}) = \mathcal{U}_{\mathcal{R}}(\mathcal{U}) = \mathcal{U}$.
3. $\mathcal{U}_{\mathcal{R}}(\mathcal{X} \cup \mathcal{Y}) = \mathcal{U}_{\mathcal{R}}(\mathcal{X}) \cup \mathcal{U}_{\mathcal{R}}(\mathcal{Y})$.
4. $\mathcal{U}_{\mathcal{R}}(\mathcal{X} \cap \mathcal{Y}) \subseteq \mathcal{U}_{\mathcal{R}}(\mathcal{X}) \cap \mathcal{U}_{\mathcal{R}}(\mathcal{Y})$.
5. $\mathcal{L}_{\mathcal{R}}(\mathcal{X} \cup \mathcal{Y}) \supseteq \mathcal{L}_{\mathcal{R}}(\mathcal{X}) \cup \mathcal{L}_{\mathcal{R}}(\mathcal{Y})$.
6. $\mathcal{L}_{\mathcal{R}}(\mathcal{X} \cap \mathcal{Y}) = \mathcal{L}_{\mathcal{R}}(\mathcal{X}) \cap \mathcal{L}_{\mathcal{R}}(\mathcal{Y})$.
7. $\mathcal{L}_{\mathcal{R}}(\mathcal{X}) \subseteq \mathcal{L}_{\mathcal{R}}(\mathcal{Y})$ and $\mathcal{U}_{\mathcal{R}}(\mathcal{X}) \subseteq \mathcal{U}_{\mathcal{R}}(\mathcal{Y})$ whenever $\mathcal{X} \subseteq \mathcal{Y}$.
8. $\mathcal{U}_{\mathcal{R}}(\mathcal{X}^c) \subseteq [\mathcal{L}_{\mathcal{R}}(\mathcal{X})]^c$ and $\mathcal{L}_{\mathcal{R}}(\mathcal{X}^c) = [\mathcal{U}_{\mathcal{R}}(\mathcal{X})]^c$.
9. $\mathcal{U}_{\mathcal{R}}[\mathcal{U}_{\mathcal{R}}(\mathcal{X})] = \mathcal{L}_{\mathcal{R}}[\mathcal{U}_{\mathcal{R}}(\mathcal{X})] = \mathcal{U}_{\mathcal{R}}(\mathcal{X})$.
10. $\mathcal{L}_{\mathcal{R}}[\mathcal{L}_{\mathcal{R}}(\mathcal{X})] = \mathcal{U}_{\mathcal{R}}[\mathcal{L}_{\mathcal{R}}(\mathcal{X})] = \mathcal{L}_{\mathcal{R}}(\mathcal{X})$.

Definition 2.5 [3] *Let $\mathcal{U}$ be the universe $\mathcal{R}$ be an equivalence relation on $\mathcal{U}$ and $\tau_{\mathcal{R}}(\mathcal{X}) = \{\mathcal{U},\phi,\mathcal{U}_{\mathcal{R}}(\mathcal{X}),\mathcal{L}_{\mathcal{R}}(\mathcal{X}),\mathcal{B}_{\mathcal{R}}(\mathcal{X})\}$, where $\mathcal{X} \subset \mathcal{U}$. Then $\tau_{\mathcal{R}}(\mathcal{X})$ satisfies the following axioms:*

1. $\mathcal{U}$ and $\phi \in \tau_{\mathcal{R}}(\mathcal{X})$.
2. The union of the members of any subcollection of $\tau_{\mathcal{R}}(\mathcal{X})$ is in $\tau_{\mathcal{R}}(\mathcal{X})$.
3. The intersection of the members of any finite subcollection of $\tau_{\mathcal{R}}(\mathcal{X})$ is in $\tau_{\mathcal{R}}(\mathcal{X})$.

That is, $\tau_{\mathcal{R}}(\mathcal{X})$ is a topology on $\mathcal{U}$ called the Nano topology on $\mathcal{U}$ with respect to $\mathcal{X}$. $(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{X}))$ is called the Nano topological space. Members of the Nano topology are called nano open sets in $\mathcal{U}$. Elements of $[\tau_{\mathcal{R}}(\mathcal{X})]^c$ are called Nano closed sets.

Definition 2.6 [3] *If $(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{X}))$ is Nano topological space with respect to A, where $A \subseteq X$. If $B \subseteq X$, then*

1. The Nano interior of the set B defined as the union of all Nano open subsets contained in B and is defined by $NInt(B)$. That is $NInt(B)$ is the greatest Nano open subset of B .
2. The Nano closure of the set B defined as the intersection of all Nano closed containing B and is denoted by $NCl(B)$. $(i.e)NCl(B)$ is the smallest Nano closed set containing B.

Here, we denoted as $(\mathcal{U},\mathcal{N})$ be a nano topological spaces, (i.e)$N = \tau_R(x)$.

Definition 2.7 *A subset P of a space $(U,\mathcal{N})$ is called*

1. nano α-open [3] if $P \subseteq n\text{-}int(n\text{-}cl(n\text{-}int(P)))$.
2. nano semi-open [3] if $P \subseteq n\text{-}cl(n\text{-}int(P))$.
3. nano pre-open [3] if $P \subseteq n\text{-}int(n\text{-}cl(P))$.
4. nano b-open [4] if $P \subseteq n\text{-}int(n\text{-}cl(P)) \cup n\text{-}cl(n\text{-}int(P))$.
5. nano β-open [5] if $P \subseteq n\text{-}cl(n\text{-}int(n\text{-}cl(P)))$.

The complements of the above mentioned sets are called their respective closed sets.

This paper are appear as follows: section 3 represents Φ_n in grill nano topological spaces. Some of its basic properties are derived. We discuss the relationship between nano topological space and grill nano topological space using Φ_n. We discussed with weak kinds of Φ_n sets in grill nano topology.μ- $\star$ -closed set, μ- $\star$ -dense in itself, μ- $\star$ -perfect and μ-co-dense are learned in grill nano topological spaces. The conclusion part is set forth in section 4.

3. Φ_n on nano topological spaces

In this section, we introduce the concept of grill on nano topological spaces as the grill function on nano topological spaces. Here, we discussed some of new open sets in the grill function on nano topological spaces.

Let $(\mathcal{U},\mathcal{N})$ be a nano topological spaces with $P,Q \subseteq \mathcal{U}$ and $\mathcal{G}$ be a grill on $\mathcal{U}$. If P is said to be grill on nano topological space then, the following conditions are true:

1. $\phi \notin \mathcal{G}$.
2. If $P \in \mathcal{G}$ and $P \subseteq Q \subseteq \mathcal{U} \Rightarrow Q \in \mathcal{G}$.
3. If $P \cup Q \in \mathcal{G}$ for $P,Q \subseteq \mathcal{U}$, then $P \in \mathcal{G}$ or $Q \in \mathcal{G}$.

It can also represent as $(\mathcal{U},\mathcal{N},\mathcal{G})$ or $(\mathcal{U},\mathcal{N},\mu)(i.e)\mathcal{G} = \mu$. The interior of nano topological space is $n\text{-}int(P)$ can be treat as μ open set. And the closure of nano topological space is $n\text{-}cl(P)$ can be consider as μ closed set.

Remark 3.1 *Let $(\mathcal{U},\mathcal{N},\mu)$ be a grill on nano topological space. The minimal grill is $\mu = \{\mathcal{U}\}$. The maximum grill is $\mu = \mathcal{P}(\mathcal{U})\backslash\{\phi\}$.*

Let $(\mathcal{U},\mathcal{N},\mu)$ be a nano topological spaces on grill and $Q_n = \{Q_n : x \in Q_n$, for every $Q_n \in (\mathcal{U},\mathcal{N},\mu)\}, \forall n \in I$ (I means Index value).

Definition 3.2 *Let $(\mathcal{U},\mathcal{N},\mu)$ be a nano topological spaces using μ on $\mathcal{U}$. We define a mapping $\Phi_n : \mathcal{P}(\mathcal{U}) \to \mathcal{P}(\mathcal{U})$ by $\Phi_n(P) = \{x \in \mathcal{U} : P \cap Q_n \in \mathcal{G}, \forall Q_n \in (\mathcal{U},\mathcal{N},\mu)\}$, $\forall P \in P(U)$ and it is denoted as μ_{Φ}-function.*

Remark 3.3 *Let $(\mathcal{U},\mathcal{N},\mu)$ be a nano topological spaces on grill and $P \subseteq \mathcal{U}$. If $P \cap Q_n \notin \mathcal{G}$, then it is consider as empty set $\{\phi\}$.*

Example 3.4 *Let* $\mathcal{U} = \{p_1, p_2, p_3, p_4\}$, $\mathcal{U}/\mathcal{R} = \{\{p_1\}, \{p_3\}, \{p_2, p_4\}\}$,$\mathcal{X} = \{p_1, p_2\} \subset \mathcal{U}$. *Then, the nano topology is* $\mathcal{N} = \{\phi, \mathcal{U}, \{p_1\}, \{p_2, p_4\}, \{p_1, p_2, p_4\}\}$.

1.If $\mu = \{\{p_4\}, \{p_1, p_4\}, \{p_2, p_4\}, \{p_3, p_4\}, \{p_1, p_2, p_4\}, \{p_1, p_3, p_4\}, \{p_2, p_3, p_4\}, \mathcal{U}\}$, then $\Phi_n(\{p_2, p_3, p_4\}) = \{\{p_2, p_3, p_4\} \in \mu\}$

2. If $\mu = \{\{p_1, p_3\}, \{p_1, p_2, p_3\}, \{p_1, p_3, p_4\}, \mathcal{U}\}$, then $\Phi_n(\{p_1, p_3\}) = \{\{p_3\} \notin \mu\}$.

Theorem 3.5 *Let* $(\mathcal{U}, \mathcal{N}, \mu)$ *be a nano topological spaces with grill* K *and* L *on* $\mathcal{U}$ *and* P, Q *are subsets of* $\mathcal{U}$. *Then*

1. $P \subset Q \Rightarrow \Phi_n(P) \subset \Phi_n(Q)$.
2. $\Phi_n(P) = cl(\Phi_n(P)) \subseteq cl(P)$.
3. $L \supseteq K$ on $\mathcal{U} \Rightarrow \Phi_n^K(P) \subseteq \Phi_n^L(P)$.
4. $\Phi_n(P) \cup \Phi_n(Q) = \Phi_n(P \cup Q)$.
5. $\Phi_n(P \cap Q) = \Phi_n(P) \cap \Phi_n(Q)$.
6. for every nano open set W, $W \cap \Phi_n(W \cap P) \subseteq \Phi_n(W \cap P)$.

Proof.

1. If $P \subset Q$, then $\Phi_n(P) \subset \Phi_n(Q)$. Let $a \in \Phi_n(P)$, then for every $Q_n \in Q_n(a), Q_n \cap P \in K$, since $Q_n \cap P \subset Q_n \cap Q$, then it shows $Q_n \cap Q \in K \Rightarrow a \in \Phi_n(Q)$.

2. First, we have prove that $\Phi_n(P) = cl(\Phi_n(P))$. So that $\Phi_n(P) = x \in (P \cap Q_n) \Rightarrow x \in P$ and $x \in Q_n \Rightarrow x \subseteq P$ and $x \subseteq Q_n \Rightarrow x \subseteq cl(P)$ and $x \in cl(Q_n) \Rightarrow x \in cl(P \cap Q_n) \Rightarrow x \in cl(\Phi_n(P)) \Rightarrow x \subseteq cl(\Phi_n(P))$. Hence, $\Phi_n(P) \subseteq cl(\Phi_n(P))$. The converse part is true. Thus, $\Phi_n(P) \supseteq cl(\Phi_n(P))$. Therefore, $\Phi_n(P) = cl(\Phi_n(P))$.

Second, we have to shows that $cl(\Phi_n(P)) \subseteq cl(P)$. So that $cl(\Phi_n(P)) = x \in cl(P \cap Q_n) \Rightarrow x \in cl(P) \cap cl(Q_n) \subseteq x \in cl(P) \Rightarrow x \subseteq cl(P)$. This shows, $cl(\Phi_n(P)) \subseteq cl(P)$.

3. For any $L \supseteq K$ on $\mathcal{U}$, $\Phi_n^K(P) \subseteq \Phi_n^L(P)$. Let $a \in \Phi_n^K(P)$, then for every $Q_n \in Q_n(a), Q_n \cap P \in K \Rightarrow Q_n \cap P \in L$. For that $a \in \Phi_n^L(P)$. This implies, $\Phi_n^K(P) \subseteq \Phi_n^L(P)$.

4. We know that(W.K.T) $P \subseteq P \cup Q$ & $Q \subseteq P \cup Q$ and then from (1), $\Phi_n(P) \subseteq \Phi_n(P \cup Q)$& $\Phi_n(Q) \subseteq \Phi_n(P \cup Q)$. Hence, $\Phi_n(P) \cup \Phi_n(Q) \subseteq \Phi_n(P \cup Q)$. Converse part is enough to prove that $\Phi_n(P \cup Q) \subseteq \Phi_n(P) \cup \Phi_n(Q)$. Let $x \in \Phi_n(P) \cup \Phi_n(Q) \Rightarrow x \in \Phi_n(P)$ or $x \in \Phi_n(Q) \Rightarrow x \subseteq P$ or $x \subseteq Q \Rightarrow x \subseteq P \cup Q \Rightarrow x \in \Phi_n(P \cup Q)$. So that $\Phi_n(P) \cup \Phi_n(Q) \subseteq \Phi_n(P \cup Q)$. It proves, $\Phi_n(P) \cup \Phi_n(Q) = \Phi_n(P \cup Q)$.

5. W.K.T $P \cap Q \subseteq P$ & $P \cap Q \subseteq Q$ and then from (1), $\Phi_n(P \cap Q) \subseteq \Phi_n(P), \Phi_n(P \cap Q) \subseteq \Phi_n(Q)$. So that $\Phi_n(P \cap Q) \subseteq \Phi_n(P) \cap \Phi_n(Q)$. It is enough to prove the converse part $\Phi_n(P) \cap \Phi_n(Q) \subseteq \Phi_n(P \cap Q)$. Let $x \in \Phi_n(P) \cap \Phi_n(Q) \Rightarrow x \in \Phi_n(P)$ and $x \in \Phi_n(Q) \Rightarrow x \subseteq P$ and $x \subseteq Q \Rightarrow x \subseteq P \cap Q \Rightarrow x \in \Phi_n(P \cap Q)$. So that $\Phi_n(P) \cap \Phi_n(Q) \subseteq \Phi_n(P \cap Q)$. Consequently, $\Phi_n(P \cap Q) = \Phi_n(P) \cap \Phi_n(Q)$.

6. If $(W \cap P) \subset P$, then $(W \cap \Phi_n(P)) \subseteq \Phi_n(P)$. Thence, $W \cap \Phi_n(W \cap P) \subseteq \Phi_n(W \cap P)$.

Definition 3.6 *Let subset* P *be a space of* $(\mathcal{U}, \mathcal{N}, \mu)$.

1. If P is said to be G_n-open (briefly, μ_Φ open set) then, $P \subseteq int(\Phi_n(P)))$.
2. If P is said to be G_n-closed (briefly, μ_c closed set) then, $P \subseteq cl(\Phi_n(P)))$.

Example 3.7 *From Example 3.4*, If $\mu = \{\{p_1\}, \{p_1, p_2\}, \{p_1, p_3\}, \{p_1, p_4\}, \{p_1, p_2, p_3\}, \{p_1, p_2, p_4\}, \{p_1, p_3, p_4\}, \{p_4\}, \{p_2, p_4\}, \{p_3, p_4\}, \{p_2, p_3, p_4\}, \mathcal{U}\}$, then the subset $P = \{p_4\}$ and

1. We get $\{p_2, p_4\}$ is μ_Φ open set.
2. We get $\{p_2, p_3, p_4\}$ is μ_c closed set.

Remark 3.8 *In a space* $(\mathcal{U}, \mathcal{N}, \mu)$, *the family of* μ_Φ *open set* $(i.e) P \subseteq int(\Phi_n(P))$ *and the family of* μ *open set* $(i.e) P \subseteq int(P)$ *are independent of each other as represent in the following example.*

Example 3.9 *Let* $\mathcal{U} = \{p_1, p_2, p_3, p_4\}$,$\frac{\mathcal{U}}{\mathcal{R}} = \{\{p_3\}, \{p_2\}, \{p_1, p_4\}\}$,$\mathcal{X} = \{p_1, p_3\} \subset \mathcal{U}$. *So, the nano topology is* $\mathcal{N} = \{\phi, \mathcal{U}, \{p_3\}, \{p_1, p_4\}, \{p_1, p_3, p_4\}\}$ *with* $\mu = \{\{p_2\}, \{p_1, p_2\}, \{p_2, p_3\}, \{p_2, p_4\}, \{p_1, p_2, p_3\}, \{p_1, p_2, p_4\}, \{p_2, p_3, p_4\}, \{p_4\}, \{p_1, p_4\}, \{p_3, p_4\}, \{p_1, p_3, p_4\}, \mathcal{U}\}$

1. If $A = \{p_3\}$, then μ open set is $\{p_3\}$ but not μ_Φ open set.
2. If $B = \{p_4\}$, then μ_Φ open set is $\{p_1, p_4\}$ but not μ open set.

Theorem 3.10 *In a space* $(\mathcal{U}, \mathcal{N}, \mu)$, *every* μ_c *closed set is a* μ *closed set.*

Proof.

Let assume that $Q_n = P$ in μ_c closed set. Then μ_c closed set is $P \subseteq cl(\Phi_n(P))) = cl(Q_n \cap P) = cl(P \cap P) = cl(P)$ by our assumption .Hence $P \subseteq cl(P)$. Therefore, it is a μ closed set.

Remark 3.11 *The converse part of the above theorem 3.10 need not be true. It can represent as a following example.*

Example 3.12 *From Example 3.9,* $C = \{p_1\}$*, then it is a* μ *closed set but not* μ_c *closed set.*

Definition 3.13 *A subset* P *of space* $(\mathcal{U}, \mathcal{N}, \mu)$ *is said to be*

1. Nano α-μ-open (briefly μ_Φ-α) if $P \subset int(cl(int(\Phi_n(P))))$.
2. Nano semi-μ-open (briefly μ_Φ-$semi$) if $P \subset cl(int(\Phi_n(P)))$.
3. Nano pre-μ-open (briefly μ_Φ-pre) if $P \subset int(cl(\Phi_n(P)))$.
4. Nano b-μ-open (briefly μ_Φ-b) if $P \subseteq int(cl(\Phi_n(P))) \cup cl(int(\Phi_n(P)))$.
5. Nano β-μ-open (briefly μ_Φ-β) if $P \subseteq cl(int(cl(\Phi_n(P))))$.

The above mentioned open sets as μ_Φ sets and their complements are called their respective closed sets as μ_c sets.

Theorem 3.14 *In a space* $(\mathcal{U}, \mathcal{N}, \mu)$*, every* μ_Φ-β *is* μ_c *closed set.*

Proof.

Let P be a subset of μ_Φ-β of $(\mathcal{U}, \mathcal{N}, \mu)$. So that $P \subseteq cl(int(cl(\Phi_n(P)))) \subseteq cl(int(\Phi_n(P))) \subseteq cl(\Phi_n(P))$. Therefore every μ_Φ-β is a μ_c closed set.

Remark 3.15 *The converse part of the above theorem 3.14 need not be true. It can represent as a following example.*

Example 3.16 *From Example 3.9,* $D = \{p_2\}$*, then it is a* μ_c *closed set but not* μ_Φ-β.

Remark 3.17 *In a space* $(\mathcal{U}, \mathcal{N}, \mu)$*, the family of* μ_Φ-α *and the family of* μ *open set are independent*

Example 3.18 *From the above remark 3.17, Lets we take the example 3.9,*

1. $\{p_3\}$ is μ open set but not μ_Φ-α.
2. $\{p_4\}$ is μ_Φ-α but not μ open set.

Theorem 3.19 *In In a space* $(\mathcal{U}, \mathcal{N}, \mu)$*, for a subset* P*, the following decisions are hold.*

1. P is μ_Φ-α iff P is both μ_Φ-pre and μ_Φ-$semi$.
2. P is μ_Φ-β iff P is both μ_Φ-pre and μ_Φ-$semi$.
3. P is μ_Φ-β iff P is μ_Φ-b.

Proof.

1. Let P be μ_Φ-α. Here we have to prove 2 parts, 1^{st}-$part$ is to prove that μ_Φ-pre and 2^{st}-$part$ is to prove that μ_Φ-$semi$. Now, we take the 1^{st}-$part$. So that $P \subseteq int(cl(int(\Phi_n(P)))) \subseteq cl(int(\Phi_n(P)))$. Hence μ_Φ-α is μ_Φ-$semi$. Now, we take the 2^{nd}-$part$ and assume that $\Phi_n(P) = int(\Phi_n(P))$. So that $P \subseteq int(cl(int(\Phi_n(P)))) = int(cl(\Phi_n(P)))$. Hence μ_Φ-α is μ_Φ-pre. Therefore, μ_Φ-α is both μ_Φ-pre and μ_Φ-$semi$.

Now we have to prove the converse part of the theorem. Let P be μ_Φ-pre and μ_Φ-$semi$ $(i.e)$ $P \subseteq int(cl(\Phi_n(P)))$ and $P \subseteq cl(int(\Phi_n(P)))$. So that $P \subseteq int(cl(cl(int(\Phi_n(P))))) = int(cl(int(\Phi_n(P))))$. Therefore, the both sets μ_Φ-pre and μ_Φ-$semi$ is μ_Φ-α, Hence the result.

2. Let P be μ_Φ-β. Here we have to prove 2 parts, 1^{st}-$part$ is to prove that μ_Φ-pre and 2^{st}-$part$ is to prove that μ_Φ-$semi$. Now, we take the 1^{st}-$part$. So that $P \subseteq cl(int(cl(\Phi_n(P)))) \subseteq cl(int(\Phi_n(P)))$. Hence, μ_Φ-β is μ_Φ-$semi$. Now, we take the 2^{nd}-$part$. So that $P \subseteq int(cl(int(\Phi_n(P)))) = int(cl(\Phi_n(P)))$ by the theorem 3.5 (2)$(i.e)\Phi_n(P) = cl(\Phi_n(P))$. Hence μ_Φ-β is μ_Φ-pre. Therefore, μ_Φ-β is both μ_Φ-pre and μ_Φ-$semi$.

Now we have to show the converse part. Let P be the both set μ_Φ-pre and μ_Φ-$semi$. So that $P \subseteq cl(int(int(cl(\Phi_n(P))))) \subseteq P = cl(int(cl(\Phi_n(P))))$. Therefore, the both sets μ_Φ-pre and μ_Φ-$semi$ is μ_Φ-β, Hence this shows the result.

3. Let P be μ_Φ-β. So that $P \subseteq cl(int(cl(\Phi_n(P)))) \subseteq cl(int(\Phi_n(P)))$ by the theorem 3.5 (2)$(i.e)\Phi_n(P) = cl(\Phi_n(P))$ $\Rightarrow cl(int(\Phi_n(P))) \subseteq cl(int(\Phi_n(P))) \cup int(cl(\Phi_n(P)))$. Therefore, μ_Φ-β is μ_Φ-b. Now we have to prove the converse part. Let P be μ_Φ-b. So that $P \subseteq cl(int(\Phi_n(P))) \cup int(cl(\Phi_n(P))) \subseteq cl(int(\Phi_n(P))) \subseteq cl(int(cl(\Phi_n(P))))$ by the theorem 3.5 (2)$(i.e)\Phi_n(P) = cl(\Phi_n(P))$. Therefore, μ_Φ-β is μ_Φ-b. Hence this proves the result.

Theorem 3.20 *In a space* $(\mathcal{U}, \mathcal{N}, \mu)$*, every* μ_Φ-α *is a* μ_Φ-b.

Proof.

Let P be a subset of μ_Φ-α of $(\mathcal{U}, \mathcal{N}, \mu)$. So that $P \subseteq int(cl(int(\Phi_n(P)))) \subseteq cl(int(\Phi_n(P))) \subseteq cl(int(\Phi_n(P))) \cup int(cl(\Phi_n(P)))$. Therefore every μ_Φ-α is a μ_Φ-b.

Remark 3.21 *The converse part of the above theorem 3.20 need not be true. It can represent as a following example.*

Example 3.22 *From Example 3.9,* $D = \{p_1, p_2, p_4\}$*, then it is a* μ_c *closed set but not* μ_Φ-β.

Definition 3.23 *A subset* P *be a nano topological space on grill* $(\mathcal{U}, \mathcal{N}, \mu)$ *is called a* μ-$\star$*-closed set if and only if (iff)* $\Phi_n(P) \subseteq P$*.* Then, the complement of closed set is called open set.

Definition 3.24 *A subset* P *of a nano topological space on* $(\mathcal{U}, \mathcal{N}, \mu)$ *is called a* μ-$\star$*-dense in itself (respectively (resp.)* μ-$\star$*-perfect) if* $P \subseteq \Phi_n(P)$ *(resp.* $P = \Phi_n(P)$*).* Then, complement of closed set is called open set.

Note: μ-$\star$-dense in itself $\Leftarrow$ μ-$\star$-perfect $\Rightarrow$ μ-$\star$-closed set

Definition 3.25 *A grill $\mathcal{G}$ in a space $(\mathcal{U}, \mathcal{N}, \mu)$ is called μ-co-dense if $Q_n \cap \mathcal{G} \neq \{\phi\}$.*

Example 3.26 *From the example 3.9, $W = \{p_1, p_2\}$ is μ- ⋆ -closed set. $X = \{p_4\}$ is μ- ⋆ -dense in itself. $Y = \{p_2\}$ is μ- ⋆ -perfect. $Z = \{p_1, p_3, p_4\}$ is μ-co-dense and also the above three examples.*

4. Conclusion

In this paper, we represent some of the open sets μ_Φ-function in grill on nano topological space. And some new results such as μ- ⋆ -closed set, μ-⋆-dense in itself, μ-⋆-perfect, and μ-co-dense are declared in grill on nano topological space. In the future, we can elaborate these concepts into grill space and other topological spaces like bitopology, neutrosophic topology, fuzzy topology, intuitionistic fuzzy graph, etc. We can develop some theories related to real-life applications

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