



# Numerical Dual Solution of stagnation point flow and Melting Heat transfer of Casson Fluid due to stretching sheet

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## 1. Introduction:

The boundary layer flow over a stretching sheet has gained much attention because of its wide applications in manufacturing processes for instance, extrusion of a polymer from a dye, chemical catalytic reactor and processes, geothermal reservoirs, etc. The pioneering work in this area has been studied by Crane [1970] who found the similarity solutions in closed form for steady two dimensional flow of an incompressible fluid flow along a stretching sheet, later this work has been extended by Gupta and Gupta [2], Carragher and Crane [3], Dutta et al. [4] by taking different parameters into consideration. The effects of viscoelastic parameter on temperature distribution of an incompressible second grade fluid over a stretching sheet were studied by Cortell [5]. The similarity solutions were obtained by considering suction/injection over a stretching sheet by Afify [6]. Hayat and Qasim [7] considered the thermal radiation, thermophoresis effects on MHD flow of Maxwell's fluid. Unsteady MHD boundary-layer flow and heat transfer due to stretching sheet in the presence of heat source or sink was done by Wubshet Ibrahim and Shanker [8]. Yohannes and shanker Bandari [9] studied on Heat and mass transfer in MHD flow of nano fluids Through a porous media due to a stretching sheet with viscous dissipation and Chemical reaction effects..

On the other hand, the study of heat transfer over a shrinking sheet is relatively a new consideration in the laminar boundary layer flows. The study on shrinking sheet was first investigated by Wang [10] by considering the stretching deceleration surface. The existence and uniqueness for viscous flow due to the shrinking sheet was discussed by Miklavcic and Wang [11] and they found that the dual solutions were reported within a particular range of the suction parameter. Different investigations were done by the researchers [12-17] by considering various parameters.

The solution of the classical problem for the dimensional fluid flow near a stagnation point flow was given by Hiemenz [18]. Chiam [19] extended the works of Hiemenz [16] replaced the solid body a stretching sheet with equal stretching and straining velocities and but he failed to get the boundary layer near the sheet. Later the reinvestigation on this work was done by Mahapatra and Gupta [20] by considering different stretching and straining velocities and they found two different kinds of boundary layers near the sheet depending on the ratio of the stretching and straining constants. Ramachandran et al. [21] investigated the steady laminar flow in two dimensional stagnation flows over heated surface by considering both cases of an arbitrary wall temperature and arbitrary surface heat flux variations. They found that a reverse flow developed in the buoyancy opposing flow region, and dual solutions are found to exist for a certain range of the buoyancy parameter. Based on the great developments in the study of stagnation point flows, few investigators studied the same flow for various fluids, like, micropolar fluid [22], power law fluid [23], nanofluid [24], upper convected Maxwell fluid [25] etc.,. On the other hand, the study of the stagnation region towards a shrinking sheet was given by Wang [26]. This problem was then extended by considering suction and injection by Bhattacharya and Layek [27] where as the unsteady stagnation point flow over a shrinking sheet with radiation effects was studied by Ali et al.[28]. Casson fluid is one kind of non Newtonian fluid and it exhibits the yield stress .The Casson fluid is a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs, and a zero viscosity at an infinite rate of shear, i.e., if a shear stress less than the yield stress is applied to the fluid, it behaves like a solid, whereas if a shear stress greater than yield stress is applied, it starts to move. The examples of Casson fluids are honey, Tomato soup and Human blood etc., . The literature pertaining to Casson fluid is presented [29-33].

The study of melting phenomena over a stretching sheet has prominent applications in industries which includes welding and magma solidification, permafrost melting and thawing of frozen ground etc., . The study of melting heat transfer over a flat surface in a laminar flow was done by Epstein and Cho [34]. This work was extended by Ishak et al.[35] by taking the moving plate into consideration. Kazmierczak [36] investigated the same work by considering the flat plate embedded in a porous medium in the presence of steady convection. The melting heat transfer of a couple stress fluids in the stagnation region towards a stretching sheet was investigated by Hayat et al. [37].The dual solutions for the melting heat transfer stagnation point flow over stretching / shrinking sheet was studied by Bachok et al.[38]. Kishore and Shanker Bandari [39] studied the stagnation point flow of a nanofluid over a stretching / shrinking sheet with nano particles as Al<sub>2</sub>O<sub>3</sub> , CuO<sub>2</sub> and some of the studies related to melting heat transfer over different fluids can be obtained from literature .

## 2. Mathematical Formulation:

Consider a steady two dimensional laminar flow of a Casson fluid over a horizontally stretching-shrinking sheet melting at a steady rate into a constant property, warm liquid of the same material, as depicted in the Fig1.

The x-axis is taken along the stretching surface in the direction of the motion with the slot as the origin, and the y-axis is perpendicular to the sheet in the outward direction towards the fluid. The flow is assumed to be confined in a region of  $y (>0)$ . It was assumed that the velocity of the ambient fluid is  $U_e = ax$  and the stretching sheet velocity is  $U_w = cx$  where  $a$  is a positive quantity,  $c$  is positive (stretching) or negative (shrinking) quantity,  $T_m$  is the temperature of the melting surface and  $T_\infty$  is the temperature of the ambient fluid where  $T_\infty > T_m$ . It is also assumed that the viscous dissipation and heat generation and absorption is neglected. The rheological equation of state for an isotropic and incompressible flow of the Casson fluid [Nakamura and Sawada (1988), Mustafa et al.(2011)] is given by

$$\tau_{ij} = 2(\mu_B + p_y/\sqrt{2\pi})e_{ij}, \pi > \pi_c,$$

$$2(\mu_B + p_y/\sqrt{2\pi_c})e_{ij}, \pi < \pi_c,$$

Where  $\mu_B$  and  $p_y$  are the plastic dynamic viscosity and the yield stress of the fluid respectively. Similarly  $\pi$  is the product of the component of deformation rate with itself,  $\pi = e_{ij}e_{ij}$ ,  $e_{ij}$  is the  $(i, j)$ -th component of the deformation rate and  $\pi_c$  is a critical value of this product based on the non-Newtonian model.

The governing equations are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = u_e \frac{\partial u_e}{\partial x} + \mu \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$\left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

Here  $u$  and  $v$  are velocities along  $x$  and  $y$  direction respectively,  $\mu$  is the dynamic viscosity parameter,  $\rho$  is density  $\beta$  ( $\beta = \mu_B \sqrt{2\pi_c / P_y}$ ) Casson parameter,  $C_p$  is specific pressure at constant pressure,  $k$  is thermal conductivity of the fluid and  $T$  is temperature.

We assume the boundary conditions of Eq.(1) to Eq.(3) are

$$u = U_w = cx, \quad T = T_m \quad \text{at } y = 0 \quad (4)$$

$$u \rightarrow U_e, \quad T \rightarrow T_\infty \quad \text{at } y \rightarrow \infty \quad (4)$$

And

$$k \left( \frac{\partial T}{\partial y} \right)_{y=0} = \rho[\lambda + C_s(T_m - T_0)]v(x, 0) \quad (5)$$

Here  $T_m$  is temperature of melting surface,  $T_\infty$  is temperature in face stream condition, where  $T_\infty > T_m$   $\lambda$  the latent heat of the considered fluid and  $C_s$  is the heat capacity of the solid surface.

In Eq.(5), it is noted that the heat conducted to the melting surface is the heat of melting and sensible heat required to raise the solid surface temperature to  $T_m$  (Epstein and Cho [22] ). One can see the details of derivations of Eq.(5) can be seen in Roberts[23].

As Eq.(2) and Eq.(3) are partial differential equations, which are unable to solve easily hence we use following similarity transformations to convert governing PDE to ODE.

$$\left. \begin{aligned} \eta &= \sqrt{\left(\frac{c}{2\nu x}\right)}y, \\ \psi &= x\sqrt{(c\nu)}f(\eta), \\ \theta(\eta) &= \frac{T - T_m}{T_\infty - T_m} \end{aligned} \right\} \quad (6)$$

Here  $\eta$  is the similarity variable,  $f(\eta)$  is the non-dimensional stream function,  $\theta(\eta)$  is the non-dimensional temperature,  $\psi$  is the stream function.

This stream function can be defined as

$$u = \frac{\partial \psi}{\partial y} \quad \& \quad v = -\frac{\partial \psi}{\partial x} \quad (7)$$

Using above in (2) and (3) we get following boundary value problems

$$\left(1 + \frac{1}{\beta}\right) f_{\eta\eta\eta} + ff_{\eta\eta} - f_\eta^2 + 1 = 0 \quad (8)$$

$$\theta_{\eta\eta} + \text{Pr} f \theta_\eta = 0 \quad (9)$$

Where suffix denotes the differentiation with respect to the similarity variable  $\eta$ ,

where  $\text{Pr} = \frac{\nu}{\alpha}$  (Prandtl number).

The corresponding boundary conditions

$$\left. \begin{aligned} f_\eta(0) = \varepsilon, \quad \text{Pr} f(0) + M \theta_\eta(0) = 0, \quad \theta(0) = 0 \\ f_\eta(\infty) = 1, \quad \theta(\infty) = 1 \end{aligned} \right\} \quad (10)$$

Where  $\varepsilon$  the moving parameter and  $M$  is the dimensionless melting parameter and are given by

$$\varepsilon = \frac{c}{a} \quad \& \quad M = \frac{C_p(T_\infty - T_w)}{\lambda + Cs(T_w - T_0)} \quad (11)$$

Here one should understand that the melting parameter  $M$  is a combination of the Stefan number  $\frac{c_p(T_\infty - T_m)}{\lambda}$  for

the liquid and solid phases respectively.

### 3. Physical quantities of engineering

Here the physical quantities of engineering interest are

### 3.1 The Skin friction co-efficient

It is defined as  $C_f = \frac{\tau_w}{\rho u_e^2}$  (12)

Where  $\tau_w = \left( \mu_B + \frac{P_y}{\sqrt{2\pi_c}} \right) \left( \frac{\partial u}{\partial y} \right)_{y=0}$  (13)

Where  $\mu_B$  is being the plastic dynamic viscosity.

Hence skin friction function is given by

$$C_f \sqrt{\text{Re}_x} = \left( 1 + \frac{1}{\beta} \right) f''(0) \quad (14)$$

### 3.2: The local Nusselt number

It is defined as  $Nu_x = \frac{xq_w}{k(T_\infty - T_w)}$  (15)

where

$$q_w = -k \left( \frac{\partial u}{\partial y} \right)_{y=0} \quad (16)$$

$$(\text{Re}_x)^{-1/2} Nu_x = -\theta'(0) \quad (17)$$

Where

$$\text{Re}_x = \frac{u_e(x)x}{\nu}$$

### 4. Numerical Procedure:

As the governing partial differential equations of the flow and heat transfer are converted into nonlinear ordinary differential equation using suitable similarity transformations along with the corresponding boundary conditions. These ordinary nonlinear differential equations are boundary value problems as they have boundary conditions. These equations may be solved by any BVP solvers. In this paper we wanted to solve these problems by Runge-Kutta Fehlberg method. So we have converted the boundary value problems into initial value problem and (Please See Ibrahim et.al [ ] for procedure) here only two initial conditions on momentum and one initial condition in energy equations are available, but we need three initial conditions on momentum and two initial condition in energy equations are needed.

Hence we have generated the missed boundary conditions i.e  $f''(0)$  and  $\theta'(0)$  by most efficient shooting method. After obtaining the missed initial conditions, the ODE's are integrated numerically using Runge-Kutta Fehlberg method

The step size and the convergence criteria have been chosen as 0.01 and  $10^{-5}$  respectively as we know the asymptotic boundary conditions of (10) are approximated by using  $\eta_{\max}$ . We have dual solutions for some governing parameters.

## Results and Discussion

After obtaining the solution of boundary value problem. We more to plot the resulting graphs to study velocity and temperature variations on the flow and energy transfer. The numerical computations have been done to calculate wall temperature gradient, critical values, and skin friction All the numerical values are tabulated in table-1 to table-5.

Table-1 represent the  $f''(0)$  value when  $\beta \rightarrow \infty$  for different values of epsilon the velocity ratio parameter.

Table-2 represent the  $f''(0)$  for different values of melting parameter.

Table -3 represent the numerical values of wall temperature gradient for different values of Pr and melting heat parameter.

Table-4 represents the critical values of velocity ratio parameter.

Table-5 represents wall temperature gradient for different values of all governing parameters arising in the flow and heat transfer.

Fig 1 shows the stagnation point flow diagram (geometric)

Fig 2 depicts the velocity distribution for different values of melting heat transfer parameter, and  $\epsilon$ . Velocity increase in parametric value of M and  $\epsilon$ .

Fig 3 shows temperature distribution for different values of **M and  $\epsilon$** . The temperature distribution increase as increase in the parameter value of **M and  $\epsilon$** .

Fig 4 shows the effect of  $\epsilon$  on the horizontal velocity and temperature for both newtonian and non-newtonian case.

Newtonian velocity profile have higher value as compared to non-newtonian flow in both cases the velocity and temperature increases.

Fig 6 and Fig 7; these figures show the effect of casson parameter in absence of melting parameter on velocity and temperature, both velocity and temperature enhances as increase in parametric values of casson fluid.

Fig 8 and Fig 9: depict the effect of melting parameter  $M$ . in presense and absence of casson parameter on velocity and temperature that is in both Newtonian and non-newtonian cases. In both cases the velocity and temperature it decreases as increase in parametric value of  $M$ .

Fig10 and Fig 11 : these figures says about the effect of  $Pr$  on velocity and temperature in both Newtonian and non-newtonian region both velocity and temperature decreases as increase in the parametric values of  $Pr$  the boundary and momentum thickness thinning occure.

Fig 12 and Fig 13 :depict the velocity and temperature profile for different values of casson parameter for both stretching and shrinking cases both velocity and temperature increases as increase in casson fluid parameter increases.

Fig 14 and Fig 15: represent the velocity distribution and temperature distribution for different valus of melting parameter, both velocity and temperature profile increase as increase in values of  $M$ .

Fig16-fig17 : show the effect of  $Pr$  on, velocity and temperature distribution for stretching and shrinking sheet cases both profile increase a increase in the parametric values of  $Pr$ .

Fig 18: shows the effect of  $\epsilon$  on velocity profile for stretching sheet case have higher values as compared to stretching sheet cases.

Fig 19: shows the effect of melting parameter on skin friction. It is equal to 0 at  $\epsilon = 1$ . And it shows that it admits the dual solutions.

Fig 20 : shows effect of  $M$  on skin friction co-efficient, It also drawn for both non- Newtonian and Newtonian case. The skin friction values for Newtonia case is lesser than non –Newtonian case.

## Conclusion

1. It is observed that the velocity and temperature profile have similar values for
  - (i) Newtonian and non-Newtonian cases
  - (ii) Streching and shrinking sheet cases.
2. This problem admits dual solution  $M$  shrinking sheet cases.
3. Fig 19 and 20 shows the existence of dual solution.

Table-1

Comparative Values of  $f''(0)$  for stretching sheet problem when  $M=0$  and  $\beta \rightarrow \infty$ 

$\varepsilon$	Bachok et al.[3]	Wang[31]	Kimiaefar et al.[32]	Present Study
0	1.2325877	1.232588	1.23258762	1.2325878
0.1	1.1465610	1.14656	1.14656098	1.1465609
0.2	1.0511300	1.05113	1.05112998	1.0511299
0.5	0.7132949	0.71330	0.71329495	1.713295
1	0	0	0	0
2	-1.8873066	-1.88731	-1.88731	-1.8873056

Table-2

Comparative values of  $f''(0)$  for different values of  $M$  when  $\beta \rightarrow \infty$ 

$\varepsilon$	Bachok et al.[3]			Present Study		
	M=1	M=2	M=3	M=1	M=2	M=3
0	1.0370034	0.9468506	0.8913811	1.0370034	0.9468505	0.8913810
0.1	0.9642521	0.8804416	0.8289482	0.9642521	0.8804412	0.8289483
0.2	0.8836747	0.8068762	0.7597516	0.8836747	0.8068762	0.7597515
0.5	0.5990895	0.5043333	0.5151721	0.5990895	0.5043334	0.5151720
1	0	0	0	0	0	0
2	-1.5804839	-1.4427473	-1.3592105	-1.5804839	-1.4427474	-1.359210

Table-3

Comparative values of  $-\theta(0)$  for different values of  $Pr$  and  $M$  when  $\varepsilon=1$  and  $\beta \rightarrow \infty$ 

Pr	M	Bachok et al.[3]	Present Study
1	0	-0.7978846	-0.79788465
	1	-0.5060545	-0.50605450
	2	-0.3826383	-0.38263831
	3	0.3119564	0.31195643
7	0	-2.1110042	-2.11100424
	1	-1.3388943	-1.33889433
	2	-1.0123657	-1.0123657
	3	-0.8253591	-0.82535911



**Table-4**  
Comparative Values of  $\theta'(0)$  for different values of Pr and M

Physical parameters				$\theta'(0)$	
Pr	M	$\varepsilon$	$\beta$	Bechok et.al[]	Present Study
1	0	1	$\infty$		0.7978846
	1				0.5060545
	2				0.38263827
	3				0.311956404
7	0	1	$\infty$		2.11100412
	1				1.33889427
	2				1.0123657
	3				0.825359066

Table 5: Critical values for  $\lambda_c$

M = 0	-1.2465
M = 1	-1.24311
M = 2	-1.2401
M = 3	-1.2366

**Table-6**  
Values of  $\theta'(0)$  for different values of governing parameters

Governing parameters				$\theta'(0)$	
Pr	M	$\varepsilon$	$\beta$		
1	0	0.5	0.5	0.56713	
	1			0.36377	
	2			0.27664	
	3			0.22638	
1	1	0.5	0.5	0.420979	
		1		0.506054	
		1.5		0.581487	
1	1	0.5	0.5	0.420979	
			1.0	0.427700	
			1.5	0.4307125	
1 5 7	1	0.5	0.5	0.420979	
					0.886350
					1.0372329

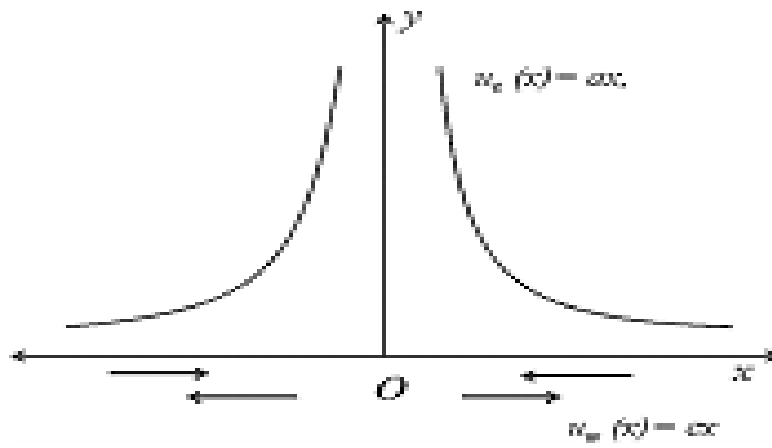


Fig 1: Stagnation point flow.

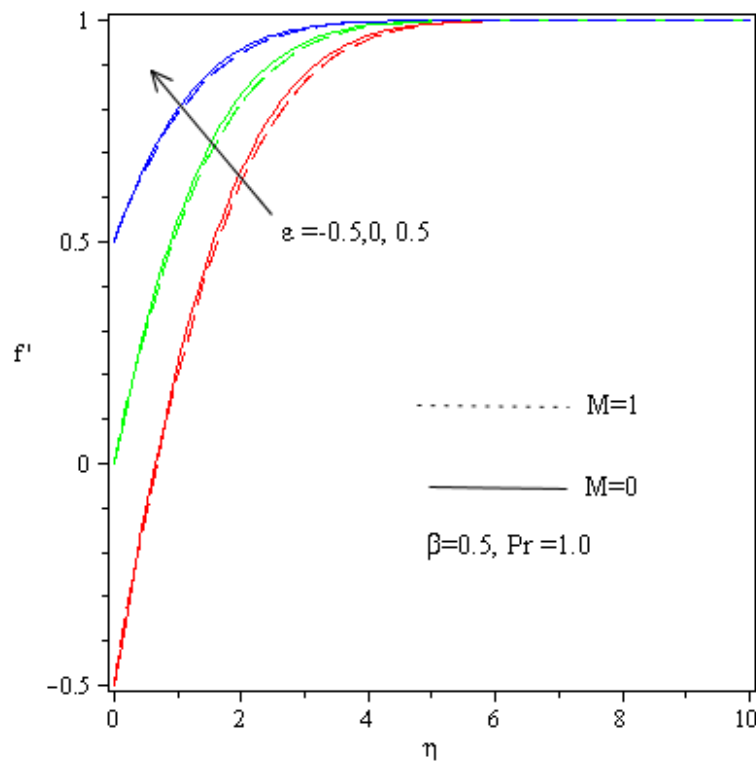


Fig.2: Velocity distribution for different values of M and  $\epsilon$

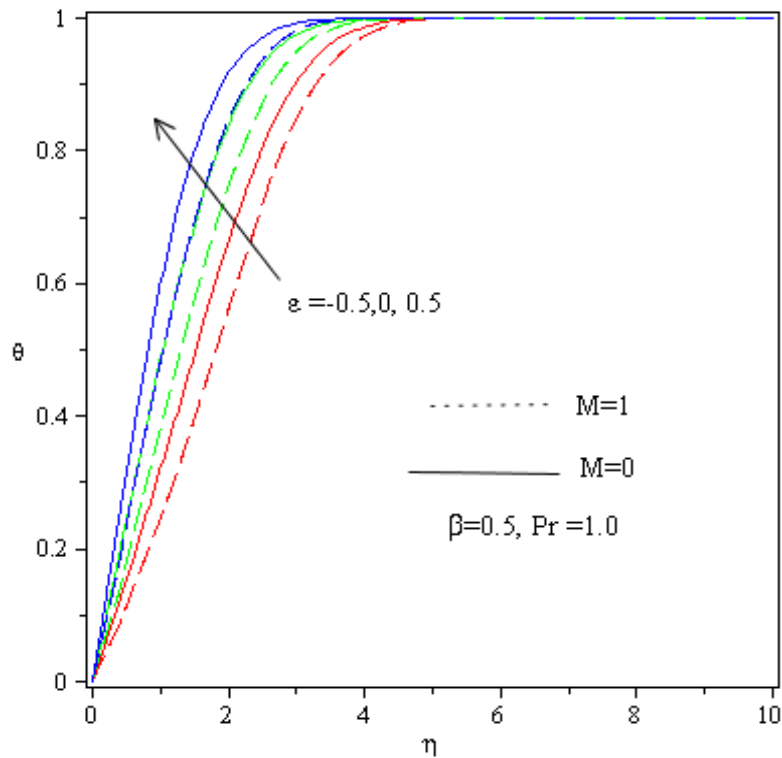


Fig.3: Temperature distribution for different values of  $M$  and  $\epsilon$

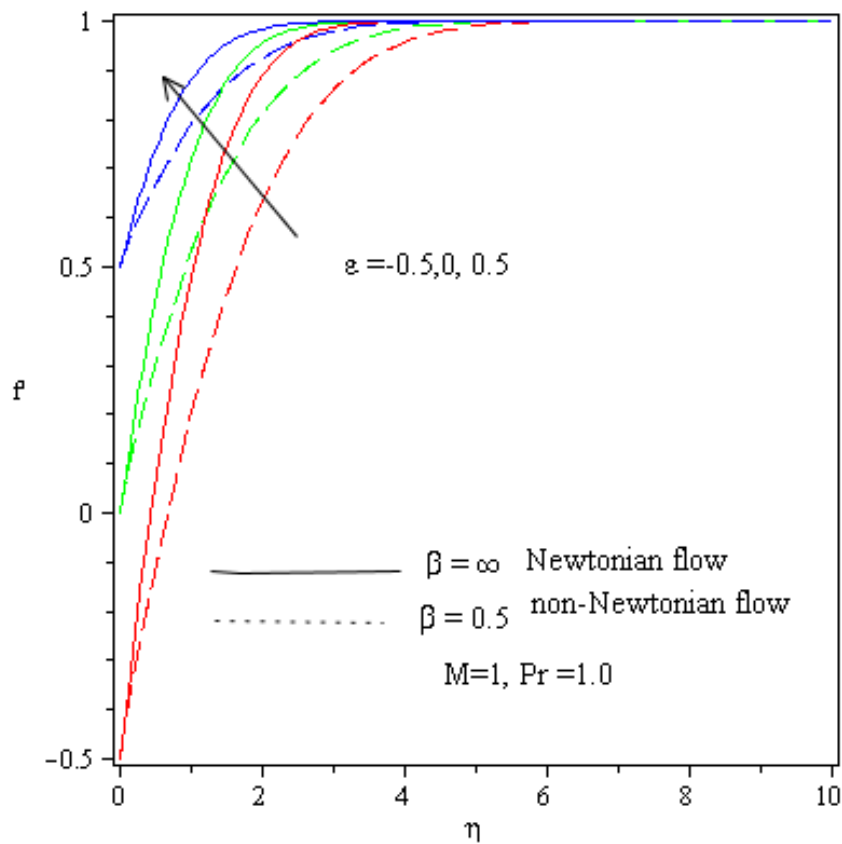


Fig.4: Velocity distribution for different values of  $\beta$  and  $\epsilon$

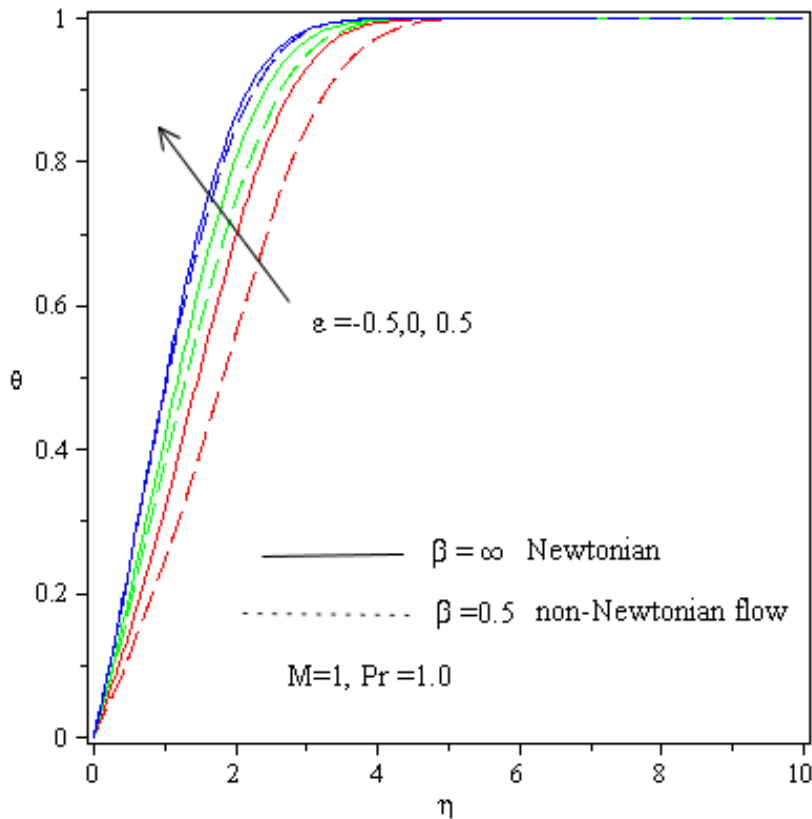


Fig.5: Temperature distribution for different values of  $\beta$  and  $\epsilon$

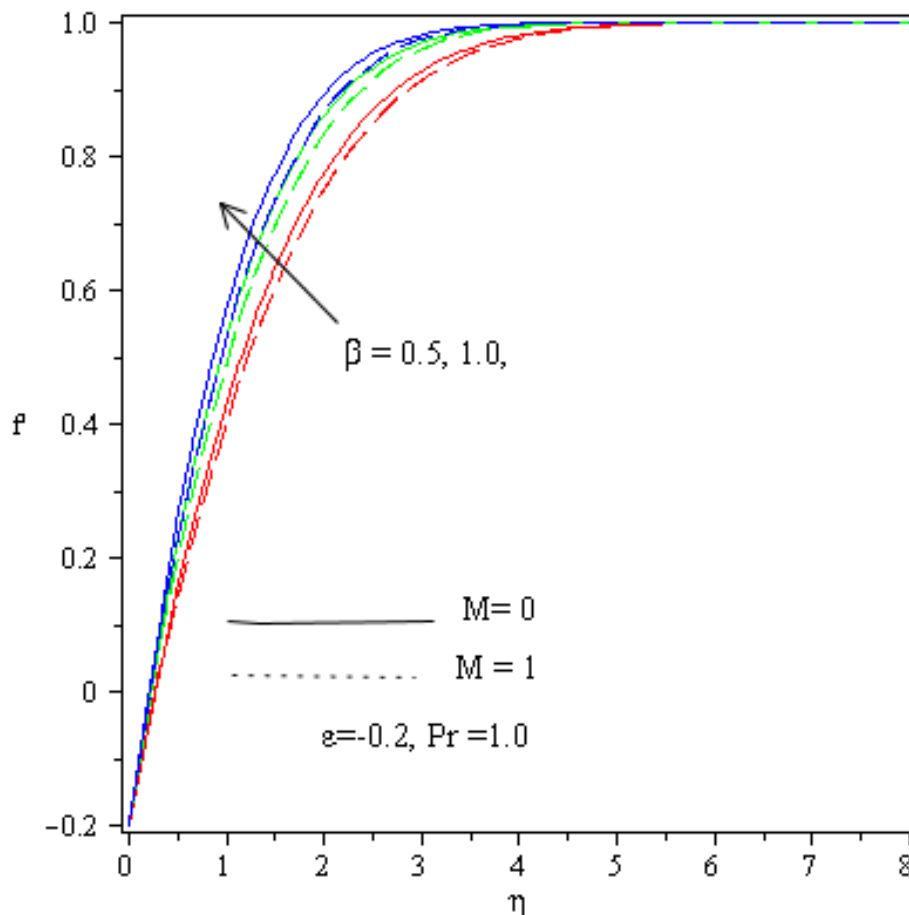


Fig.6: Velocity distribution for different values of  $\beta$  and  $M$

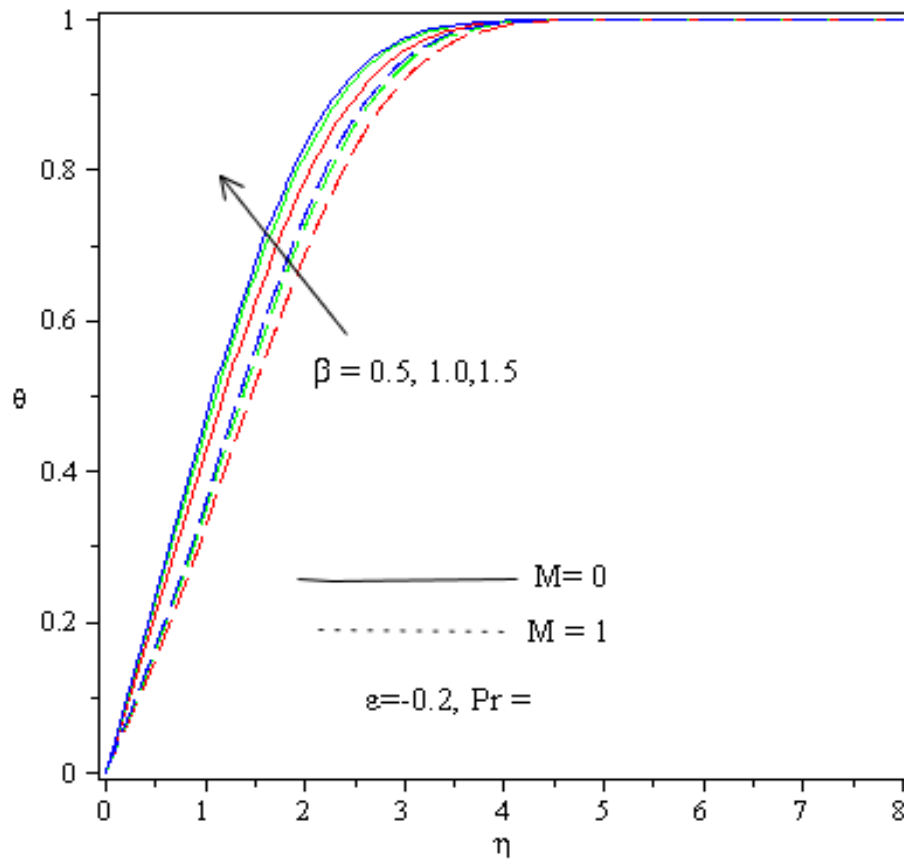


Fig.7: Temperature distribution for different values of  $\beta$  and  $M$

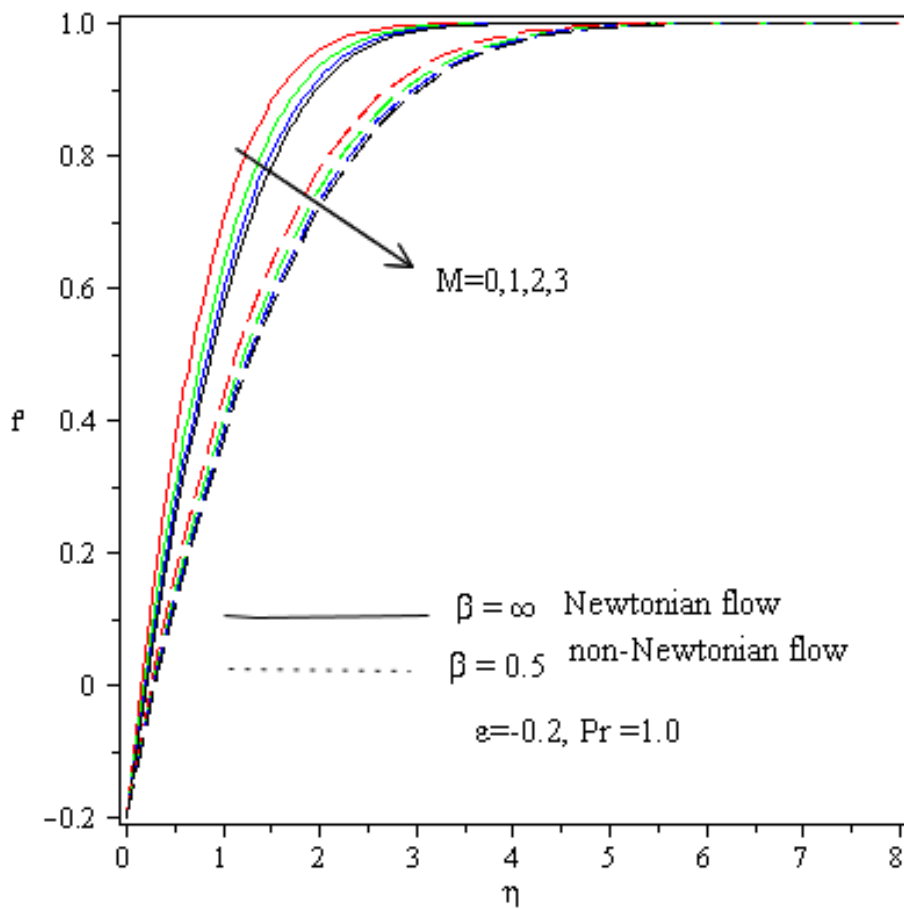


Fig.8: Velocity distribution for different values of  $M$

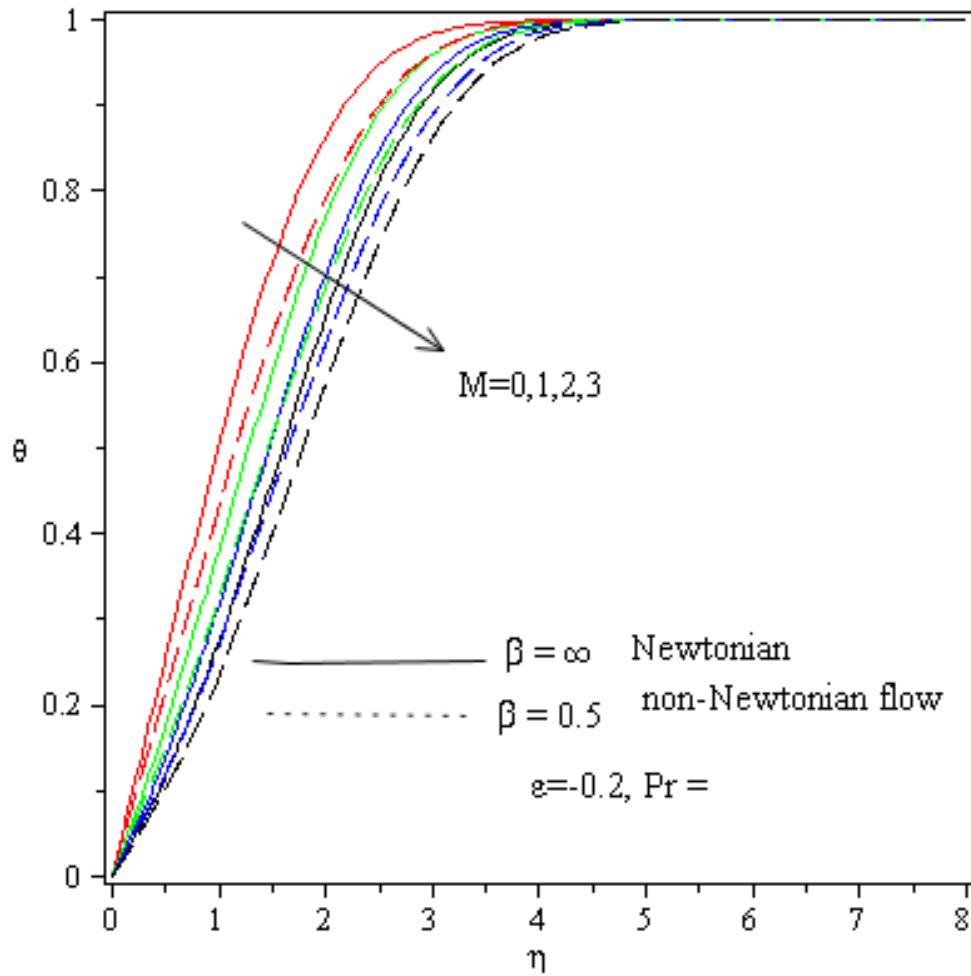


Fig.9: Temperature distribution for different values of M

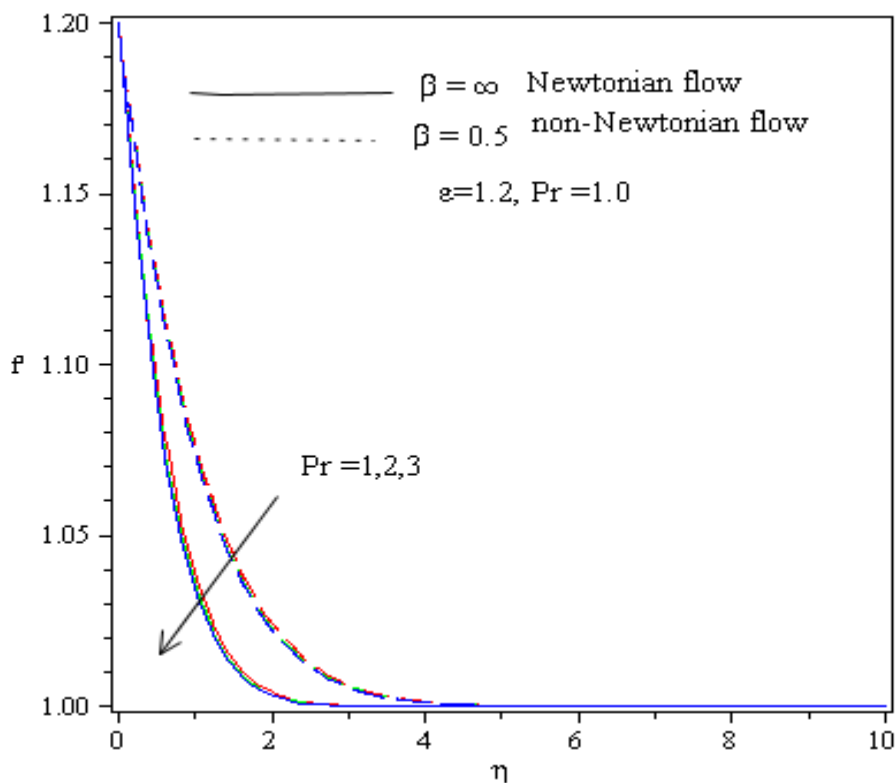


Fig.10: velocity distribution for different values of Pr.

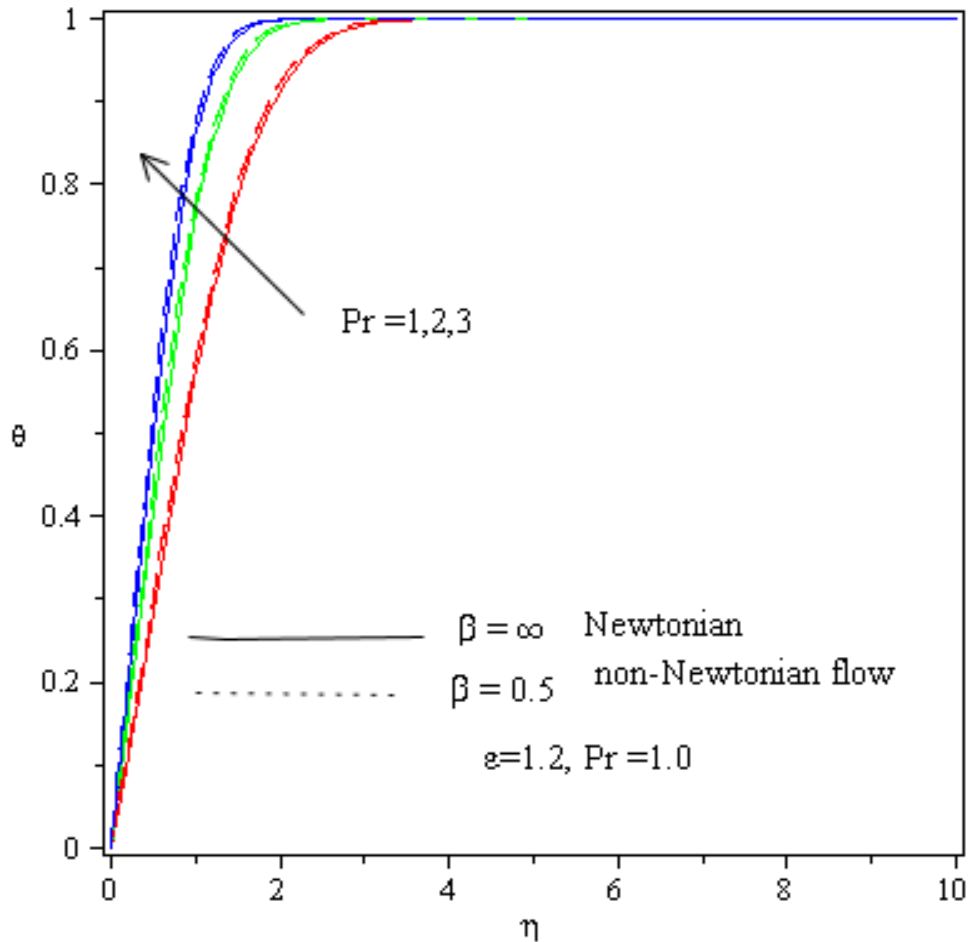


Fig.11: Temperature distribution for different values of Pr.

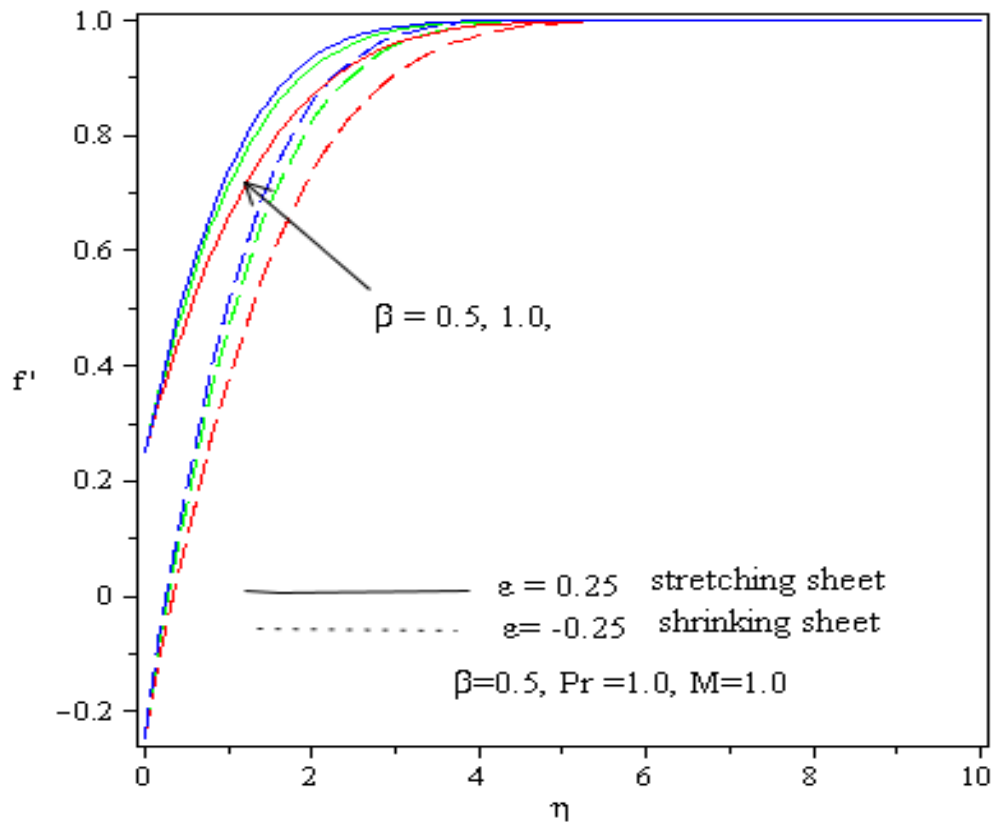


Fig.12: Velocity distribution for different values of  $\beta$  and  $\epsilon$   
(For Stretching and shrinking sheet)

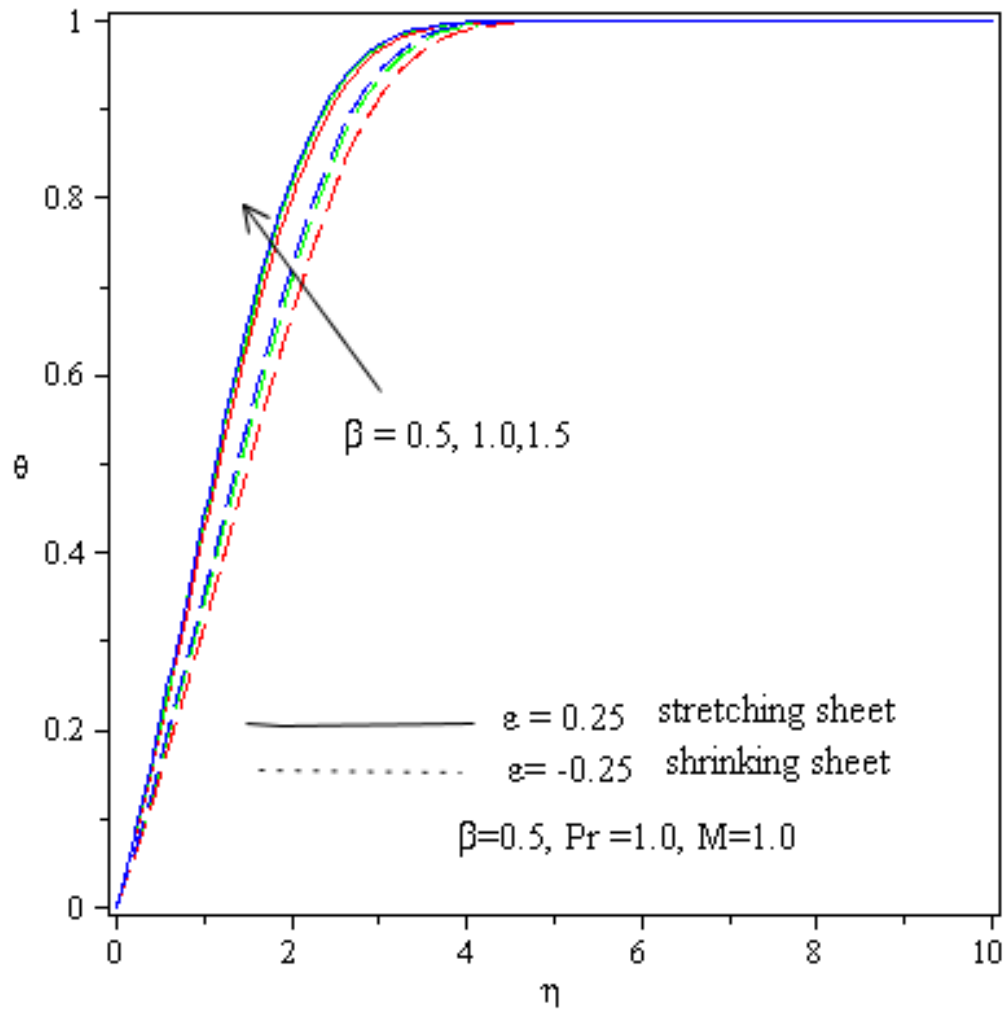


Fig.13: Temperature distribution for different values of  $\beta$  and  $\epsilon$  (For Stretching and shrinking sheet)

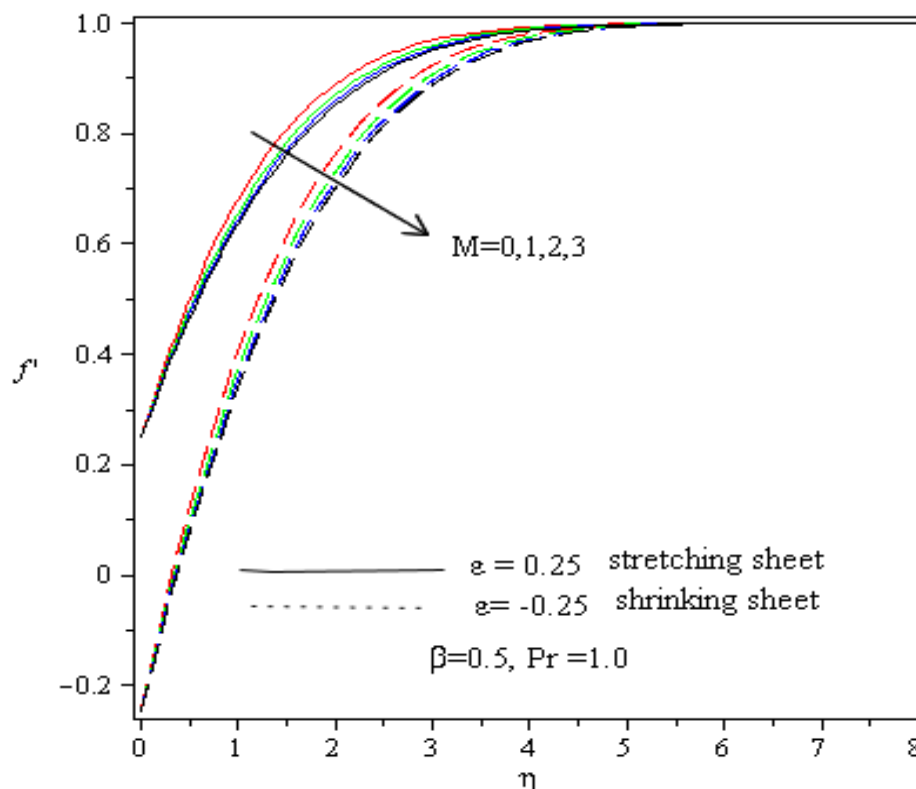


Fig.14: velocity distribution for different values of  $M$  (For Stretching and shrinking sheet)



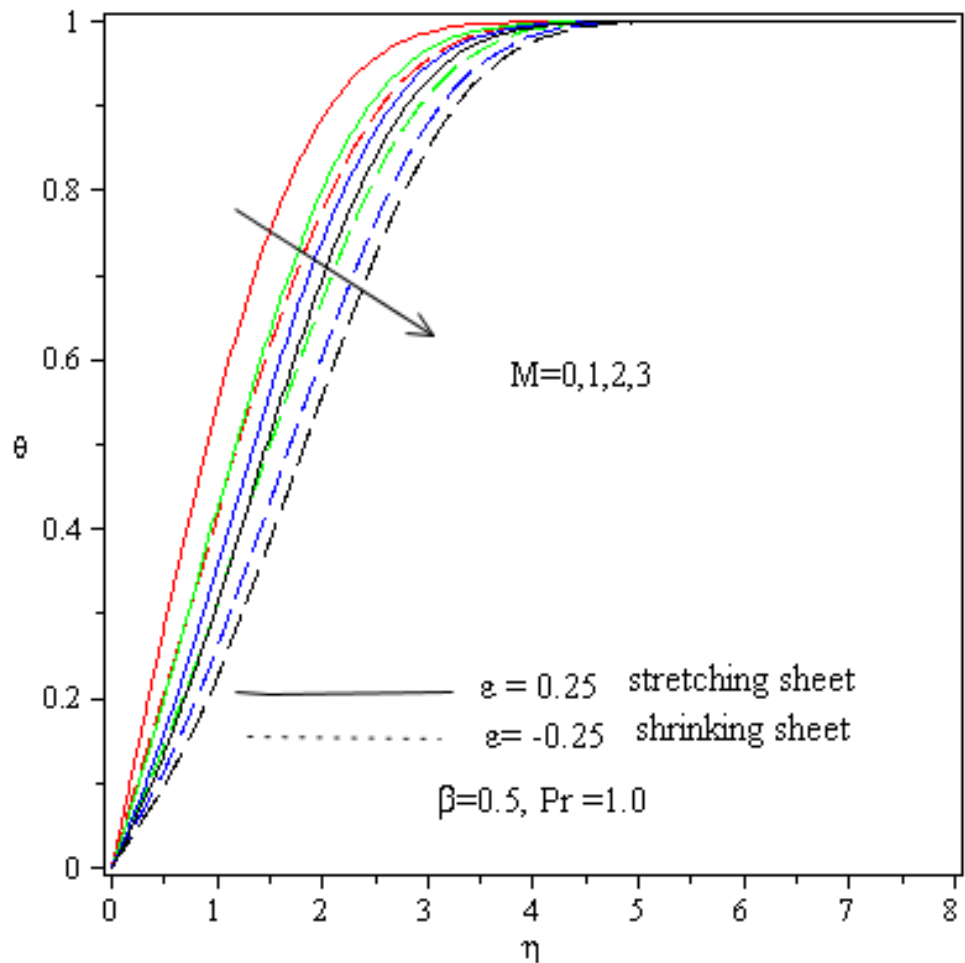


Fig.15: Temperature distribution for different values of M (For Stretching and shrinking sheet)

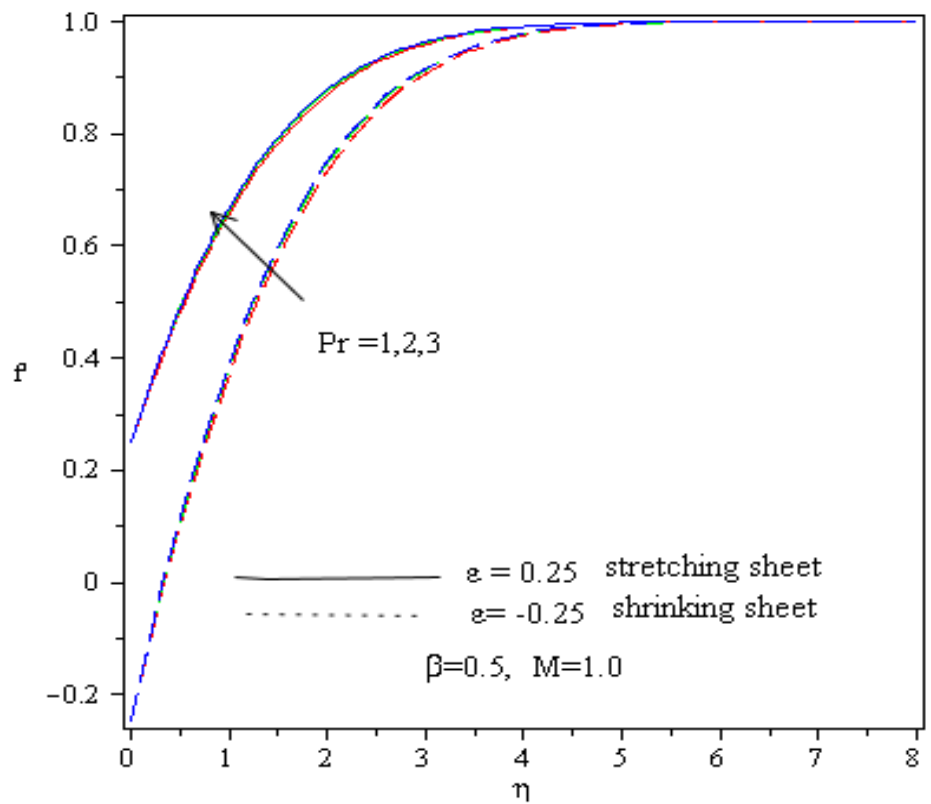


Fig.16: velocity distribution for different values of Pr (For Stretching and shrinking sheet)

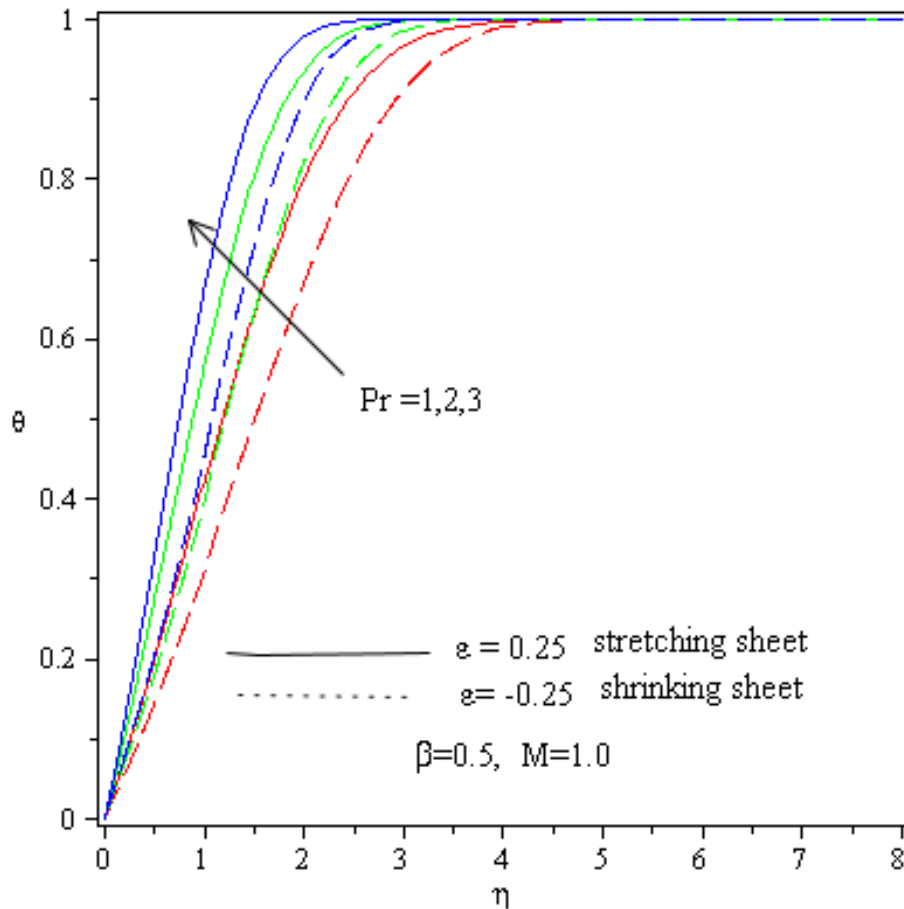


Fig.17: Temperature distribution for different values of Pr (For Stretching and shrinking sheet)

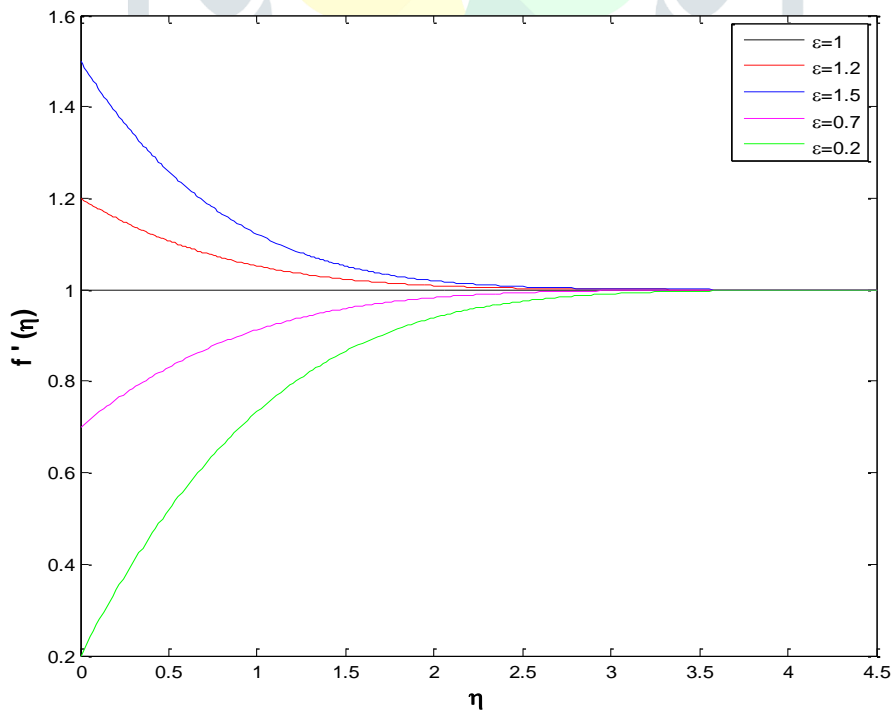


Fig 18: velocity distribution with various values of  $\epsilon$

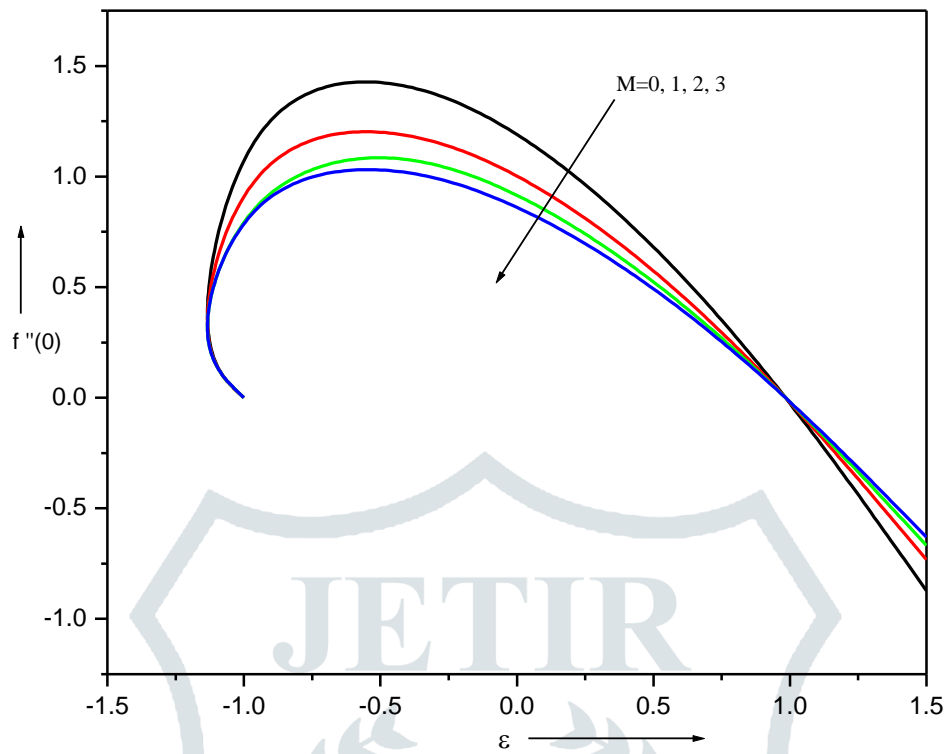


Fig 19. Skin friction variation for different values of M (For Newtonian Case)

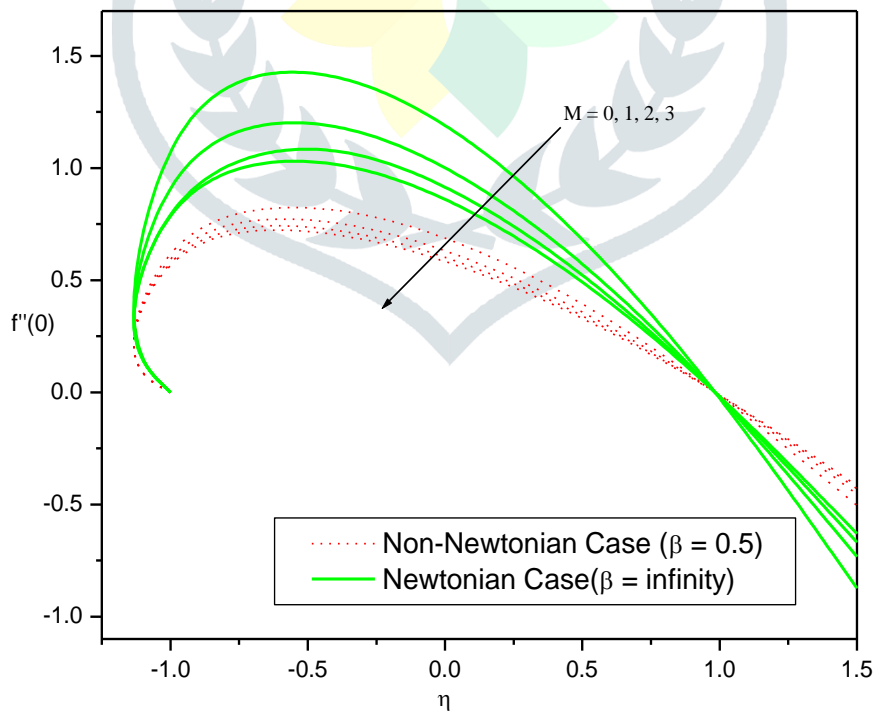


Fig 20. Skin friction variation for different values of M

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