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# CHARACTERIZATION OF CIRCULAR ARC GRAPHS AS CONNECTED GRAPHS, PATHS AND GENERALIZED STAR GRAPHS USING SOME PARAMETERS WITH NEIGHBORHOODS IN G.

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## Abstract:

Circular arc graphs are a new class of intersection of graphs defined on a set of arcs on a circle. A graph is a circular arc graph, if it is the intersection graph of a finite set of arcs on a circle. That is, there exists one arc for each vertex of a graph G and two vertices in graph G are adjacent if and only if the corresponding arcs intersect. In this paper, we have discussed about the characterization of circular arc graphs as connected graphs, paths and generalized star graphs using some parameters with neighborhoods in G.

**Index terms:** Graph, Circular arc family, Circular arc graph, path graphs, generalized 3 star graphs, generalized double 3 star graphs, neighborhood of vertex.

## 1. Introduction:

Graph theory is rapidly moving into the main stream of mathematics mainly due to its applications in many fields which include biochemistry, electrical engineering, computer science, and operation research. Although graph theory is one of the younger branches of mathematics, it is fundamental to a number of applied fields.

Graph theory originated with the Konigsberg Bridge Problem, which Leonhard Euler (1707-1783) solved in 1736.Over the past 60 years; there has been a great deal of exploration in the area of graph theory. High-speed digital computer is one of the main reasons for the recent growth of interest in graph theory and its applications. Graphs are among the most ubiquitous models of both natural and human-made structures. They can be used to model many types of relations and process dynamics in Physical, Biological and Social systems. Many problems of practical interest can be represented by graphs and can be solved using graph theory. So, one can confidently put forward that a mere act of thinking about a problem in terms of a graph will certainly suggest insights and probable solution methods.

The rules and the moves of the queen in the game of chess are well-known. A queen can go further any number of squares horizontally, vertically or diagonally. Chess lovers of Europe in 1850 have hit upon the idea of determining the minimum number of queens, to be placed on the board so as to allow the queen to attack or occupy all the squares. Virtual problems in life, when changed into graph problems exhibit some remarkable features, giving rise to special categories of graphs namely Interval graphs, circle graphs, Circular-arc graphs, Circular-arc overlap graphs etc.

Circular arc graphs are a new class of intersection graphs, defined for a set of arcs on a circle. A graph is circular-arc graph, if it is the intersection graph of a finite set of arcs on a circle. That is, there exists one arc for each vertex of G and two vertices in G are adjacent in G, if and only if the corresponding arcs intersect. A vertex is said to dominate another vertex if there is an edge between the two vertices. If we bend the arc into a line, then the family of arcs is transformed into a family of intervals. Therefore, every interval graph is a circular arc graph, where the opposite is always not true. The combinational structures in CAG are varied and extensive, where it finds an application in many other fields such as Biology, Genetics, Traffic control, and Computer science and particularly useful in cyclic scheduling and computer storage allocation problems etc.

## 2. Preliminaries:

The graph G is an ordered pair (V, E) where V is a non empty set consisting of elements called vertices and E is a set consisting of unordered pair of V called edges. A graph with vertex set V and edge set E is denoted by G (V, E). Vertices are also known as nodes, edges are also called as lines and ties or links. The undirected edge e determined by the pair of vertices u,v is denoted by (u,v) and the directed edge determined by the vertices u, v is denoted by {u,v}. The number of vertices present in a graph is called the order of the graph and is denoted by O (G). The number of edges present in a graph is called the size of the graph. Two vertices u and v are said to adjacent if there exist an edge between them. Two distinct edges $e_1$ and $e_2$ are said to be adjacent if they have a vertex in common. If e = (u,v), then the edge e is said to be incident with the vertices u and v, and u, v are said to be incident with the edge e. Number of edges that are incident on the vertex of the graph is said to be the degree of the vertex of the graph. Neighborhood of a vertex v belonging to the vertex set v of a graph G is the set of all vertices adjacent to v (including v). It is denoted by nbd[v]. A walk W in a graph G is a finite non null sequence. W= $v_0, e_1, v_1, e_2 \ldots \ldots e_k, v_k$ .Whose terms are alternatively vertices and edges such that for $1 \leq i \leq k$, the end vertices of $e_i$ are $v_{i-1}$, $v_i$ and it is denoted as $(v_0,v_k)$-walk.

Vertices with which a walk begins and ends are called as terminal vertices of the walk. The vertex with which a walk begins is called the initial vertex of the walk and the vertex with which the walk ends is called the final vertex of the walk. Vertices other than the terminal vertices are called intermediate vertices. A walk that begins and ends at the same vertex is called a closed walk. A walk that is not closed is called an open walk. An open walk in which no vertex appears more than once is called a path. The number of edges present in a path is called its length. A graph G of order greater than or equal to two is said to be connected if there is at least one path between every pair of vertices. Circular arc graphs are a new class of intersection graphs, defined for a set of arcs on a circle. That is, there exists one arc for each vertex of G and two vertices in G are adjacent in G, if and only if the corresponding arcs intersect. A vertex is said to dominate another vertex if there is an edge between the two vertices. Let A = $\{A_1,A_2,\ldots \ldots A_k\}$ be a circular arc family on a circle, where all the arcs together cover the entire circle. An arc begins at the end point $p_i$ and ends at the end point $q_i$ considered in the clockwise direction is denoted by $(p_i,q_i)$. Two arcs $A_i$ and $A_j$ are said to intersect each other if they have non-empty intersection. A representation of a graph with arcs helps in the solving of combinatorial problems on the graph.

A connected graph of order m with three vertices of degree 1 one vertex of degree 3 and (n-4) vertices of degree 2 is called a generalized 3-star graph. A connected graph G of order m with two vertices of degree 3, four vertices of degree 1 and (m-6) vertices of degree 2 is called a generalized double 3 – star graph.

## 3. Main theorems:

**Theorem 1:**

Let A= $\{A_1,A_2,A_3\ldots\ldots\ldots\ldots A_m\}$ be a circular arc family corresponding to a circular arc graph G. where $A_n= [a_n,b_n]$ ($a_n$ is the starting of arc and $b_n$ is the end of the arc) G be a connected graph if and only if there does not exist an arbitrary arc $A_h$ where $A_2 \leq A_h \leq A_{m-1}$ such that $b_n < a_h$ , for all n=1,2,3,……..h-1.

**Proof:**

Let A = $\{A_1,A_2,A_3\ldots\ldots A_m\}$ be a circular arc family corresponding to a circular arc graph G. First let G be a connected graph. Here it has to be shown that there does not exit an arbitrary arc $A_h$ where $A_2 \leq A_h \leq A_{m-1}$ such that

$b_n < a_h$ , for all n=1,2,3,…… h-1 …………………..1.1.

By the definition of circular arc family

$a_h < a_{h+1} < a_{h+2} < \ldots\ldots\ldots < a_m$ ……………………1.2

From 1.1 and 1.2

$b_1 < a_h < a_{h+1} < a_{h+2} < \ldots\ldots\ldots < a_m$

$b_2 < a_h < a_{h+1} < a_{h+2} < \ldots\ldots\ldots < a_m$

$b_3 < a_h < a_{h+1} < a_{h+2} < \ldots\ldots\ldots < a_m$

……………………………………

……………………………………

$b_{h-1} < a_h < a_{h+1} < a_{h+2} < \ldots\ldots < a_m$

Therefore no arc in the set $\{A_1, A_2, A_3, \ldots \ldots A_h\}$ will intersect in the set $\{A_h, A_{h+1}, A_{h+2}, \ldots\ldots A_n\}$.

Let $V_n$ be the vertex of the circular arc graph G corresponding to $A_n = [a_n, b_n]$ where n=1,2,……..m. Therefore no vertex in the set $\{V_1, V_2, V_3 \ldots\ldots V_{h-1}\}$ is adjacent to a vertex belongs to the set $\{V_h, V_{h+1}, V_{h+2} \ldots\ldots V_m\}$.

There does not exists an edge between the vertex belonging to set $\{V_1, V_2, V_3 \ldots\ldots V_{h-1}\}$ and the vertex belonging to $\{V_h, V_{h+1}, V_{h+2} \ldots\ldots V_m\}$.There does not exists any path between the vertex belonging to the set of vertices $\{V_1, V_2, V_3 \ldots\ldots V_{h-1}\}$ and the vertex belonging to the set of vertices $\{V_h, V_{h+1}, V_{h+2} \ldots\ldots V_m\}$.

Therefore, the graph G is not a connected graph which contradicts that the graph G is a connected graph. Therefore our assumption that there exists an arbitrary arc $A_h$ where $A_2 \le A_h \le A_{m-1}$, such that $b_n < a_h$ for n=1,2,3,………..h-1 is wrong.

Hence if G is a connected graph there does not exists an arc $A_h$ where $A_2 \le A_h \le A_{m-1}$ satisfying the condition $b_n < a_h$, for all n=1,2, 3,………h-1.

Conversely, let us assume that there does not exist an arbitrary arc $A_h$, where $A_2 \le A_h \le A_{m-1}$ satisfying the condition $b_n < a_h$ for all n=1, 2, 3…. h-1. Implies there exist a path between pair of vertices in the graph G. Therefore, G is a connected graph. Hence the required result.

**Theorem: 2**

Let G be a circular arc graph of order m≥2 such that there does not exist an arbitrary arc $A_h$ where $A_2 \le A_h \le A_{m-1}$ satisfying the condition $b_n < a_h$ for all n=1, 2, 3……..h-1.Then G is a path graph if and only if $\sum_{n=1}^{m} d_n^2 \le 4P$ where P is the size of the graph and $d_n$ is the degree of the vertex $V_n$ of the graph G where n=1, 2, 3…….m.

**Proof:**

Let G be a circular arc graph of order m≥2, such that there does not exist an arbitrary arc $A_h$ where $A_2 \le A_h \le A_{m-1}$ satisfying the condition $b_n < a_h$ for all n=1, 2, 3……..h-1.

By the theorem 1, G is a connected graph

$P \ge m-1$ ---------------------------2.1

Where P is a size of the graph

For any graph G $\sum_{n=1}^{m} d_n = 2P$ ---------------------- 2.2

Where $d_n$ is the degree of the vertex $V_n$ for n=1, 2, 3……. m.

First let us assume that the graph G satisfying the condition $\sum_{n=1}^{m} d_n^2 < 4P$ --------2.3

Now $\sum_{n=1}^{m} (d_n-2)^2 = \sum_{n=1}^{m} d_n^2 - 4\sum_{n=1}^{m} d_n + 4m$

$< 4P - 4(2P) + 4m$

$< 4m - 4P$

$\le 4(1)$ (from 2.1)

$\le 4$

$\sum_{n=1}^{m} (d_n-2)^2 \le 4$

$0 \le \sum_{n=1}^{m} (d_n-2)^2 \le 4$

The inequality gets satisfied when $0 \le d_n \le 4$ for all n=1, 2, 3………m ---------------2.4

Implies that $d_n$=1 or 2 or 3 for all n=1,2,3………m furthermore at most three $d_n$ values can differ from 2.

Therefore, the degree sequence that satisfies 2.1, 2.2, 2.3, and 2.4 is $1^2, 2^{m-2}$.

For any directed graph the degree sequence $1^2$, $2^{m-2}$ defines a path graph.

So, for any connected graph if $\sum_{n=1}^{m} d_n^2 \leq 4P$ then the graph is a path graph.

Conversely,

Let G be a path graph, then

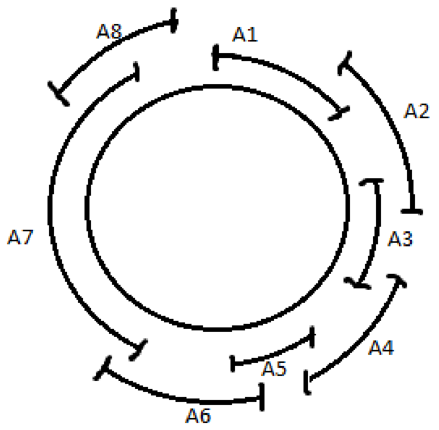
$$\sum_{n=1}^{m} d_n^2 = 2(1)^2 + (m-2)2^2$$
$$= 2 + 4m - 8$$
$$= 4m - 6 \leq 4P - 2 \leq 4P$$

Therefore $\sum_{n=1}^{m} d_n^2 \leq 4P$

Hence the required result is obtained.

**Illustration 2.1:**

Let the circular arc family be A= {$A_1$, $A_2$, $A_3$, … $A_8$} corresponding to the circular arc graph G as follows




Circular-arc family A

nbd [$A_1$] = {$A_1$,$A_2$}

nbd [$A_2$] = {$A_1$,$A_2$,$A_3$}

nbd [$A_3$] = {$A_2$,$A_3$,$A_4$}

nbd [$A_4$] = {$A_3$,$A_4$,$A_5$}

nbd [$A_5$] = {$A_4$,$A_5$,$A_6$}

nbd [$A_6$] = {$A_5$,$A_6$,$A_7$}

nbd [$A_7$] ={$A_6$,$A_7$,$A_8$}

nbd [$A_8$] ={$A_7$,$A_8$}

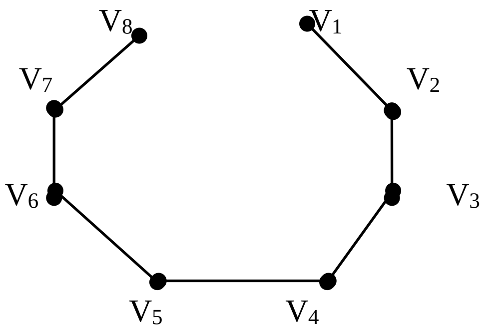
Therefore, there does not exist an arbitrary arc $A_h$, where $A_2 \leq A_h \leq A_{m-1}$ satisfying the condition $b_n < a_h$ for all n=1, 2, 3……. h-1. Therefore, G is a connected graph

Hence $\sum_{n=1}^{8} d_n^2 \leq 4P$

**Verification:**

Here O (G), m=8 and size of G, P=7

The graph G in the case will be as follows




Circular-arc graph G

here d(1)=1,d(2)=2,d(3)=2,d(4)=2,d(5)=2,d(6)=2,d(7)=2 and d(8)=1

$\sum_{n=1}^{8} \quad d_n^2 = 26$

But 4P=4(7) =28

Clearly, $\sum_{n=1}^{m} \quad d_n^2 \leq 4P$

Hence the theorem is verified

**THEOREM 3:**

Let A= {$A_1$, $A_2$, $A_3$ ......... $A_m$} be the circular arc family corresponding to a circular arc graph G of order m≥5. Then there exist an arbitrary arc $A_h \in A$ and contained in $A_{h-1}$ satisfying the conditions

(i) $nbd[A_{h-1}]=\{A_{h-2},A_{h-1},A_h,A_{h+1}\}$
(ii) $nbd[A_h]=\{A_{h-1},A_h\}$
(iii) $nbd[A_{h+1}]=\{A_{h-1},A_{h+1},A_{h+2}\}$
(iv) $nbd[A_m]=\{A_{m-1},A_m,A_{m+1}\}$ for all m except for m=1, $A_{h-1}$, $A_h$, $A_{h+1}$ and $A_m$
(v) $nbd[A_1]=\{A_1,A_2\}$ and $nbd[A_m]=\{A_{m-1},A_m\}$ for $A_3 \leq A_h \leq A_{m-2}$

(Or)

(vi) $nbd[A_{h-1}]=\{A_{h-2},A_{h-1},A_h,A_{h+1}\}$
(vii) $nbd[A_h]=\{A_{h-1},A_h\}$
(viii) $nbd[A_{h+1}]=\{A_{h-1},A_{h+1},A_{h+2}\}$
(ix) $nbd[A_m]=\{A_{m-1},A_m,A_{m+1}\}$ for all m except for m=1, $A_{h-1}$, $A_h$, $A_{h+1}$ and $A_m$
(x) $nbd[A_1]=\{A_1,A_2\}$ and $nbd[A_m]=\{A_{m-1},A_m\}$ for h=m-1

If and only if

$$\sum_{n=1}^{m} \quad d_n^2 = 4m-4$$

**Proof:**

Let A={$A_1$, $A_2$, $A_3$ ..........$A_m$} be a circular arc family corresponding to circular arc graph G of order m and size p. let G be a connected graph. Since G a circular graph there exist a one to one corresponding between the vertex set V= {$V_1$,$V_2$,$V_3$ ........$V_m$} of the graph G and the circular arc family A. let $V_h$ be the vertex corresponding to the arc $A_h$ of the arc graph G for l=1,2,3………m and let $d_1$,$d_2$,$d_3$……….$d_m$ be the degree of the vertices $V_1$,$V_2$,$V_3$ ........$V_m$ respectively

Here, G is a connected graph.

P≥m-1 ---------------------------3.1

Where P is a size of the graph

For any graph G $\sum_{n=1}^{m} \quad d_n = 2P$ --------------------3.2

First let us suppose that there exist an arbitrary arc $A_h$, where $A_h$ is contained in $A_{h-1}$ and $A_3 \leq A_h \leq A_{m-2}$ satisfying the conditions

(i) $nbd[A_{h-1}]=\{A_{h-2},A_{h-1},A_h,A_{h+1}\}$
(ii) $nbd[A_h]=\{A_{h-1},A_h\}$
(iii) $nbd[A_{h+1}]=\{A_{h-1},A_{h+1},A_{h+2}\}$
(iv) $nbd[A_m]=\{A_{m-1},A_m,A_{m+1}\}$ for all m except for $m=1,A_{h-1},A_h,A_{h+1}$ and $A_m$
(v) $nbd[A_1]=\{A_1,A_2\}$ and $nbd[A_m]=\{A_{m-1},A_m\}$ for $A_3 \leq A_h \leq A_{m-2}$

By (i) we have

$(V_{h-2}, V_{h-1}),(V_{h-1},V_h),(V_{h-1},V_{h+1})$ are edges of the graph G

By (ii) it is clear that

$(V_{h-1}, V_h)$ is the edge of the graph G

By (iii) we have

$(V_{h-1}, V_{h+1})$ and $(V_{h+1},V_{h+2})$ are the edges of the graph G

By (iv) it is evident that

$(V_1,V_2),(V_2,V_3),(V_3,V_4),\ldots\ldots(V_{h-3},V_{h-2}),(V_{h-2},V_{h-1}),(V_{h+1},V_{h+2}),$

$(V_{h+2},V_{h+3}),(V_{h+3},V_{h+4}),\ldots\ldots(V_{m-2},V_{m-1})$ and $(V_{m-1},V_m)$, are the remaining edges of the graph G. Therefore, the distinct edges of the graph are

$(V_1,V_2),(V_2,V_3),(V_3,V_4),\ldots\ldots(V_{h-3},V_{h-2}),(V_{h-2},V_{h-1}),(V_{h+1},V_{h+2}),$

$(V_{h+2},V_{h+3}),(V_{h+3},V_{h+4}),\ldots\ldots(V_{m-2},V_{m-1})$ and $(V_{m-1},V_m)$.

Clearly,

$d(V_1)=1,d(V_2)=2,d(V_3)=2\ldots\ldots d(V_{h-2})=2,d(V_{h-1})=3,d(V_h)=1,d(V_{h+1})=2,d(V_{h+3})=2\ldots\ldots d(V_{h-1})=2$,and $d(V_m)=1$

Implies G is a connected graph with one vertex of degree 3 three vertex of degree 1 and (m-4) vertex of degree 2. Hence G is a generalized 3-star graph.

Now, let us suppose that there exist an arbitrary arc $A_h \in A$ such that $A_h$ is contained in $A_{h-1}$ and $A_h=A_{m-1}$ satisfying the conditions (vi),(vii),(viii),(ix)and (x) stated in the theorem. Then the distinct edges in the graph are $(V_1,V_2),(V_2,V_3),(V_3,V_4),\ldots\ldots(V_{h-3},V_{h-2}),$ $(V_{h-2},V_{h-1})(V_{h-1},V_h)(V_{h-1},V_{h+1})$ that is the distinct edges of the graph are $(V_1,V_2),(V_2,V_3),(V_3,V_4),\ldots\ldots(V_{m-3},V_{m-2}),(V_{m-2},V_{m-1}),(V_{m-1},V_m)$.

Clearly,$d(V_1)=1,d(V_2)=2,d(V_3)=2\ldots\ldots d(V_{l-2})=2,d(V_{l-1})=3,d(V_l)=1$and $d(V_{l+1})=d(V_m)=1$.

Therefore, G is a connected graph with one vertex of degree 3 three vertices of degree 1 and (m-4) vertices of degree 2. Hence G is a generalized 3-star graph. In both the cases, the graph G is generalized 3-star graph and also

$$\sum_{n=1}^{m} d_n^2 = 3(1)^2+3^2+(m-4)2^2$$

$$=3+9+4m-16$$

$$= 4m-4$$

Therefore $\sum_{n=1}^{m} d_n^2=4m-4$

Conversely,

Let $\sum_{n=1}^{m} d_n^2 =4m-4$ -------------------3.3

Now $\sum_{n=1}^{m} (d_n-2)^2=\sum_{n=1}^{m} d_n^2-4\sum_{n=1}^{m} d_n+4m$

$=4m-4-8p+4m$

$=8(m-p)-4$

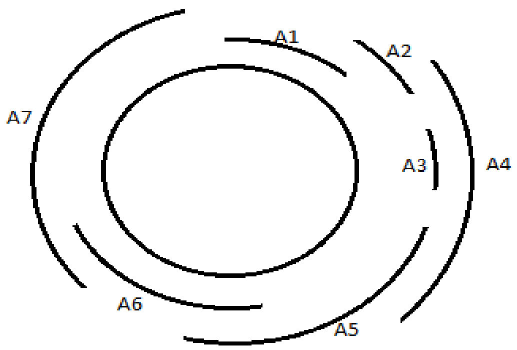
$\leq 8-4$

$\sum_{n=1}^{m} (d_n\text{-}2)^2 \leq 4$ ------------------------- 3.4

The unique degree sequence satisfying 3.3 and 3.4 is the sequence $1^3,2^{m-4},3$. For a connected graph, this degree sequence uniquely determines a generalized 3-star of order m. For that, one vertex must of degree 3, three vertices of degree 1 and the remaining (m-4) vertices must be of degree 2.Hence, conditions (i),(ii),(iii),(iv) and (v) must hold for some arbitrary arc $A_h \in A$,where $A_h$ is contained in $A_{h-1}$ and $A_3 \leq A_h \leq A_{m-2}$(or)conditions (vi),(vii),(viii),(ix)and (x) must hold for some arbitrary arc $A_h$ is contained in $A_{h-1}$ and $A_h = A_m$

**Illustrations:**

Let the circular arc family A= {$A_1$, $A_2$, $A_3$ ………..$A_7$} be the connected graph G be as follows




Circular-arc family A

nbd [$A_1$]={$A_1$,$A_2$}

nbd [$A_2$]={$A_1$,$A_2$,$A_4$}

nbd [$A_3$]={$A_3$,$A_4$}

nbd[$A_4$]={$A_2$,$A_3$,$A_4$,$A_5$}

nbd[$A_5$]={$A_4$,$A_5$,$A_6$}

nbd[$A_6$]={$A_5$,$A_6$,$A_7$}

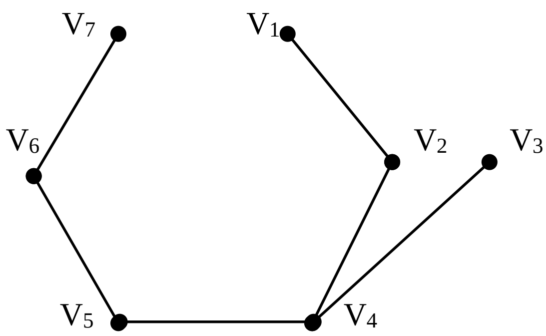
nbd[$A_7$]={$A_6$,$A_7$}

clearly, the conditions (i),(ii),(iii),(iv)and (v) stated in the theorem are satisfied by the circular arc graph G for $A_l$=$A_3$.

Hence $\sum_{n=1}^{m} d_n^2$=4m-4

**Verification:**

Here, O (G) m=7.the circular arc graph G in this case as follows




Circular-arc Graph G

Here d(1)=1,d(2)=2,d(3)=1,d(4)=3,d(5)=2,d(6)=2 and d(7)=1.

Clearly, G is a generalized 3-star graph.

Also $\sum_{n=1}^{m} d_n^2$=1+4+1+9+4+4+1

=24

And 4m-4=4(7)-4 =24

Therefore $\sum_{n=1}^{m} d_n^2$=4m-4.

Hence the theorem is verified.

**Theorem 4:**

Let A={$A_1$, $A_2$, $A_3$,………$A_m$}be a circular arc family corresponding to the connected circular arc graph G of order n≥8.If there exists an arbitrary arcs $A_p$, $A_q$∈A for 3<$A_p$<$A_{p+3}$<$A_q$<m-2 and contained in $A_{p-1}$ and $A_{q-1}$ respectively satisfying the conditions

(i) nbd [$A_{p-1}$] = {$A_{p-2}$,$A_{p-1}$,$A_p$,$A_{p+1}$} and nbd[$A_{q-1}$] = [$A_{q-2}$,$A_{q-1}$,$A_q$,$A_{q+1}$}
(ii) nbd [$A_p$] = {$A_{p-1}$,$A_p$} and nbd [$A_q$] = {$A_{q-1}$,$A_q$}
(iii) nbd [$A_{p+1}$] = {$A_{p-1}$ ,$A_{p+1}$,$A_{p+2}$} and nbd [$A_{q+1}$]={$A_{q-1}$,$A_{q+1}$,$A_{q+2}$}
(iv) nbd [$A_n$]= {$A_{n-1}$, $A_n$ ,$A_{n+1}$}for all m except for n=1,p-1,q-1,p,q,p+1,q+1,m
(v) nbd [$A_1$]={$A_1$,$A_2$} and nbd[$A_m$] ={$A_{m-1}$, $A_m$}

Then $\sum_{n=1}^{m} d_n^2$=4m-2

**Proof:**

Let A= {$A_1$, $A_2$, $A_3$, ……… $A_m$} be a circular arc family corresponding to the connected circular arc graph G of order n≥8.let G be a connected graph.

Since G is a circular arc graph there exists a one to one correspondence between the vertex set V= {$V_1$, $V_2$, $V_3$,………$V_m$} of G and circular arc family A. Let $V_h$ be the vertex corresponding to the arc $A_h$ of the circular arc graph G. here G is a connected graph. Therefore

P≥m-1

Where P is the size of the graph

For any graph G

$$\sum_{n=1}^{m} d_n^2 = 2P$$

First,

Let us suppose that there exist arbitrary arcs $A_p$, $A_q$∈ A for 3≤$A_p$≤$A_{p+3}$≤$A_q$≤m-2 and contained in $A_{p-1}$ and $A_{q-1}$ respectively satisfying the conditions (i),(ii),(iii),(iv) and (v) stated in the theorem

By (i) we have

($V_{p-2}$,$V_{p-1}$),($V_{p-1}$ ,$V_p$),($V_{p-1}$,$V_{p+1}$) and ($V_{q-2}$,$V_{q-1}$),($V_{q-1}$,$V_q$),($V_{q-1}$,$V_{q+1}$) are the edges of the graph G.

By (ii) we can say that

($V_{p-1}$ , $V_p$) and ($V_{q-1}$,$V_q$) are the edges of the graph G.

By (iii) it is clear that

($V_{p-1}$,$V_{p+1}$),($V_{p+1}$,$V_{p+2}$) and ($V_{q-1}$,$V_{q+1}$),($V_{q+1}$,$V_{q+2}$) are also the edges of the graph G.

By (iv) it is evident that

($V_{n-1}$, $V_n$) ($V_n$, $V_{n+1}$) are the edges of the graph

By (v) it is clear that

($V_1$,$V_2$)($V_2$,$V_3$)($V_3$,$V_4$)………($V_{p-3}$,$V_{p-2}$)($V_{p-2}$,$V_{p-1}$)($V_{p+1}$,$V_{p+2}$)($V_{p+2}$,$V_{p+3}$)….($V_{q-2}$,$V_{q-1}$) ($V_{q+1}$,$V_{q+2}$)($V_{q+2}$,$V_{q+3}$)………….($V_{m-2}$,$V_{m-1}$)($V_{m-1}$,$V_m$) are the remaining edges of the graph G.

The distinct edges of the graph are

$(V_1, V_2)(V_2, V_3)(V_3,V_4)$………………$(V_{p-3},V_{p-2})(V_{p-2},V_{p-1})(V_{p-1},V_p)(V_{p+1},V_{p+2})(V_{p+2},V_{p+3})$

……….$(V_{q-2},V_{q-1})(V_{q+1},V_{q+2})(V_{q+2},V_{q+3})$………………$(V_{m-2},V_{m-1})(V_{m-1},V_m)$.

$d(V_1)=1,d(V_2)=2,d(V_3)=2,$……… $d(V_{p-2})=2,d(V_{p-1})=3,d(V_p)=1,d(V_{p+1})=2,d(V_{p+2})=2,$….

$d(V_{q-2})=2,d(V_{q-1})=3,d(V_q)=1,d(V_{q+1})=2,$……….. $d(V_{m-1})=2,d(V_m)=1$.

Therefore, G is a connected graph with two vertices of degree 3 four vertices of degree 1 and (m-6) vertices of degree 2. Hence G is a generalized double 3 star graph.

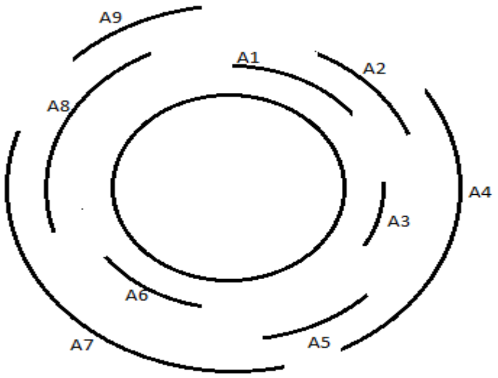
Here $\sum_{n=1}^{m} d_n^2=4(1)^2+2(3)^2+(m-6)2^2$

$=4+18+4m-24$

$= 4m-2.$

**Illustration:**

Let circular arc family A= $\{A_1, A_2, A_3\ldots\ldots A_9\}$ of the given connected circular arc graph G




Circular-arc family A

$nbd[A_1]= \{A_1,A_2\}$

$nbd[A_2] = \{A_1,A_2,A_4\}$

$nbd[A_3] = \{A_3,A_4\}$

$nbd[A_4] = \{A_2,A_3,A_4,A_5\}$

$nbd[A_5] = \{A_4,A_5,A_7\}$

$nbd[A_6] = \{A_6,A_7\}$

$nbd[A_7] = \{A_5,A_6,A_7,A_8\}$

$nbd\ [A_8] = \{A_7,A_8,A9\}$

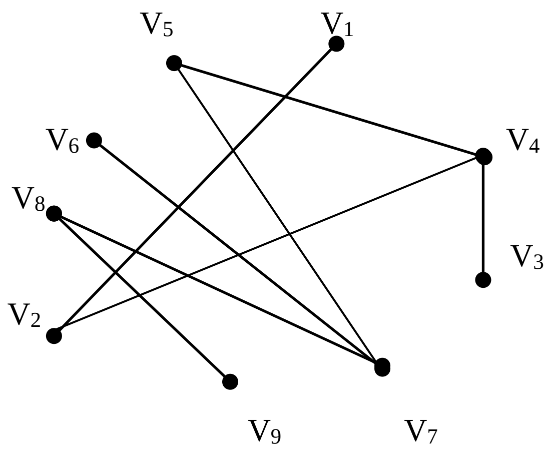
$nbd[A_9] = \{A_8,A_9\}$.

Clearly for $A_p= A_3$ and $A_q=A_6$ the circular arc family satisfies all the five conditions stated in the theorem. Hence

$$\sum_{n=1}^{m} d_n^2 =4m-2.$$

**Verification:**

Here O (G) m=9, the circular arc graph G is shown the figure 3




Circular-arc graph G

$d(V_1)=1, d(V_2)=2, d(V_3)=1, d(V_4)=3, d(V_5)=2, d(V_6)=1, d(V_7)=3, d(V_8)=2, d(V_9)=1$

Therefore, G is a generalized double 3-star graph

$\sum_{n=1}^{9} d_n^2=1+4+1+9+4+1+9+4+1$

$=34$

$4m-2 =4(9)-2$

$=34$

$\sum_{n=1}^{9} d_n^2=4m-2$

Hence the theorem is verified.

## 4. CONCLUSION:

Circular arc graphs are reach in combinatorial structures and have found applications in several disciplines such as Biology, Ecology, Traffic control, Genetics and Computer Science and particularly useful in Cyclic scheduling and computer storage allocation problems. In this paper, we characterized Circular arc graphs as complete graphs, path graphs, generalized 3 star graphs and generalized double 3 star graphs using some parameters with neighborhoods of graph G.

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