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Various Methods Adapted To Solve Integro-Differential Equations

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Abstract

In this paper we outlines the various methods to solve Integro-Differential Equations .There are several methods to solve IDE's namely Adomian Decomposition method, Variational Iteration method, Taylors method. The Adomian Decomposition method is powerful and efficient methods that both give approximations of higher accuracy and closed form solutions if existing.The variational iteration method gives several successive approximation through using the iteration of the correction functional. The Taylor series can be used to calculate the value of an entire function at every point, if the value of the function, and of all of its derivatives, are known at a single point.

Keywords : Integro-Differential equation, Adomian Decomposition method, Variational iteration method, Laplace method, Taylors method.

Introduction

Integro–differential equation are run across in various fields of sciences. The integro-differential equations contain both integral and differential operators. The derivatives of unknown functions may appear to any order.

$$u^{(n)}(x) = f(x) + \int_{g(x)}^{h(x)} k(x,t)F\big(u(t)\big)\,dt,$$

This is a general form of integro-differential equation where $u^{(n)}(x)$ *is* n^{th} derivative of u(x). The kernel k(x,t) and function f(x) are given real value functions and F(u(x)) is a function of u(x). It can be classified into two types namely if the limits of integration in above equation are constant then this equation is called Fredholm integro-differential equation and if the at least one limit in above integral equation is variable then this equation is called Volterra integro-differential equation. Linearity and nonlinearity of above equation is depending on the function $F(u(x))$[1].

In recent years, there has been an increasing interest in the integral and integro-differential equations. The integro-differential equations play an important role in many branches of linear and non-linear functional analysis and their applications in the theory of engineering, physics, mechanics, chemistry, biology, economics and electrostatics [2, 3].

Following number of methods are used to solve Integro –differential equations.

Useful Methods :

1. Adomian Decomposition method :

Adomian Decomposition method (ADM) was introduced and developed by George Adomian.The Adomian decomposition method is a well-known systematic method for practical solution of linear or nonlinear and deterministic or stochastic operator equations, including ordinary differential equations, partial differential equations, integral equations, integro-differential equations, to mention but few. The Adomian decomposition method is a powerful technique, which provides efficient algorithms for analytic approximate solutions and numerical simulations for real-world applications in the applied sciences and engineering. The Adomian decomposition method gives solution in an infinite series of components. [3-6]

Basic Ideas of Adomian Decomposition Method (ADM)

Consider the differential equation of the form

$$Lu + Ru + Nu = g(x) \qquad 1.1$$

Where L is the linear operator which is highest order derivative, R is the remainder of linear operator including derivatives of less order than L, Nu represents the non- linear terms and $g(x)$ is the source term. Solving for Lu

$$Lu = g(x) - Ru - Nu \qquad 1.2$$

since L is invertible, so applying the inverse operator L^{-1} to both side of equation (2)

$$L^{-1}Lu = L^{-1}[g(x)] - L^{-1}Ru - L^{-1}Nu \qquad 1.3$$

After integrating source term and combining it with the terms arising from given conditions of the problem, a function f(x) is defined in the equation

$$U = f(x) - L^{-1}(Ru) - L^{-1}(Nu) \qquad 1.4$$

The non- linear operator $Nu = Fu$ is represented by an infinite series of specifically generated Adomian polynomials for the specific non linearity. Assuming Nu is analytic

$$F(u) = \sum_{k=0}^{\infty} A_k \qquad 1.5$$

Where $A_{k's}$ are given as

$A_0 = F(u_0)$

$A_1=u_1F'(u_0)$

$A_2=u_2F'(u_0)+\frac{1}{2!}u_1^2F''(u_0)$ . . .

The polynomial $A_{m's}$ are generated for all kinds of non-linearity so that they depend only on u_0 to $u_{m's}$ by the following alogarithm

$$Am=\frac{1}{k!}\frac{d^k}{d\lambda^k}\{N\sum_{n=0}^{\infty}(\lambda^n y_n)\} \quad (1.6)$$

where λ is a parameter .

2. Variational Iteration method

The variational iteration method (VIM) developed in 1999 by He in [7,8,9] will be used to study the linear wave equation, nonlinear wave equation, and wave-like equation in bounded and unbounded domains.The variational iteration method (VIM) is one of the well-known semi-analytical methods for solving linear and nonlinear ordinary as well as partial differential equations.

The VIM is used to solve effectively, easily, and accurately a large class of non-linear problems with approximations which converge rapidly to accurate solutions. The method gives rapidly convergent successive approximations of the exact solution if such a solution exists; otherwise a few approximations can be used for numerical purposes.

Basic Ideas of Variational Iteration Method

Idea of variational iteration method depends on the general Lagrange's multiplier method[10,11,12]

Consider the differential equation

$$Lu+Nu=g(t) \quad 2.1$$

Where L and N are linear and nonlinear operators respectively and g(t) is given contineous function.He introduced the Variational Iteration method where a correction functional for equation (2.1) can be written as

$$u_{n+1}(t)=u_n(t)+\int_0^t \lambda[Lu_n(s)+N\tilde{u}_n(s)-g(s)]ds \quad 2.2$$

where λ is general Lagranges multiplier which can be identified optimally via the Variational theory and $\tilde{u}_n$ is a restricted variation, which means $\delta\tilde{u}_n=0$.

It is obvious now that the main step of He's Variational Iteration method require first the determination of the Lagrangian multiplier λ that will be identified optimally. Having determined the Lagrangian multiplier, the successive approximations.

U_{n+1}, $n \geq 0$ of the solution u will be readily obtained upon using any selective functions u_0, consequently ,the solution,

$$u=\lim_{n\to\infty} un \quad 2.3$$

the equation 2.2 gives several approximations and therefore the exact solution is obtained at the limit of the resulting successive approximations.

3. The Taylors Method

Taylor series are named after Brook Taylor, who introduced them in 1715.In recent years, the numerical methods for linear integro-differential equations have been extensively studied by many authors. Taylor method is a powerful technique for solving integro-differential equations.[13]

Taylors series is an expansion of a function into an infinite series of a variable x or into a finite series plus a remainder term Taylor series have wide reaching applications across mathematics, physics, engineering and other sciences.[14,15]

Basic Ideas of Taylors Method

The Taylor formula of *n*th order has the following form :

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^n}{n!}(x-a)^n + R \qquad 3.1$$

Where

$$R=\frac{f^{n+1}(s)}{n+1}(x-a), \qquad a < s < x$$

When using n terms it approximates the value of the functions $f(x)$ with the error R, which is function of $(x-a)^{n+1}$

To solve the Integro –Differential equation for a solution in the interval $[a, b]$, the interval must be divided into r subintervals , by $\{ x_0, x_1, \ldots\ldots\ldots x_r \}$

Where

$$x_{k-1} < x_k \quad , k = 1,2,3, \ldots\ldots\ldots r$$

$$x_0 = a, x_r = b \text{ and } h_k = x_k - x_{k-1}$$

Taylors formula will be written in the following form

$$g(x_{k+1}) = g(x_k) + g'(x_k)h_{k+1} + \frac{1}{2!}g''(x_k)(h_{k+1})^2 + \cdots + \frac{1}{n!}g^n(x_k)(h_{k+1})^n$$

where $k = 0,1, \ldots\ldots f - 1.$

the values of $g(x_k),\ g'(x_k),\ g''(x_k), \cdots \ \cdots,\ g^n(x_k)$

for each iteration will be obtained from the previous iteration and form the Integro-Differential equation.

CONCLUSION :

By applying the ADM, one can construct approximate solutions to algebraic equations, fractional ordinary differential equations fractional partial differential equations. The main advantage of the method is that it can be used directly to solve, all types of differential equations with homogeneous and inhomogeneous boundary conditions.The main advantage of the VIM method lies in its flexibility and ability to solve nonlinear equations easily. The VIM has no specific requirements, such as linearization, small parameters, etc. for nonlinear operators.Taylor series expansion is an awesome concept, not only the world of mathematics, but also in optimization theory, function approximation and machine learning.

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