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Applications of Kharrat-Toma Transform in Handling Population Growth and Decay Problems

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Abstract.
Many researchers used various integral transforms to solve the population growth and decay problems. In this paper we use recently developed Kharrat-Toma transform to solve the problems on population growth and decay.

Key Words: Growth problems, Decay Problem, Integral transforms, Kharrat-TomaTransform.

1. Introduction

Many quantities in the universe grow or decay at a rate proportional to their size. For example a colony of bacteria may double or triple in a hour. If the size of the colony after t hours is given by $y(t)$, then we can express this information in the mathematical language in the form of a first order differential equation,

$$\frac{dy}{dt} = 2y$$

The quantity y that grows or decays at a rate proportional to its size is governed by a first order differential equation,

$$\frac{dy}{dt} = ky$$

If $k < 0$ then the above equation is called the law of natural decay and if If$k > 0$ then the above equation is called the law of natural growth. This equation is solved by separation of variable method. Integral transforms plays an important role in solving differential equations.

Recently, Integral transforms are one of the most useful and simple mathematical technique for obtaining the solutions of advance problems occurred in many fields like science, Engineering, technology, commerce and economics. To provide exact solution of problem without lengthy calculations is the important feature of integral transforms.

Due to this important feature of the integral transforms many researchers are attracted to this field and are engaged in introducing various integral transforms. Recently, Kushare and Patil [1] introduced new integral transform called as Kushare transform for solving differential equations in time domain. Further, Savita Khakale and Dinkar Patil [2] introduced Soham transform. As researchers are interested in introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields, various equations in different domain. In January 2022, Sanap and Patil [3] used Kushare transform for obtaining the solution of the problems on Newton's law of Cooling.

In April 2022 D. P. Patil, et al [4] solved the problems on growth and decay by using Kushare transform. D.P. Patil [5] also used Sawi transform in Bessel functions. Further, Patil [6] evaluate improper integrals by using Sawi transform of error functions. Laplace transforms and Shehu transforms are used in chemical sciences by Patil [7]. Dinkar Patil [8] solved wave equation by using Sawi transform and its convolution theorem. Using Mahgoub transform, parabolic boundary value problems are solved by D .P. Patil [9].

D .P. Patil used double Laplace and double Sumudu transforms to obtain the solution of wave equation [10]. Dr. Patil [11] also obtained dualities between double integral transforms. Kandalkar, Gatkal and Patil [12] used Kushare transform to solve the system of differential equations. D. P. Patil [13] solved boundary value problems of the system of ordinary differential equations by using Aboodh and Mahgoub transforms. Double Mahgoub transformed is used by Patil [14] to solve parabolic boundary value problems.

Laplac, Sumudu , Aboodh , Elazki and Mahagoub transforms are compared and used it for solving boundary value problems by Dinkar Patil [15]. D. P. Patil et al [16] solved Volterra Integral equations of first kind by using Emad-Sara transform. Futher Patil with Tile and Shinde [17] used Anuj transform and solved Volterra integral equations for first kind. Rathi sisters and D. P. Patil [18] solved system of differential equations by using Soham transform. Vispute, Jadhav and Patil [19] used Emad Sara transform for solving telegraph equation. Kandalkar, Zankar and Patil [20] evaluate the improper integrals by using general integral transform of error function. Dinkar Patil, Prerana Thakare and Prajakta Patil[21] obtained the solution of parabolic boundary value problems by using double general integral transform. Dinkar Patil used Emad- Falih transform for solving problems based on Newton's law of cooling [22]. D. P. Patil et al [23] used Soham transform to obtain the solution of Newton's law of cooling. Dinkar Patil et al [24] used HY integral transform for handling growth and Decay problems, D. P. Patil et al used HY transform for Newton's law of cooling [25]. D. P. Patil et al [26] used Emad-Falih transform for general solution of telegraph equation. Dinkar Patil et al [27] introduced double kushare transform. Recently, D. P. Patil et al [28] solved population growth and decay problems by using Emad Sara transform. Alenzi transform is used in population growth and decay problems by patil et al [29]. Thete et al [30] used Emad Falih transform for handling growth and decay problems. Nikam, Patil et al [31] used , Kushare transform of error functions in evaluating improper integrals. Wagh sisters and Patil used Kushare [32] and Soham [33] transform in chemical Sciences. Malpani, Shinde and Patil[34] used Convolution theorem for Kushare transform and applications in convolution type Volterra integral equations of first kind. Raundal and Patil [35] used double general integral transform for solving boundary value problems in partial differential equations. Rahane, Derle and Patil [36] generalized Double rangaig integral transform. Emad A. Kuffi et al [37][38] developed and used SEE transform to solve Volterra integro-differential equation. Patil et al [39] used Kushare transform to solve Volterra Integro-Differential equations of first kind.

(2) Preliminary:

In this section we state preliminary concepts which are required to solve the growth and decay problems.

Definition: The Kharrat-Toma integral transform[40] and inversion is defined by.

$$B[f(t)] = G(S) = s^3 \int_0^\infty f(t)\, e^{\frac{-t}{s^2}} dt \quad , \quad t \geq 0$$

$$f(\mathrm{t}) = \mathrm{B}^{-1}[\mathrm{G(s)}] = \mathrm{B}^{-1}\left[\mathrm{s}^3 \int_0^\infty \mathrm{f(t)}\, \mathrm{e}^{\frac{-\mathrm{t}}{\mathrm{s}^2}} \mathrm{dt}\right]$$

The B integral transform states that, if $f(t)$ is piecewise continuous on $[0, +\infty)$ and has exponential order. The B^{-1} will be the inverse of the B integral transform.

Theorem (1):[Sufficient Condition for Existence of a Kharrat-Toma Transform]:[40] The Kharrat-Toma transform $B[f(t)]$ exists if the function $f(t)$has an exponential order and $\int_0^b |f(t)|dt$ exists for any $b > 0$.

Kharrat-Toma Transform of exponential function:

$$B(e^{at}) = s^3 \int_0^\infty e^{\frac{-t}{s^2}} e^{at} dt$$

$$= s^3 \int_0^\infty e^{-\left(\frac{1}{s^2}-a\right)t} dt$$

$$= s^3 \left[\frac{e^{-\left(\frac{1-as^2}{s^2}\right)}}{-\left(\frac{1-as^2}{s^2}\right)} \right]_0^\infty$$

$$= \left[0 + \frac{1}{\left(\frac{1+as^2}{s^2}\right)} \right]$$

$$= \frac{s^5}{1-as^2}$$

Kharrat-Toma transform of some fundamental function:

Sr.No	Functions	$KHA - Toma$ Transform
1	1	s^5
2	t	s^7
3	e^{at}	$\frac{s^5}{1-as^2}$
4	t^n	$s^{2n+5}n!$
5	$\sin at$	$\frac{as^7}{1+a^2s^4}$
6	$\cos at$	$\frac{s^5}{1+a^2s^4}$
7	$\sinh at$	$\frac{as^7}{1-a^2s^4}$
8	$\cosh at$	$\frac{s^7}{1-a^2s^4}$

Theorem (2): Kharrat-Toma transform of Derivatives:

Let $B[f(t)] = G(s)$, then

$$B[f'(t)] = \frac{1}{t^2} G(s) - s^3 f(0)$$

(3) Applications of Kharrat-Toma transform in population growth and decay problems:

In this section, we use Kharrat-Toma transform to solve growth and decay problem:

(3.1) Kharrat-Toma transform for growth problem:

Inthis sectionwe use Kharrat-Toma transform for growth problem as follows:

The population(growth of a plant, or a cell, or an organ, or a species) is governed by first order linear differential equation, This problem can be written in the mathematical form as:

$$\frac{dN}{dt} = P.N(t) \qquad (1)$$

With initial condition,

$$N(t_\circ) = N_\circ \qquad (2)$$

Where P is a positive real number, N is the amount of population at time t and $N_\circ$ is the initial population at time $t_\circ$.

Applying Kharrat-Toma transfom on both side of equation (1)

$$B\left\{\frac{dN}{dt}\right\} = P.B\{N(t)\}$$

Now, applying the property, Kharrat-Toma transform of derivative of function, on above equation,

$$\frac{1}{s^2}[G(s)] - s^3[N(0)] = P.B\{N(t)\}$$

Since, $t_\circ = 0$, $N = N_\circ$

$$\frac{1}{s^2}[G(s)] - P[G(s)] = s^3 N_\circ$$

$$\therefore G(s)\left[\frac{1}{s^2} - P\right] = s^3 N_\circ$$

$$\therefore G(s) = N_\circ\left(\frac{s^3.s^2}{1 - s^2 P}\right)$$

$$\therefore G(s) = N_\circ\left(\frac{s^5}{1 - s^2 P}\right)$$

Applying inverse Kharrat-Toma on above equation,

$$(B)^{-1}[G(s)] = (B)^{-1}\left[\frac{N_\circ s^5}{1 - s^2 P}\right]$$

$$\Rightarrow N(t) = N_\circ (B)^{-1}\left[\frac{s^5}{1 - s^2 P}\right]$$

$$\Rightarrow N(t) = N_\circ e^{Pt}$$

Which is required amount of population at time t .

(3.2) Kharrat-Toma transformation for decay problem:

In this section, we use Kharrat-Toma transform for decay problem which is given as follows:

The decay problem of the substance is governed by the first order linear ordinary differential equation.

$$\frac{dN}{dt} = -P.N(t)(3)$$

With initial condition as,

$$N(t_\circ) = N_\circ \qquad (4)$$

Where N is the amount of substance at time t ; P is positive real number and $N_\circ$ is the initial amount of the substance at time, $t_\circ$. In equation (3), the negative sign to R.H.S is taken because of the mass is decreasing with time and so the derivative $\frac{dN}{dt}$ must be negative.

Applying Kharrat-Toma transform on both side of the equation (3)

$$B\left\{\frac{dN}{dt}\right\} = -P.(B)\{N(t)\}$$

Now applying the property, Kharrat-Toma transform of derivative of the function of above equation,

$$\frac{1}{s^2}G(s) - s^3 N(0) = -P.(B)\{N(t)\}$$

Since, $t_\circ = 0$, $N = N_\circ$,

$$\frac{1}{s^2}[G(s)] + P.[G(s)] = N_\circ s^3$$

$$\therefore G(s)\left[\frac{1}{s^3}+P\right]=N_{\circ}s^3$$

$$\therefore G(s)=N_{\circ}\left(\frac{s^3.s^2}{1+s^2P}\right)$$

$$\therefore G(s)=\frac{N_{\circ}s^5}{1+s^2P}$$

Applying inverse Kharrat-Toma transform on both side of equation (3)

$$(B)^{-1}[G(s)]=(B)^{-1}\left[\frac{N_{\circ}s^5}{1+s^2P}\right]$$

$$\Rightarrow N(t)=N_{\circ}(B)^{-1}\left[\frac{N_{\circ}s^5}{1+s^2P}\right]$$

$$\Rightarrow N(t)=N_{\circ}e^{-Pt}$$

Which is required amount of substance at time t.

(4) Applications: In this section, we solve some problems on population growth and decay.

Application (1): The population of the city grows at the rate proportional to the number of people presently living in the city. If after two years the population has doubled and after three years the population is 20000, Estimate the number of people initially in the city.

Solution :

This problem can be written in mathematical form as :

$$\frac{dN}{dt}=PN(t)(5)$$

Where N denote the number of people living in the city at any time t and P is the constant of proportionality. Consider, $N_{\circ}$ is the number of people initially living in the city at time, $t=0$.

Applying Kharrat-Toma transform of the derivative of function, in above equation,

$$B\left\{\frac{dN}{dt}\right\}=P.B\{N(t)\}$$

Now, apply the property of Kharrat-Toma transform of derivative function in above equation,

$$\frac{1}{s^2}[G(s)]-s^3[N(0)]=P.B\{N(t)\}$$

$$\therefore N=N_{\circ},t=0$$

$$\frac{1}{s^2}[G(s)-P.[G(s)]]=s^3N_{\circ}$$

$$\Rightarrow G(s)\left[\frac{1}{s^2}-P\right]=s^3N_{\circ}$$

$$\therefore G(s)=\left(\frac{N_{\circ}s^5}{1-s^2P}\right)$$

Applying inverse Kharrat-Toma transform on above equation,

$$(B)^{-1}[G(s)]=N_{\circ}(B)^{-1}\left[\frac{s^5}{1-s^2P}\right]$$

$$\Rightarrow N(t)=N_{\circ}e^{Pt}(6)$$

Now, at $t=2,N=2N_{\circ}$

$$\therefore 2N_{\circ}=N_{\circ}e^{2P}$$

$$\therefore 2=e^{2P}$$

$$\therefore P=\frac{1}{2}\log_e 2$$

$$\therefore P=0.3466$$

Now, put $t=3,N=20000$

Put this value in equation (6),

$$\therefore 20000=N_{\circ}e^{3P}$$

$$\therefore 20000 = N_{\circ}e^{3(0.3466)}$$
$$\therefore 20000 = N_{\circ}(2.8287)$$
$$\therefore N_{\circ} = 7070.3857 \approx 7070$$

Which is required number of people living in the city, initially.

Application (2) :

A radioactive substance is known to decay at a rate proportional to the amount present. If initially there is 100 miligrams of the radioactive substance present and after two hours it is observed that the radioactive substance has lost 10 percent of its original mass, find half life of the radioactive substance.

Solution:

This problem can be written in the following form:

$$\frac{dN}{dt} = -P.N(t)(7)$$

Where, N denotes the amount of radioactive substance at time t and P is the proportionality constant.

Consider $N_{\circ}$ is the initial amount of the radioactive substance at time $t = 0$

Applying Kharrat-Toma transform on both side of equation (7),

$$B\left\{\frac{dN}{dt}\right\} = -P.B\{N(t)\}$$

Now, applying the property, Kharrat-Toma transform of derivative of function, on above equation

$$\frac{1}{s^2}[G(s)] + P[G(s)] = N(0)s^3$$

Since, $N = N_{\circ}, t = t_{\circ}$

$$\therefore G(s)\left[\frac{1}{s^2} + P\right] = N_{\circ}s^3$$

$$\Rightarrow G(s) = \left[\frac{N_{\circ}s^5}{1 + s^2P}\right]$$

Since, $t = 0, N_{\circ} = 100$

$$G(s) = \left[\frac{100s^5}{1 + s^2P}\right]$$

Now, applying inverse Kharrat-Toma on above equation,

$$(B)^{-1}[G(s)] = (B)^{-1}\left[\frac{100s^5}{1 + s^2P}\right]$$

$$= 100(B)^{-1}\left[\frac{s^2}{1 + s^2P}\right]$$

$$= 100e^{-Pt} \qquad (8)$$

Now, by the given condition in the problem at $t = 2$, the radioactive substance has lost 10 percent of its original mass 100 miligrams.

So,

$$N = 100 - 10 = 90$$
$$\therefore 90 = 100e^{-2P}$$
$$\therefore e^{-2P} = 0.9$$
$$\therefore -2P = \log_e 0.9$$
$$\therefore P = \frac{-1}{2}\log_e 0.9$$
$$\therefore P = 0.0527$$

We required half time of radioactive substance(t) .

When $N = \frac{N_{\circ}}{2} = \frac{100}{2} = 50$

Substitute this value in equation(8),

$$\therefore 50 = 100e^{-Pt}$$
$$\therefore 50 = 100e^{-0.0527t}$$
$$\therefore 0.5 = e^{-0.0527t}$$
$$\therefore -0.0527t = \log_e(0.5)$$
$$\therefore 0.0527t = -0.6932$$
$$\therefore t = 13.1537\ hours$$

This is required half life time of the radioactive substance.

(5) Conclusion:

We have successfully used Kharrat-Toma transform to solve the growth and decay problems.

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