



## EQUITABLE COLOR CLASS DOMINATION NUMBER OF SOME SPECIAL GRAPHS AND SOME DERIVED PATH GRAPHS

<sup>1</sup>A.Esakkimuthu and <sup>2</sup>S.Mari Selvam

<sup>1</sup> Assistant Professor of Mathematics, <sup>2</sup> Research Scholar of Mathematics.

<sup>1,2</sup> Department of Mathematics, Kamaraj College, Thoothukudi, Tamilnadu, India.

**Abstract:** Let  $G = (\mathcal{V}_p, \mathcal{E}_q)$  be a connected graph. Assume a group CC containing colors. Let  $\tau: \mathcal{V}_p(G) \rightarrow CC$  be an equitable colorable function. A dominating subset  $\mathcal{S}_p$  of  $\mathcal{V}_p$  is called an equitable color class dominating set if the number of dominating nodes in each color class is equal. The least possible cardinality of an equitable color class dominating set of  $G$  is equitable color class domination number itself. It is indicated by  $\gamma_{ECC}(G)$ . In this paper we explore the equitable color class domination number of some special graphs and some derived path graphs.

**Keywords:** Domination, Dominating Set, Equitable Coloring, Color Class(CC), Equitable Color Class, Equitable Color Class Dominating set.

**I INTRODUCTION:** In graph theory, domination and coloring are the most important theories and they are still a very active field of research. The domination theory was introduced in the 1950s but it was highly researched from the mid-1970s [6] and [7]. oystein ore introduced the terms “Dominating Set” and “Domination Number” in 1962 [12]. Graph coloring is a special case of graph labeling. The first results about graph coloring deal almost exclusively with planar graphs in the form of the coloring of maps. It has been studied as an algorithmic problem since the early 1970s. In this paper, we considered all the graphs are finite, undirected with no loops. Let  $G = (\mathcal{V}_p, \mathcal{E}_q)$  be a graph.  $\mathcal{V}_p$  is a set of nodes it is finite and non-empty and  $\mathcal{E}_q$  is a set of edges. The degree of a node  $v$  is denoted by  $\deg(\mathcal{V}_p)$ . The minimum and maximum degree of a graph  $G$  are denoted by  $\delta(G)$  and  $\Delta(G)$  respectively [4].

### II DEFINITIONS AND NOTATIONS:

**Definition 2.1:** [6] In a graph  $G = (\mathcal{V}_p, \mathcal{E}_q)$ , a subset  $\mathcal{S}_p$  of nodes is a dominating set if every node in  $\mathcal{V}_p - \mathcal{S}_p$  is adjacent to some node in  $\mathcal{S}_p$ . The least possible cardinality of the dominating set of  $G$  is called its domination number and it is indicated by  $\gamma(G)$ .

**Definition 2.2:** [11] In a graph  $G$ , adjacent nodes don't ordain the same color is known as proper coloring. The least possible number of colors used to color a graph  $G$  is known as its chromatic number and it is indicated by  $\chi(G)$ .

**Definition 2.3:** [16] A subset of nodes ordained to the same color is known as a color class.

**Definition 2.4:** [11] In a graph, adjacent nodes don't have the same color and the difference between the cardinality of color classes is  $\leq 1$  is called an equitable coloring graph. The least possible number of colors used to equitably color a graph  $G$  is known as its equitable chromatic number and it's indicated by  $\chi_E(G)$ .

**Notation 2.5:** Let  $\mathcal{X}$  be any real number. Then  $\lfloor \mathcal{X} \rfloor$  indicates the greatest integer  $\leq \mathcal{X}$  and  $\lceil \mathcal{X} \rceil$  indicate the smallest integer  $\geq \mathcal{X}$ .

**Notation 2.6:** If  $a, b$  be the integers and  $n > 0$  then  $a \equiv b \pmod{n}$  indicates  $n|a-b$ .

**Definition 2.7:** [9] The ordinary subdivision graph is obtained from the graph by inserting a new node of degree 2 on each edge of the graph and it is indicated by  $\mathcal{S}(G)$ .

**Definition 2.8:** [9] The Splitting graph of  $G$ ,  $\mathcal{S}'(G)$  is obtained from  $G$  by adding for each vertex  $v$  of  $G$  a new vertex  $v'$  so that  $v'$  is adjacent to every vertex that is adjacent to  $v$ .

**Definition 2.9:** [9] The Shadow graph  $\mathcal{D}_2(G)$  of a connected graph  $G$  is constructed by taking two copies of  $G$ ,  $G'$  and  $G''$  and joining each vertex  $u'$  in  $G'$  to the neighbors of the corresponding vertex  $u''$  in  $G''$ .

III PRIMARY RESULTS:

**Definition 3.1:** [5] Let  $G = (\mathcal{V}_p, \mathcal{E}_q)$  be a connected graph. Assume a group CC containing colors. Let  $\tau: \mathcal{V}_p(G) \rightarrow CC$  be an equitably colorable function. A dominating subset  $\mathcal{S}_p$  of  $\mathcal{V}_p$  is called an equitable color class dominating set if the number of dominating nodes in each color class is equal. The least possible cardinality of an equitable color class dominating set of  $G$  is the equitable color class domination number itself. It is indicated by  $\gamma_{ECC}(G)$ .

**Theorem 3.2:** The equitable color class domination number of a butterfly graph ( $n > 1$ )  $\gamma_{ECC}(\mathcal{BF}_n) = n + 2$ .

**Proof:** Let  $\mathcal{BF}_n$  be the butterfly graph with  $\mathcal{V}_p(\mathcal{BF}_n) = \{u, v, w, u_i, v_i : 1 \leq i \leq n\}$  and  $\mathcal{E}_q(\mathcal{BF}_n) = \{wu, wv, wu_i, wv_i : 1 \leq i \leq n\} \cup \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n - 1\}$ .  $|\mathcal{V}_p(\mathcal{BF}_n)| = 2n + 3$ .  $\chi_E$  of  $\mathcal{BF}_n$  is  $n + 2$ . Let  $\tau: \mathcal{V}_p(\mathcal{BF}_n) \rightarrow CC$  be an equitably colorable function where  $CC = \{1, 2, \dots, n, n + 1, n + 2\}$ . Ordain the color  $i$  to the nodes  $u_i$  and  $v_i$  where  $1 \leq i \leq n$ . Ordain the color  $n + 1$  to the nodes  $u$  and  $v$ . Ordain the color  $n + 2$  to the node  $w$ . Now we choose  $w, u$  and  $u_i$  ( $1 \leq i \leq n$ ) nodes as dominating nodes. The number of dominating nodes in each color class is same and one. Hence,  $\gamma_{ECC}(\mathcal{BF}_n)$  is  $n + 2$ .

**Example 3.3:**

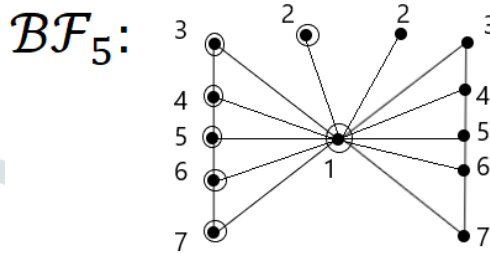


Fig.3.1

Dominating set =  $\{w, u, u_1, u_2, u_3, u_4, u_5\}$ .

$$\gamma_{ECC}(\mathcal{BF}_5) = 7.$$

**Theorem 3.4:** The equitable color class domination number of lotus inside circle graph  $\gamma_{ECC}(\mathcal{LC}_n) = 4\lceil \frac{n}{6} \rceil$ .

**Proof:** Let  $\mathcal{LC}_n$  be the lotus inside circle graph with  $\mathcal{V}_p(\mathcal{LC}_n) = \{u, u_i, v_i : 1 \leq i \leq n\}$ . Let  $u$  be the center node of  $\mathcal{K}_{1,n}$  and  $u_1, u_2, \dots, u_n$  be the pendent vertices of  $\mathcal{K}_{1,n}$ . Let  $v_1, v_2, \dots, v_n$  be the vertices of the cycle  $\mathcal{C}_n$  such that  $u_1$  is adjacent with  $v_n$  and  $v_1, u_i$  is adjacent with  $v_{i-1}$  and  $v_i$  ( $2 \leq i \leq n$ ).  $|\mathcal{V}_p(\mathcal{LC}_n)| = 2n + 1$ .  $\chi_E$  of  $\mathcal{LC}_n$  is  $n + 2$ . Let  $\tau: \mathcal{V}_p(\mathcal{LC}_n) \rightarrow CC$  be an equitably colorable function where  $CC = \{1, 2, 3, 4\}$ .

**Case 1:**  $n$  is odd

Ordain the color 1 to the nodes  $u$  and  $v_i$  ( $i$  is odd and  $1 \leq i \leq n - 1$ ), 2 to the nodes  $u_n$  and  $v_i$  ( $i$  is even), 3 to the nodes  $v_n$  and  $u_i$  ( $i$  is odd and  $1 \leq i \leq n - 1$ ) and 4 to the nodes  $u_i$  ( $i$  is even). Now we choose dominating nodes as  $v_{3i+1}$  and  $u_{3i+2}$  ( $i = 0, 1, 2, \dots, \lceil \frac{n}{3} \rceil - 1$ ). If  $3i+1$  or  $3i+2 > n$  for  $i = \lceil \frac{n}{3} \rceil - 1$  then replacing the dominating nodes  $v_{3i+1}$  by  $v_{n-1}$  and  $u_{3i+2}$  by  $u_{n-2}$ . If  $3i+1$  or  $3i+2 > n$  for  $i = \lceil \frac{n}{3} \rceil$  and  $\lceil \frac{n}{3} \rceil - 1$  then replacing the dominating nodes  $v_{3i+1}$  by  $v_{n-2}$  or  $v_{n-1}$  and  $u_{3i+2}$  by  $u_{n-1}$  or  $u_{n-4}$ . These nodes are dominates all other nodes and the number of dominating nodes in each color class are same and  $\lceil \frac{n}{6} \rceil$ . The total number of dominating nodes is  $\lceil \frac{n}{6} \rceil + \lceil \frac{n}{6} \rceil + \lceil \frac{n}{6} \rceil + \lceil \frac{n}{6} \rceil = 4\lceil \frac{n}{6} \rceil$ . Hence, when  $n$  is odd  $\gamma_{ECC}$  of any  $\mathcal{LC}_n$  is  $4\lceil \frac{n}{6} \rceil$ .

**Case 2:**  $n$  is even

Ordain the color 1 to the nodes  $u$  and  $v_i$  ( $i$  is odd), 2 to the nodes  $v_i$  ( $i$  is even), 3 to the nodes  $u_i$  ( $i$  is odd) and 4 to the nodes  $u_i$  ( $i$  is even). Now we choose dominating nodes as  $v_{3i+1}$  and  $u_{3i+2}$  ( $i = 0, 1, 2, \dots, \lceil \frac{n}{3} \rceil - 1$ ). If  $3i+1$  or  $3i+2 > n$  for  $i = \lceil \frac{n}{3} \rceil - 1$  then replacing the dominating nodes  $v_{3i+1}$  by  $v_n$  and  $u_{3i+2}$  by  $u_{n-1}$ . These nodes are dominates all other nodes and the number of dominating nodes in each color class are same and  $\lceil \frac{n}{6} \rceil$ . The total number of dominating nodes is  $\lceil \frac{n}{6} \rceil + \lceil \frac{n}{6} \rceil + \lceil \frac{n}{6} \rceil + \lceil \frac{n}{6} \rceil = 4\lceil \frac{n}{6} \rceil$ . Hence, when  $n$  is even  $\gamma_{ECC}$  of any  $\mathcal{LC}_n$  is  $4\lceil \frac{n}{6} \rceil$ .

**Example 3.5:**

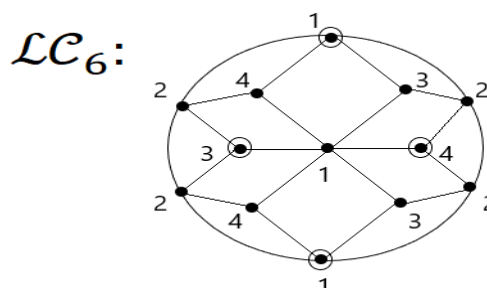


Fig.3.2

$$\text{Dominating set} = \{v_1, v_4, u_2, u_5\}.$$

$$\gamma_{ECC}(\mathcal{H}_6) = 4.$$

**Theorem 3.6:** The equitable color class domination number of a jewel graph  $\gamma_{ECC}(\mathcal{J}_n) = 2 + \lfloor \frac{n}{2} \rfloor$ .

**Proof:** Let  $\mathcal{J}_n$  be the jewel graph with  $\mathcal{V}_p(\mathcal{J}_n) = \{u_j, v_i : 1 \leq j \leq 4, 1 \leq i \leq n\}$ . Let  $u_1, u_2, u_3, u_4$  be the vertices of the cycle  $\mathcal{C}_4$  and  $v_i (1 \leq i \leq n)$  is adjacent to  $u_1$  and  $u_2$ .  $|\mathcal{V}_p(\mathcal{J}_n)| = n + 4$ .  $\chi_E$  of  $\mathcal{J}_n$  is  $2 + \lfloor \frac{n}{2} \rfloor$ . Let  $\tau: \mathcal{V}_p(\mathcal{J}_n) \rightarrow CC$  be an equitably colorable function where  $CC = \{1, 2, \dots, \lfloor \frac{n}{2} \rfloor, \lfloor \frac{n}{2} \rfloor + 1, \lfloor \frac{n}{2} \rfloor + 2\}$ .

**Case 1:**  $n$  is even

Ordain the color 1 to the nodes  $u_1$  and  $u_3$ , 2 to the nodes  $u_2$  and  $u_4$ . Now ordain the balance  $\frac{n}{2}$  colors to the balance  $n$  nodes  $v_i$  so each color will ordain 2 nodes. Now we choose dominating nodes as any one node from each color class. These nodes are dominates all other nodes and the number of dominating nodes in each color class are same and one. Hence, when  $n$  is even  $\gamma_{ECC}$  of any  $\mathcal{J}_n$  is  $2 + \lfloor \frac{n}{2} \rfloor$ .

**Case 2:**  $n$  is odd

Ordain the color 1 to the nodes  $u_1$  and  $u_3$ , 2 to the nodes  $u_2$  and  $u_4$ . Now ordain the balance  $\lfloor \frac{n}{2} \rfloor$  colors to the balance  $n$  nodes so  $\lfloor \frac{n}{2} \rfloor - 1$  number of colors will ordain 2 nodes and the balanced one color will ordain 3 nodes. Now we choose dominating nodes as any one node from each color class. These nodes are dominates all other nodes and the number of dominating nodes in each color class are same and one. Hence, when  $n$  is odd  $\gamma_{ECC}$  of any  $\mathcal{J}_n$  is  $2 + \lfloor \frac{n}{2} \rfloor$ .

**Example 3.7:**

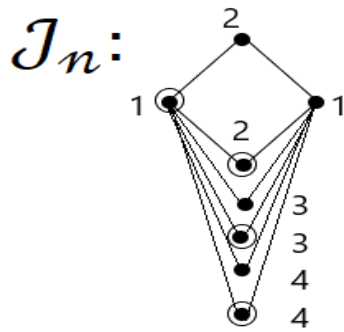


Fig.3.3

$$\text{Dominating set} = \{u_1, u_4, v_2, v_4\}$$

$$\gamma_{ECC}(\mathcal{J}_4) = 4.$$

**Theorem 3.8:** The equitable color class domination number of a jahangir graph  $\gamma_{ECC}(\mathcal{J}_{(3,n)}) = 3 \lfloor \frac{n}{2} \rfloor$ .

**Proof:** Let  $\mathcal{J}_{(3,n)}$  be the jahangir graph with  $\mathcal{V}_p(\mathcal{J}_{(3,n)}) = \{v, v_i, u_i, w_i : 1 \leq i \leq n\}$ . Let  $v$  and  $v_i (1 \leq i \leq n)$  be the nodes of the wheel  $\mathcal{W}_n$ ,  $u_i$  and  $w_i$  is placed the edge between the nodes  $v_i$  and  $v_{i+1} (1 \leq i \leq n-1)$ ,  $u_n$  and  $w_n$  is placed the edge between the nodes  $v_n$  and  $v_1$ .  $|\mathcal{V}_p(\mathcal{J}_{(3,n)})| = 3n + 1$ .  $\chi_E$  of  $\mathcal{J}_{(3,n)}$  is 3. Let  $\tau: \mathcal{V}_p(\mathcal{J}_{(3,n)}) \rightarrow CC$  be an equitably colorable function where  $CC = \{1, 2, 3\}$ . Ordain the color 1 to the nodes  $v_i$ , 2 to the nodes  $u_i$ , 3 to the nodes  $w_i$  and  $v$ .

**Case 1:**  $n$  is even

Choose dominating nodes as  $v_i, u_j, w_i (i$  is odd,  $j$  is even,  $1 \leq i, j \leq n)$ . These nodes are dominates all other nodes and the number of dominating nodes in each color class are same and  $\lfloor \frac{n}{2} \rfloor$ . Hence, when  $n$  is even  $\gamma_{ECC}$  of any  $\mathcal{J}_{(3,n)}$  is  $3 \lfloor \frac{n}{2} \rfloor$ .

**Case 2:**  $n$  is odd

Choose dominating nodes as  $v_i, u_j, w_i, u_n (i$  is odd,  $j$  is even,  $1 \leq i, j \leq n)$ . These nodes are dominates all other nodes and the number of dominating nodes in each color class are same and  $\lfloor \frac{n}{2} \rfloor$ . Hence, when  $n$  is even  $\gamma_{ECC}$  of any  $\mathcal{J}_{(3,n)}$  is  $3 \lfloor \frac{n}{2} \rfloor$ .

Example 3.9:

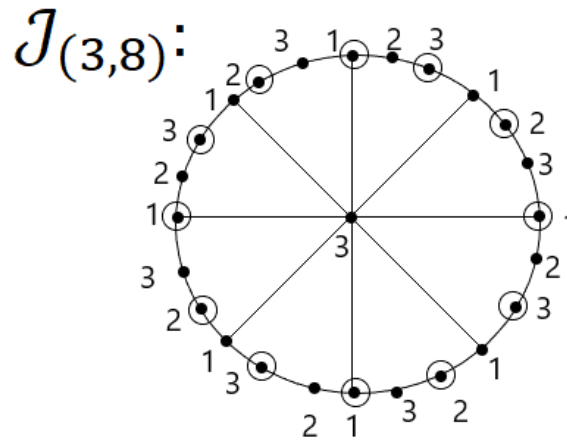


Fig.3.4

Dominating set =  $\{v_1, v_3, v_5, v_7, u_2, u_4, u_6, u_8, w_1, w_3, w_5, w_7\}$ .

$$\gamma_{ECC} (J_{(3,8)}) = 12.$$

**Theorem 3.10:** The equitable color class domination number of dumbbell graph ( $n > 2$ )  $\gamma_{ECC}(\mathcal{D}\mathcal{B}_n) = \begin{cases} 3 \lfloor \frac{2n}{9} \rfloor, & n \text{ is odd} \\ 2 \lfloor \frac{n}{3} \rfloor, & n \text{ is even} \end{cases}$

**Proof:** Let  $\mathcal{D}\mathcal{B}_n$  be the dumbbell graph with  $\mathcal{V}_p(\mathcal{D}\mathcal{B}_n) = \{u_i, v_i : 1 \leq i \leq n\}$ .  $v_i, u_i$  are the nodes of two cycles  $\mathcal{C}_n$  and joint the nodes  $v_1$  and  $u_1$ .  $|\mathcal{V}_p(\mathcal{D}\mathcal{B}_n)| = 2n$ .

**Case 1:**  $n$  is odd

If  $n$  is odd then  $\chi_E$  of  $\mathcal{D}\mathcal{B}_n$  is 3. Let  $\tau: \mathcal{V}_p(\mathcal{D}\mathcal{B}_n) \rightarrow CC$  be an equitably colorable function where  $CC = \{1, 2, 3\}$ . Ordain the color 1 to the nodes  $v_{9i+k}, u_{9i+l}$  ( $k = \{1, 6, 8\}, l = \{3, 5, 7\}$  and  $i = 0, 1, \dots, \lfloor \frac{n}{9} \rfloor - 1$ ), 2 to the nodes  $v_{9i+k}, u_{9i+l}$  ( $k = \{2, 4, 9\}, l = \{1, 6, 8\}$  and  $i = 0, 1, \dots, \lfloor \frac{n}{9} \rfloor - 1$ ), 3 to the nodes  $v_{9i+k}, u_{9i+l}$  ( $k = \{3, 5, 7\}, l = \{2, 4, 9\}$  and  $i = 0, 1, \dots, \lfloor \frac{n}{9} \rfloor - 1$ ).

**Sub case 1:**  $n = 18m-3, m \in \mathbb{N}$ .

Replacing the colors between the nodes  $v_{n-1}$  and  $v_n, u_{n-1}$  and  $u_n$ .

**Sub case 2:**  $n = 18m-1, m \in \mathbb{N}$ .

Replacing the color of the nodes  $v_n$  from 1 to 2 and  $u_n$  from 2 to 3.

**Sub case 3:**  $n = 18m+1, m \in \mathbb{N}$ .

Replacing the color of the nodes  $v_n$  from 1 to 3 and  $u_n$  from 2 to 1.

Now choose dominating nodes as  $v_{9i+k}, u_{9i+l}$  ( $k = \{1, 4, 7\}, l = \{3, 6, 9\}$  and  $i = 0, 1, \dots, \lfloor \frac{n}{9} \rfloor - 1$ ). These nodes are dominates all other nodes and the number of dominating nodes in each color class are same and  $2 \lfloor \frac{n}{9} \rfloor$ . Hence, when  $n$  is odd  $\gamma_{ECC}$  of any  $\mathcal{D}\mathcal{B}_n$  is  $3 \lfloor \frac{n}{9} \rfloor$ .

**Case 2:**  $n$  is even

If  $n$  is even then  $\chi_E$  of  $\mathcal{D}\mathcal{B}_n$  is 2. Let  $\tau: \mathcal{V}_p(\mathcal{D}\mathcal{B}_n) \rightarrow CC$  be an equitably colorable function where  $CC = \{1, 2\}$ . Ordain the color 1 to the nodes  $v_i, u_j$  ( $i$  is odd,  $j$  is even), 2 to the nodes  $v_i, u_i$  ( $i$  is even,  $j$  is odd). Now choose dominating nodes as by the following cases.

**Sub case 1:**  $n \equiv 0 \pmod{6}$

$v_1, v_4, v_7, \dots, v_{n-2}$  and  $u_3, u_6, u_9, \dots, u_n$ .

**Sub case 2:**  $n \equiv 2 \pmod{6}$

$v_1, v_4, v_7, \dots, v_{n-1}$  and  $u_1, u_3, u_6, \dots, u_{n-2}$ .

**Sub case 3:**  $n \equiv -2 \pmod{6}$

$v_1, v_4, v_7, \dots, v_{n-3}, v_{n-1}$  and  $u_1, u_3, u_6, \dots, u_{n-1}$ .

These nodes are dominates all other nodes and the number of dominating nodes in each color class are same and  $\lfloor \frac{n}{3} \rfloor$ . Hence, when  $n$  is even  $\gamma_{ECC}$  of any  $\mathcal{D}\mathcal{B}_n$  is  $2 \lfloor \frac{n}{3} \rfloor$ .

**Example 3.11:**

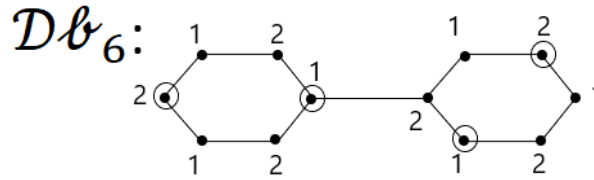


Fig.3.5

$$\text{Dominating set} = \{v_1, v_4, u_3, u_6\}$$

$$\gamma_{ECC}(\mathcal{D}b_6) = 4.$$

**Theorem 3.12:** The equitable color class domination number of subdivision path  $\mathcal{S}(\mathcal{P}_n)$  ( $n > 1$ ) is  $2\lfloor \frac{n}{3} \rfloor$ .

**Proof:**

Let  $\mathcal{S}(\mathcal{P}_n)$  be the subdivision of path with  $\mathcal{V}_p(\mathcal{S}(\mathcal{P}_n)) = \{v_i(1 \leq i \leq n), u_i(1 \leq i \leq n-1)\}$ .  $v_i$  are the nodes of path  $\mathcal{P}_n$  and  $u_i$  are the nodes insert between  $v_i$  and  $v_{i+1}$ .  $|\mathcal{V}_p(\mathcal{S}(\mathcal{P}_n))| = 2n-1$ .  $\chi_E$  of  $\mathcal{S}(\mathcal{P}_n)$  is 2. Let  $\tau: \mathcal{V}_p(\mathcal{S}(\mathcal{P}_n)) \rightarrow CC$  be an equitably colorable function where  $CC = \{1,2\}$ . Ordain the color 1 to the  $v_i$  nodes and 2 to the  $u_i$  nodes. Now we choose dominating nodes as  $v_{3i+3}$  and  $u_{3i+1}$  ( $i = 0, 1, \dots, \lfloor \frac{n}{3} \rfloor - 1$ ). Suppose  $3i + 3$  or  $3i + 1 > n$  for  $i = \lfloor \frac{n}{3} \rfloor - 1$  then replace the dominating node from  $v_{3i+3}$  and  $u_{3i+1}$  to  $v_n$  and  $u_{n-1}$ . These nodes are dominates all other nodes and the number of dominating nodes in each color class are same and  $\lfloor \frac{n}{3} \rfloor$ . Hence,  $\gamma_{ECC}$  of any  $\mathcal{S}(\mathcal{P}_n)$  is  $2\lfloor \frac{n}{3} \rfloor$ .

**Example 3.13:**

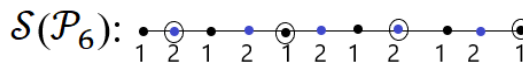


Fig 3.6

$$\text{Dominating set} = \{v_3, v_6, u_1, u_4\}.$$

$$\gamma_{ECC}(\mathcal{S}(\mathcal{P}_6)) = 4.$$

**Theorem 3.14:** The equitable color class domination number of splitting graph of path  $\mathcal{S}'(\mathcal{P}_n)$  ( $n > 1$ ) is  $2\lfloor \frac{n}{4} \rfloor$ .

**Proof:**

Let  $\mathcal{S}'(\mathcal{P}_n)$  be the splitting graph of path with  $\mathcal{V}_p(\mathcal{S}'(\mathcal{P}_n)) = \{v_i, u_i : 1 \leq i \leq n\}$ .  $v_i$  are the nodes of path  $\mathcal{P}_n$  and  $u_i$  are the newly added nodes.  $|\mathcal{V}_p(\mathcal{S}'(\mathcal{P}_n))| = 2n$ .  $\chi_E$  of  $\mathcal{S}'(\mathcal{P}_n)$  is 2. Let  $\tau: \mathcal{V}_p(\mathcal{S}'(\mathcal{P}_n)) \rightarrow CC$  be an equitably colorable function where  $CC = \{1,2\}$ . Ordain the color 1 to the nodes  $v_i, u_i$  ( $i$  is odd) and 2 to the nodes  $v_i, u_i$  ( $i$  is even). Now we choose dominating nodes as  $v_{4i+2}$  and  $v_{4i+3}$  ( $i = 0, 1, \dots, \lfloor \frac{n}{4} \rfloor - 1$ ). Suppose  $4i + 3 > n$  for  $i = \lfloor \frac{n}{4} \rfloor - 1$  then replace the dominating node from  $v_{4i+3}$  to  $v_{n-1}$ . Suppose  $4i + 2$  and  $4i + 3 > n$  for  $i = \lfloor \frac{n}{4} \rfloor - 1$  then replace the dominating nodes from  $v_{4i+2}$  and  $v_{4i+3}$  to  $v_{n-1}$  and  $v_n$ . These nodes are dominates all other nodes and the number of dominating nodes in each color class are same and  $\lfloor \frac{n}{4} \rfloor$ . Hence,  $\gamma_{ECC}$  of any  $\mathcal{S}'(\mathcal{P}_n)$  is  $2\lfloor \frac{n}{4} \rfloor$ .

**Example 3.15:**

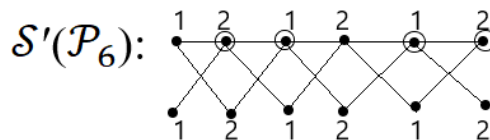


Fig 3.7

$$\text{Dominating set} = \{v_2, v_3, v_5, v_6\}.$$

$$\gamma_{ECC}(\mathcal{S}'(\mathcal{P}_6)) = 4.$$

**Corollary 3.16:** The equitable color class domination number of shadow graph of path  $\mathcal{D}_2(\mathcal{P}_n)$  ( $n > 1$ ) is  $2\lfloor \frac{n}{3} \rfloor$ .

**Proof:**

Let  $\mathcal{D}_2(\mathcal{P}_n)$  be the shadow graph of path with  $\mathcal{V}_p(\mathcal{D}_2(\mathcal{P}_n)) = \{v_i(1 \leq i \leq n), u_i(1 \leq i \leq n-1)\}$ .  $v_i$  are the nodes of path  $\mathcal{P}_n$  and  $u_i$  are the nodes of copy of path  $\mathcal{P}_n$ . Joining each node  $v_i$  to the neighbours of the corresponding node  $u_i$ .  $|\mathcal{V}_p(\mathcal{D}_2(\mathcal{P}_n))| = 2n$ .  $\chi_E$  of



$\mathcal{D}_2(\mathcal{P}_n)$  is 2. Let  $\tau: \mathcal{V}_p(\mathcal{D}_2(\mathcal{P}_n)) \rightarrow CC$  be an equitably colorable function where  $CC = \{1,2\}$ . Ordain the color to the nodes and choosing dominating node is follows from the theorem 3.14.

**Example 3.17:**

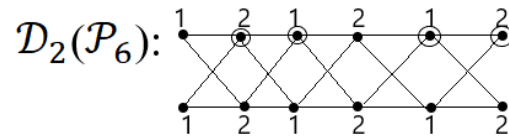


Fig 3.8

Dominating set =  $\{v_2, v_3, v_5, v_6\}$ .

$$\gamma_{ECC}(\mathcal{D}_2(\mathcal{P}_6)) = 4.$$

#### REFERENCES:

- [1] S. Arumugam and Jay Bagga, "Theoretical Computer Science and Discrete Mathematics", Springer Science and Business Media LLC, 2017
- [2] S. Athisayanathan, R. Ponraj and M.K. Karthik Chidambaram, "Group A cordial labeling of Graphs", International Journal of Applied Mathematical Sciences, Vol 10, No.1(2017), pp 1-11 (ISSN 0973-0176).
- [3] M Changat and S Das, "Algorithms and Discrete Applied Mathematics" Springer Science and Business Media LLC, 2016
- [4] G. Chartrand, Lesniak, Graphs and Digraphs, fourth ed., CRC Press, Boca Raton, 2005.
- [5] A.Esakkimuthu and S.Mari Selvam, Equitable color class domination number of some cycle related graphs, International Journal of Research and Analytical Reviews (IJRAR), Volume 9, Issue 4, December 2022, (E-ISSN 2348-1269, P- ISSN 2349-5138).
- [6] T. W. Haynes, S. T. Hedetniemi and P. J. Slater, "Domination in Graphs-Advanced Topics", New York: Dekker, 1998.
- [7] T. W. Haynes, S. T. Hedetniemi and P. J. Slater, "Fundamentals of domination in Graphs", New York: Dekker, 1998.
- [8] S. T. Hedetniemi and R. C. Laskar, "Topics on domination", North Holland 1991.
- [9] Karthik Chidambaram, M. K, S. Athisayanathan and R. Ponraj , Group  $\{1,-1,i,-i\}$  cordial labeling of Some Splitting graphs, Global Journal of Engineering Science and Researches, Vol.4(11), November 2017, pp 83-88 (ISSN: 2348-8034, Impact factor: 4.022).
- [10] P. C. Lisna, M. S. Sunitha. "b-Chromatic sum of a graph" Discrete Mathematics, Algorithms and Applications, 2016
- [11] K. W. Lih, "The equitable coloring of graphs", Handbook of combinatorial optimization, 1998, (pp. 2015-2038).
- [12] O. Ore, Theory of Graphs, Amer. Math. Soc. Colloquium Pub., Amer. Math. Soc., Providence. Rhode Island, 38(1962), 206.
- [13] Pardalos, Panos M., Ding-Zhu Du, and Ronald L. Graham, eds. Handbook of combinatorial optimization. New York: Springer, 2013.
- [14] Hu, Y.F.. "Algorithms for scheduling with applications to parallel computing", Advances in Engineering Software, 1997
- [15] A. Sugumaran, E. Jayachandran, "Domination number of some graphs", International Journal of Scientific Development and Research (IJS DR), ISSN: 2455-2631 Volume 3, Issue 11, November 2018.
- [16] A. Vijayalekshmi, A. E. Prabha. "Introduction of color class dominating sets in graphs", Malaya Journal of Matematik, 2020
- [17] B. Zhou, "On the maximum number of dominating classes in graph coloring", Open Journal of Discrete Mathematics, 2016 Mar 31, 6(2):70-3.