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A Comparative Study on the Analysis of Multi-Component Signals using WVD and its Variants

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***Abstract*— Time Frequency distributions (TFD), which indicate the energy content of a signal as a function of both time and frequency, are powerful tools for time-varying signal analysis. The lack of a single distribution that is "best " for all applications has resulted in a proliferation of TFDs, each corresponding to a different, fixed mapping from signals to the time-frequency plane. A major drawback of all fixed mappings is that, for each mapping, the resulting time frequency representation is satisfactory only for a limited class of signals. In this paper, we compare the different TFDs of Cohen's class-Wigner-Ville, Choi-Williams, Zhao-Atlas-Marks and Born-Jordan distributions by applying them to multi-component signals.**

***Keywords*— Time-Frequency Distribution, Wigner-Ville, Choi-Williams, Zhao-Atlas-Marks, Born-Jordan, Cohen's class, Quadratic TFD, Kernel, Ambiguity Function.**

I. INTRODUCTION

Non-stationarity brings new challenges for signal processing area. Natural direction to calculate only the spectrum of the signal can be insufficient, providing only general information with loss of time-varying nature of analysed phenomena. The violation of main assumption of spectral analysis, the stationarity, can be solved by introducing the joint time-frequency domain ([1], [2], [3]).

Time –frequency signal analysis & processing (TFSAP) concerns the analysis & processing of signals with time varying frequency content. Such signals are best represented by a time frequency distribution (TFD), which shows how the energy of the signal is distributed over the two-dimensional time-frequency space. By processing the signals, we get to understand the features produced by concentration of signal energy in two dimensions-time and frequency, whereas signals represented by using either time or frequency representation doesn't give us the information of the other dimension. We require TFDs for the analysis of non-stationary signals (signals having time-varying frequency content) like sinusoidal FM, linear FM, musical performance, transients etc. These non-stationary signals occur in telecommunications, radar, sonar, vibration analysis, speech processing and medical diagnosis. By using time-frequency representations (TFR), the interpretation of signals is easier. It helps in detecting multiple signals, and also their nature. Also TFD's are useful in extracting signal from noise. The frequency spectrum of the signal doesn't provide an accurate method of analysing the signal. Through the use of TFDs, the analysis of the signal is greatly simplified [4], [5].

TFRs have been classified as linear, bilinear (quadratic), adaptive and averaged TFRs. Linear TFRs that satisfies the linearity superposition principle includes Short-time Fourier transform (STFT), Gabor Transform and the Wavelet transform. The standard method for studying time-varying signals is the STFT method that is based on the assumption that for a short-time, signal can be considered stationary. The spectrogram utilizes a short-time window whose length is chosen so that over the length of the window, the signal is stationary. Then, the Fourier transform of this windowed signal is calculated to obtain the energy distribution along the frequency direction at the time corresponding to the centre of the window. The crucial drawback of this method is that the length of the window is related to the frequency resolution. Increasing the window length leads to improving frequency resolution but it means that the non-stationarity occurring during this interval will be smeared in time and frequency. This inherent relationship between time and frequency resolution becomes more important when one is dealing with signals whose frequency content is changing rapidly. A time-frequency characterization that would overcome above drawback led to the development of non-parametric, bilinear transformations [3].

A Quadratic TFR (QTFR) is one that satisfies the quadratic superposition principle. For QTFRs, the windowing techniques are not required because the objective is to form energy distributions so that the signal energy can be distributed in the TF plane. QTFRs often overcome the TF resolution problem that limits the linear TFRs. Some important QTFRs are Wigner-Ville Distribution (WVD), Choi-Williams Distribution (CWD), Zhao-Atlas-Marks Distribution (ZAMD), and Born-Jordan Distribution (BJD). All these TFD's are members of Cohen's bilinear class.

The paper is organized as follows. In the next section we define the class of all quadratic time-frequency distributions covariant to time-shifts and frequency shifts; and each distribution is described in brief. In Section III we analyse the performance of these distributions by applying various simulated multi-component signals in MATLAB. The conclusion is given in Section IV.

II. Cohen's Generalization

Cohen defined a general class of bilinear transformation (TFC) introducing kernel function, $\Phi_{\omega t}(\theta,\tau)$. The significance of Cohen's work is to reduce the problem of designing time-dependent spectrum to the selection of the kernel function. Each distribution in Cohen's class can be interpreted as the two-dimensional Fourier transform of a weighted version of the symmetric ambiguity function (AF) of the signal to be analysed. That is, if *TFC (t,ω)* is a bilinear TFD, then

$$TFC(t,\omega)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(\theta,\tau)\Phi(\theta,\tau)e^{-j\theta t-j\omega t}\,d\theta d\tau \qquad (1)$$

where

$$A(\theta,\tau)=\int_{-\infty}^{+\infty} x*(u-\frac{\tau}{2})x(u+\frac{\tau}{2})e^{j\theta u}\,du \qquad (2)$$

where t-time, ω-angular frequency, τ-time lag, θ-angular frequency lag, and u- additional integral time variable. The weighting function Φ(θ,τ) is called the kernel of the distribution or parameterization function. The properties of a particular bilinear TFD are completely determined by its kernel function. Since the AF is a bilinear function of the signal, it exhibits cross terms(c-t) which are undesirable, which when allowed to pass into the TFD, can reduce auto component resolution, obscure the true signal features, and make interpretation of the distribution difficult. Therefore the kernel is often selected to weight the AF such that the auto terms (a-t), which are centred at the origin of the (θ,τ) ambiguity plane, are passed, while the cross terms, which are located away from the origin are suppressed[4], [5].

For a given class of signals, we say that a TFD offers good performance if, for each signal in the class, it achieves a high degree of both cross-component suppression and auto component concentration, and provides an accurate representation of the time-frequency content of the signal. An important theoretical and practical goal of time-frequency analysis is to define a TFD that attains good performance for a large class of different signals. The most prominent influence of cross-terms is then observed in case of Wigner-Ville distribution, where the kernel Φ(θ,τ)=1. Applying Gaussian kernel (CWD), "sinc" kernel (BJD) or cone-shaped kernel (ZAMD) brings smoothing effect on the equation level.

Next we briefly describe each time-frequency distribution. Detailed descriptions, properties and derivations of the TFDs can be found from [1]-[6].

A. Wigner-Ville Distribution

Wigner –Ville Distribution (WVD) is the most important and simplest of Cohen's class of Bilinear TFRs. It may be defined as

$$WVD(t,\omega)=\int_{-\infty}^{+\infty} x(t+\frac{\tau}{2})x*(t-\frac{\tau}{2})e^{-j\omega\tau}\,d\tau \qquad (3)$$

where x (t) is a possibly complex-valued analytical signal. Due to the quadratic nature of the WVD, its discrete version may be affected by spectral aliasing, in particular if the signal x is real valued and sample at the Nyquist rate. A solution to this problem consists in using the analytic signal by taking the Hilbert transform of the signal. As its bandwidth is half the one of the real signal, the aliasing will not take place in the useful spectral domain of this signal. Also since the spectral domain is divided by two, the number of components in the time-frequency plane is also divided by two. Consequently, the number of interference terms decreases significantly. WVD has a number of mathematical properties considered desirable in a TFR. In particular, the WVD is always real-valued, it preserves time, and frequency shifts and satisfies the marginal properties. That is if the WVD is summed over frequency at a fixed time, a value equal to the energy at that point is obtained [4], [5], [6], [7].

Despite the desirable properties of the Wigner distribution, its use in practical applications has been limited by one un-desirable property; namely, the presence of cross terms. The Wigner-Ville distribution of the sum of two signals x(t) +y(t)

$$WVD_{x+y}(t,\omega)=WVD_x(t,\omega)+2\mathrm{Re}[WVD_{x,y}(t,\omega)] +WVD(t,\omega) \qquad (4)$$

has a "cross-term" 2Re[$WVD_{x,y}$] in addition to the two auto-components, where the cross-Wigner distribution is defined as

$$WVD_{x,y}(t,\omega)=\int_{-\infty}^{+\infty} x(t+\frac{\tau}{2})y*(t-\frac{\tau}{2})e^{-j\omega\tau}\,d\tau \qquad (5)$$

Cross-terms lie between two auto-terms and are oscillatory, with their frequencies increasing with increasing distance in time-frequency between the two components. The cross-terms can have a peak value as high as twice that of the auto-terms. These interference terms are troublesome since they may overlap with auto-terms and thus make it difficult to visually interpret the WVD image. One way to attenuate these interferences is to smooth the distribution in time and in frequency, according to their structure. But the consequence of this is a decrease of the time and frequency resolutions, and more generally a loss of theoretical properties. Hence these terms must be present or the good properties of the WVD (marginal properties, instantaneous frequency and group delay, localization, unitarity etc) cannot be satisfied. Actually there is a trade-off between the quantity of interferences and the number of good properties [6].

B. Choi-Williams Distribution(Exponential)

It was first proposed by Hyung-III Choi and William J. Williams in 1989. This distribution function adopts a kernel to suppress the cross-term interference of WVD. The CWD is a shift-invariant transform. It is a smoothed version of the Wigner-Ville distribution through a kernel function defined by

$$\Phi(\theta,\tau)=\frac{|\overline{\pi\sigma}}{|\tau|}e^{-\pi^2\sigma t^2/\tau^2} \qquad (6)$$

The Choi-Williams distribution is then defined as

$$CWD(t,\omega)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\sqrt{\pi\sigma}}{|\tau|}e^{\frac{-\pi^2\sigma(t-u)^2}{\tau^2}} z(u+\frac{\tau}{2})z*(u-\frac{\tau}{2}) e^{-j\omega\tau}\,du d\tau \qquad (7)$$

where z (t) is a complex-valued analytical signal. The smoothing of the distribution is controlled by the constant σ which is real and positive. It is a scaling factor to control its attenuation rate and σ < 10 is often preferred. As σ→∞, the CWD will simply converge to the WVD, as the kernel goes to 1. Inversely, the smaller σ, the better is the reduction of the interferences [8], [9]. The "cross"-shape of the kernel (Fig. 1) of CWD implies that the efficiency of this distribution strongly depends on the nature of the analysed signal.

The CWD has a coarser time-frequency resolution than the WVD because the CWD also blurs the auto-terms when the CWD reduces the cross-terms. Because the exponential kernel function doesn't reduce the values of the ambiguity function on the horizontal axis or the vertical axis, the CWD preserves the cross-terms on the horizontal axis and the vertical axis. In other words, the CWD doesn't suppress the cross-terms that two auto-terms with the same time centre or frequency centre generate whereas it suppresses the cross-term interference between two signal components that have a large difference in central time or central frequency. Therefore for large values of the signal length, this requires a long computation time and more memory.

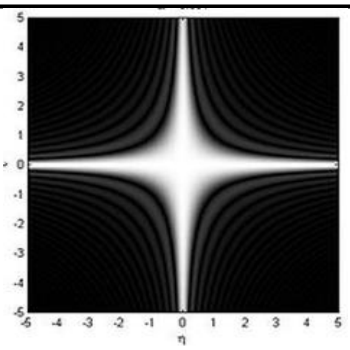


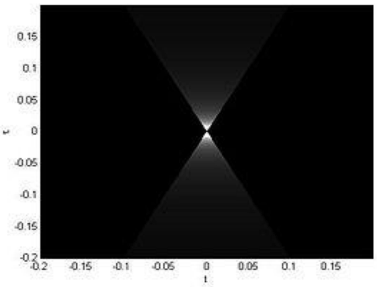
Fig. 1 Kernel of Choi-Williams Distribution

C. Zhao-Atlas-Marks Distribution (Windowed Sinc)

It was first proposed by Yunxin Zhao, Lee. E. Atlas and Robert J. Marks in 1990. Its kernel function is defined by

$$\Phi(\theta,\tau)=w(\tau)\,rect\,\frac{t}{2\tau/a} \tag{8}$$

Where 'a' is a constant which is real and positive and w (τ) is a window function. Since its kernel function in θ, τ domain looks like two cones; it is named as cone-shaped kernel distribution (Fig. 2). ZAMD is defined as



$$ZAMD(t,\omega)=\int_{-\infty}^{\infty}\int_{t-|\frac{\tau}{a}|}^{t+|\frac{\tau}{a}|} w(\tau)\,z(u+\frac{\tau}{2})\,z*(u-\frac{\tau}{2})\,e^{-j\omega\tau}\,du\,d\tau \tag{9}$$

Here 'a' is often taken as 2. The advantage of this special kernel function is that it can completely remove the cross-term between two components that have same centre frequency, but on other hand, it cannot reduce cross-terms that two auto-terms with the same time centre generate [10]. The ZAMD produces a good resolution in time and frequency domains in the case of constant frequency. The ZAMD method reduces the interference resulting from the cross-terms present in multi-component signals. It is useful in resolving close spectral peaks and capturing non-stationary and multi-component signals. For signals with changing frequency, the ZAM distribution displays ghosting around the signal. The ghosting increases when the rate of frequency change increases. So ZAM is not an ideal transform.

Fig. 2 Kernel of Zhao-Atlas-Marks Distribution

D. Born-Jordan Distribution(Sinc)

The Born-Jordan distribution is a shift-invariant, kernel smoothed Wigner-Ville distribution. The kernel function is defined by

$$\Phi(\theta,\tau)=\frac{1}{|2\alpha\tau|}\,rect\,\frac{t}{2\alpha\tau} \tag{10}$$

which defines the Born-Jordan distribution as:

$$BJD(t,\omega)=\int_{-\infty}^{\infty}\int_{t-|\alpha\tau|}^{t+|\alpha\tau|}\frac{1}{2\alpha\tau}\,z(u+\frac{\tau}{2})\,z*(u-\frac{\tau}{2})\,e^{-j\omega\tau}\,du\,d\tau \tag{11}$$

It has been observed that the BJ distribution is a special case of ZAM with w(τ)=a/|2τ| and a=1/α. The BJD performs well for signals with constant frequency [9], [11], [12]. Fig. 3 shows the kernel of BJ Distribution in the ambiguity plane.

Since the entire distributions mentioned above try to reduce the interferences, these are called Reduced Interference distributions.

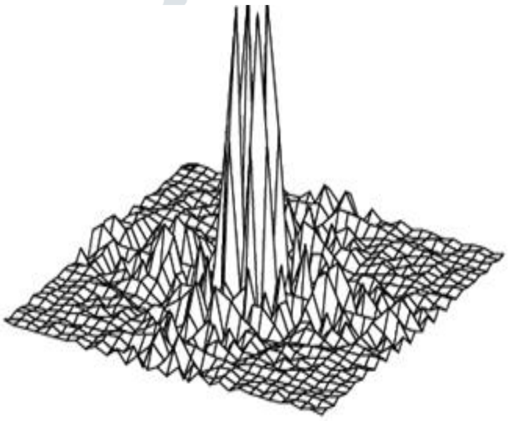


Fig. 3 Kernel of Born-Jordan Distribution in ambiguity plane

III. Simulation Results

In this section the different distributions for a set of four multi-component signals is analysed by using their MATLAB simulations.

A. Multiple Linear FM

First two linear frequency modulated signals (lfm), also called chirp signals is applied. Let one be an up lfm and the other a down lfm which overlap each other. Noise is added to the signals.

Fig. 4 shows the different Time-Frequency Distributions of two lfm's. By applying a noise level of 0.5 it can be seen that WVD is clear but has lots of cross-terms. The resolution of CWD is better than ZAMD and BJD. When applying a noise level of 1.2 the WVD is not at all visible (Fig. 5). It is hard to distinguish the signal when the background noise is increased. Comparing with BJD, CWD provides better resolution whereas ZAMD is worse.

B. Multiple Up-Lfm

Here we apply two up linear FM signals whose starting frequencies and ending frequencies are different. For a noise level of 0.5, CWD provides nearly perfectly resolved signals in time and frequency (see in Fig. 6). A strong smeared cross-term is visible with the CWD. BJD is the second best transform whereas the cross-terms are greatly reduced for ZAMD. But it displays a grainy target with poor frequency resolution. As noise level increases, the signal becomes totally invisible using WVD, while it is moderately blurred with ZAMD. BJD provides better resolution and is less grainy compared to CWD.

C. Sinusoidal signal combine with Up-Lfm

This signal contains a sinusoidal signal combined with a chirp signal. The transforms are shown in Fig. 7.

The CWD and BJD provide a nearly perfectly resolved target signal in time and frequency. For CWD, the output is smeared, whereas ZAMD provides less cross-term interference, but poor resolution. WVD has significant cross-term interference.

D. Two-Tonal Signals

The final signal consists of two overlapping sinusoidal signals; overlapping in time. Both sinusoidal signals are having different frequencies. From Fig. 8 it is clear that as the noise margin increases, WVD becomes totally invisible and the signal is difficult to identify. Even though CWD provides better resolution, it contains more cross terms than BJD and ZAMD. The noisy background in ZAMD makes it slightly difficult to identify the signal. BJD is comparatively better than others

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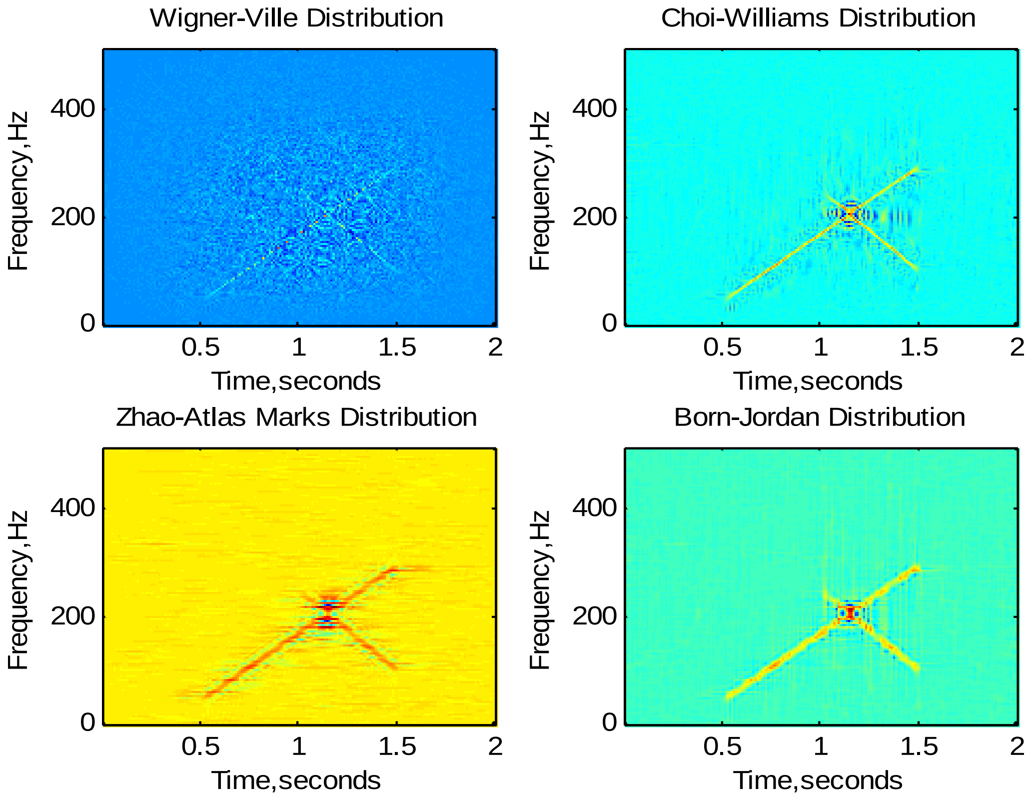

Fig. 4 Different Time-Frequency Distributions for multiple LFM with NL=0.5

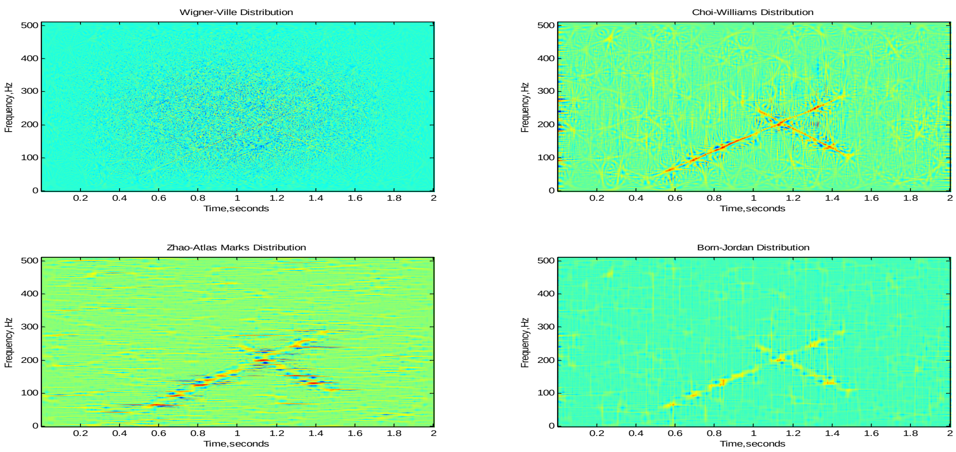

Fig. 5 Different Time-Frequency Distributions of multiple LFM with NL=1.2

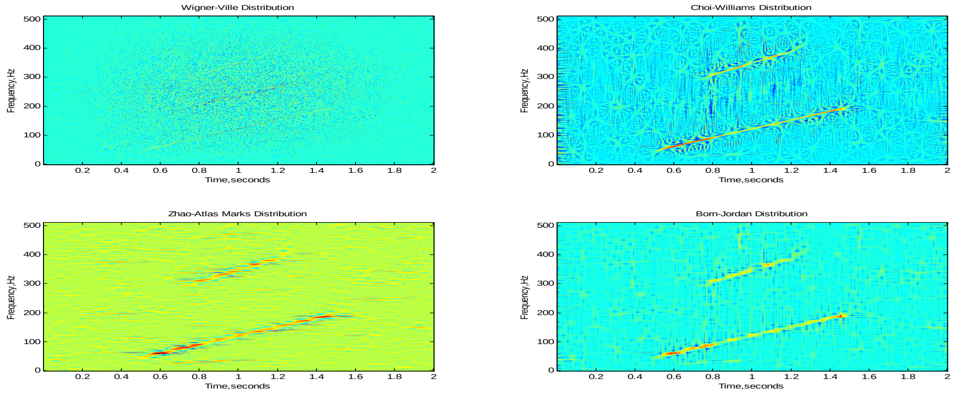

Fig. 6 Different Time-Frequency Distributions of two up LFM with NL=1.2

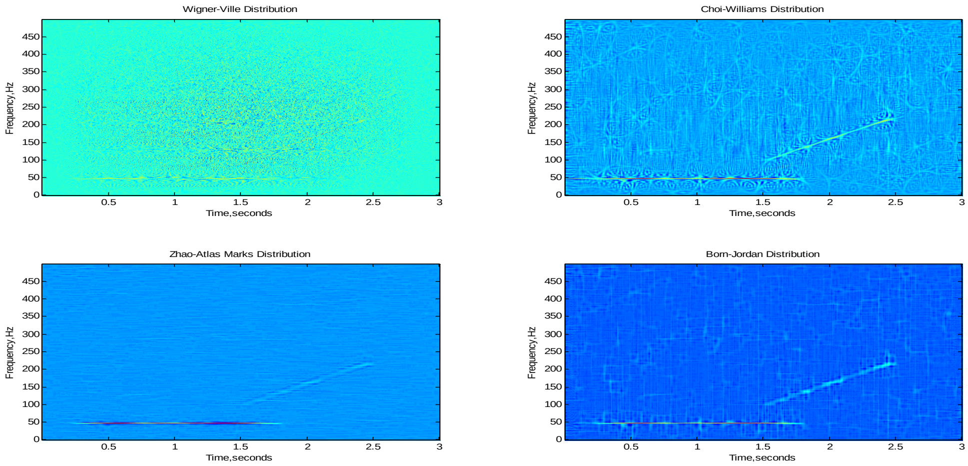

Fig. 7 Different Time-Frequency Distributions of a Sine combined with a chirp for NL=1.2

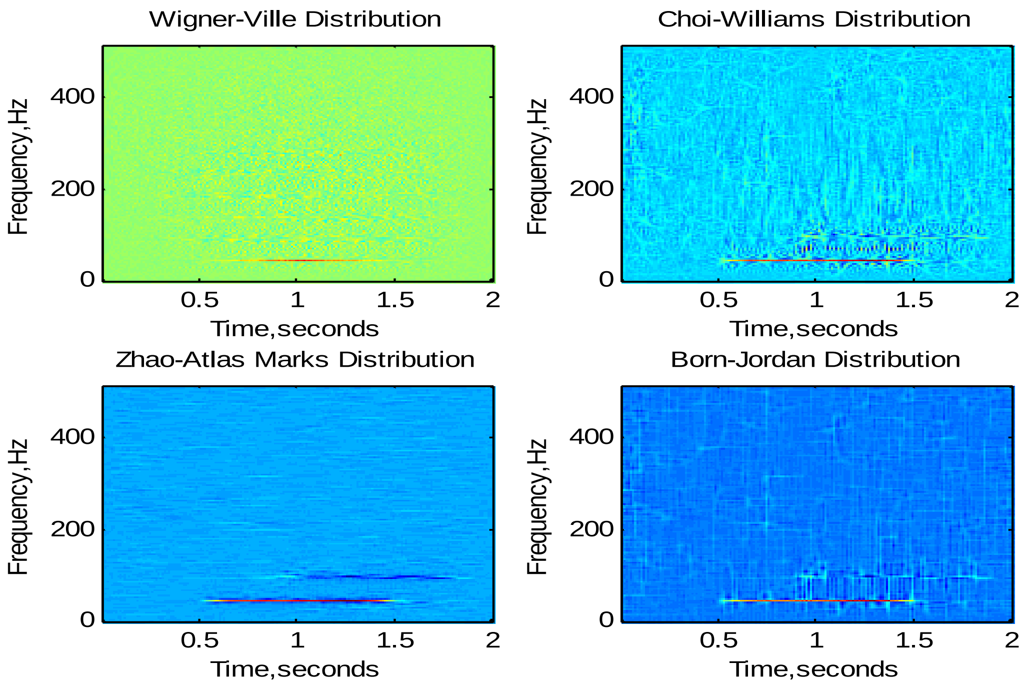

Fig. 8 Different Time-frequency Distributions for a two-tonal signal for NL=1.2

IV. CONCLUSION

In this paper, we have tried to compare the Cohen's class TFDs for a set of multi-component signals and we have found that, even though WVD provides better resolution, as the noise margin increases, the cross-terms of WVD makes the signal impossible to be identified, whereas its variants like CWD, ZAMD and BJD reduces the cross-components to a minimum.

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