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SEQUENTIAL PROCEDURES FOR THE SIMULTANEOUS ESTIMATION OF PARAMETERS

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Abstract: A class of sequential procedures for simultaneous estimation of the parameter (s) of an absolutely continuous population is developed in the presence of an unknown nuisance parameter. The proposed class is proved to be 'asymptotically efficient and consistent'.

Index Words: Asymptotic risk efficiency, Stopping time

INTRODUCTION

In some cases, it is preferable to use a preassigned coverage probability to contain the population's parameters within predetermined bounds. These circumstances give rise to issues with population parameter estimation. A sequential method for creating a fixed-size semi-circular region for the simultaneous estimation of the mean and variance of a univariate normal population was created by Mukhopadhyay[7] in 1981. He proved the sequential procedure's asymptotic effectiveness and consistency. In order to simultaneously estimate the mean vector and an unknown scalar multiplier of the covariance matrix of a multivariate normal population, Chaturvedi (1985) [3]extended the findings for the multivariate case. He adopted a fixed sample size approach because it does not address this estimation issue. his estimation is not addressed by the technique. According to ChowRobbin's definition of "asymptotically efficient and consistent,"[5] the proposed approach was shown to be true. Under certain continuous probability models, a distributional relationship between the estimator (s) of the parameter(s) of interest and those of the nuisance parameter(s) involved therein is seen. Chaturvedi, Pandey S K, and Gupta M developed a class of sequential methods [4](1991). This class for the concurrent estimate of the parameter(s) of an absolute continuous population is demonstrated to be "asymptotically efficient and consistent" in the current study. The suggested class may be used to solve multiple simultaneous estimating issues using a variety of cases.

1.2 THE SET-UP OF THE ESTIMATION PROBLEM

Let $\{\mathbf{X}_i\}, i = 1, \ldots$ be a sequence of iid rv's from a t-variate $(t \geqslant 1)$absolutely continuous)population $f(\mathbf{x}; \boldsymbol{\theta}, \Psi)$, where $\theta$is a $t \times 1$ vector of unknown parameter(s) of interest and Ψ is a scalar nuisance parameter assumed to be unknown. Denoting by R^t and R^+, respectively, he t-dimensional Euclidean space and the positive-half of the real line, let $\left(\theta', \Psi\right)' \in R^t \times R^+$. Given a random sample $\mathbf{X}_1, \ldots, \mathbf{X}_n$ of size $n(\geqslant t+1)$, let $\hat{\theta}_n = \hat{\theta}(\mathbf{X}_1, \ldots, \mathbf{X}_n)$ and $\hat{\Psi}_n = \hat{\Psi}(\mathbf{X}_1, \ldots, \mathbf{X}_n)$ be the estimators of θ and Ψ, respectively. [4]We, further, make the following assumptions:

(A,): There exist a known $t \times t$ positive definite matrix Q, a number $\delta \in (0,1]$ and an inpeger ≥ 1) such that

$$n[\psi^{-1}(\theta_n - \theta)' Q(\theta_n - \theta)]^{\delta} \sim \chi^2_{(r)}$$

where $\chi^2_{(\mathrm{r})}$ denotes a chi-square variate with r degrees of freedom.[4]

(A_2) : For all $n \geqslant t+1, \hat{\theta}_n$ and $\hat{\Psi}_n$ are stochastically independent.

(A_3) : There exist integers $s(\geqslant 1)$ such that for all $n \geqslant s+1$.

$r(n-s)\hat{\Psi}_n/\Psi = \sum_{j=1}^{n-s} Z_j^{(r)}$

where $Z_j^{(r)}$'s are iid rv's with $Z_j^{(r)} \sim \chi^2_{(r)}$.[4]

(A_4): $\hat{\Psi}_n$ is a consistent estimator of ψ.

For preassigned d > 0 and 0 < α < 1

to construct a confidence region R_n of maximum width 2d such that P($\xi \in R_n$) ≥ α . For & $\xi_n = \left(\underline{\hat{\theta}_n}', \psi_n\right)'$,

$$Q^* = \begin{pmatrix} Q & 0 \\ 0 & 1 \end{pmatrix}$$

we define

$$R_n = [\underline{Z} = \left(\underline{a}', b\right)' : \underline{a} = (a_1, \cdots\cdots, a_t)', b > 0 \text{ and} \left(\hat{\xi}_n - \underline{Z}\right)' Q^* \left(\hat{\xi}_n - \underline{Z}\right) \leq d^2 \Big] \quad \ldots\ldots\ldots\ldots\ldots(1.1)$$

Denoting by I_t ,a txt Identity matrix, let us define a$(t+1)X(t+1)$ positive definite matrix

$$\Sigma = \begin{pmatrix} I_t & 0 \\ 0 & \dfrac{2\psi^2}{q} \end{pmatrix}$$

Furthermore, let λ = maximum eigen value of Σ. It can be verified that the region

$$R_n^* = \left[\underline{Z} = \left(\underline{a}', b\right)', \underline{a} = (a_1, \ldots, a_t)', b > 0 \text{ and } \lambda \left(\hat{\xi}_n - \underline{Z}\right)' \Sigma^{-1} Q^* \left(\hat{\xi}_n - \underline{Z}\right) \leq d^2\right] \quad \ldots\ldots\ldots\ldots..(1.2)$$

is contained in R_n. Utilizing (A_1) and (A_3), we obtain from [1.1]and [1.2] that

$$P(\xi \in R_n) \geq P(\xi \in R_n^*)$$
$$= P\left[\lambda\left(\hat{\xi}_n - \underline{Z}\right)' \Sigma^{-1} Q^* \left(\hat{\xi}_n - \underline{Z}\right) \leq d^2\right].$$

$$= P\left[\psi^{-1}\left(\underline{\hat{\theta}}_n - \underline{\theta}\right) \cdot Q\left(\underline{\theta}_n - \underline{\theta}\right) + \frac{q}{2\psi^2}\left(\hat{\psi}_n - \psi\right)^2 \leq \frac{d^2}{\lambda}\right]$$

$$= P\left[n^{-1}x^2_{(r)} + (n-s)^{-1}x^2_{(1)} \leq \lambda^{-1}d^2\right]$$
$$\geq P\left[(n-s)^{-1}x^2_{(r)} + (n-s)^{-1}x^2_{(1)} \leq \lambda^{-1}d^2\right]$$
$$\geq P\left[x^2_{(r+1)} \leq \lambda^{-1}(n-s)d^2\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.3$$

let ' a ' be any constant such that

$P\left[x^2_{(r+1)} \leq a\right] = \alpha$…………………(1.4)

from (1.3) and (1.4) it follows that for known λ, in order to achieve $P(\xi \in R_n) \geq \alpha$, the minimum value of sample size required $n \geq n^{**}$

$$s+\frac{a\lambda}{d^2}$$

but, in the ignorance of λ, no fixed size procedu achieves the goals of "preassigned with and coverage probability" simultaneously for all values of λ. In such a situation, we utilize a class of sequential procedures [4] which is discussed in the following section.

THE CLASS C_R^* OF SEQUENTIAL PROC EDUPES

Let $\hat{\lambda}_n$ be the maximum eigen value of

$$\Sigma_n=\begin{pmatrix}\hat{\psi}_n I_t & 0\\ 0 & \frac{2\psi_n^2}{q}\end{pmatrix}.$$

The stopping time $N\equiv N(d)$ is the smallest positive integer $n\geq m\geq \max\cdot\{s+1,t+1\}$ such that

$$n\geq s+\left(\frac{a_n\hat{\lambda}_n}{d^2}\right)\ldots\ldots..(2.1)$$

where $\{a_n\}, n=1,2,\ldots\ldots$ is a sequence of positive numbers converging to ' a '. We construct the region R_N for ξ .The following theorem establishes the results that the sequential procedure (2.1) is 'asymptotically efficient and consistent' in view of Chow and Robbins (1965).

Theorem : N is a well − defined stopping rule [4]…………..(2.2)

$$\lim_{d\to 0}\left(\frac{N}{n^{**}}\right)=1\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2.3)$$

$$\lim_{d\to 0}E\left(\frac{N}{n^{**}}\right)=1\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2.4)$$

$$\lim_{d\to 0}P(\xi\in R_N)\geq\alpha\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2.5)$$

Proof : Without any loss of generality, let $\hat{\lambda}_n=\hat{\psi}_n$. Denoting $v_n=\frac{q(n-s)\hat{\psi}_n}{\psi}$ and using (A_3). it follows from the definition of N that

$$\begin{aligned}P(N\geq n)\ &\leq P\left[n\leq s+\left(\frac{a_n\hat{\psi}_n}{d^2}\right)\right]\\ &=P[2q(n-s)\}^{-\frac{1}{2}}\{v_n-\mathrm{q}(\mathrm{n}-\mathrm{s})\}]\\ &\leq\left[\left\{\left(\frac{q}{2}\right)(n-s)\right\}^{\frac{1}{2}}\left\{\frac{(n-s)a}{(n^{**}-\mathrm{s})a_n}-1\right\}\right]\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (2.6)\end{aligned}$$

Since $\left\{2q^{(n-s)}\right\}^{-\frac{1}{2}}\{v_n-q(n-\mathrm{s})\}\xrightarrow{L}N(0,1)$

$n\to\infty$ and from Zacks$(1971,\mathrm{p}.561)$, $1-\Phi(x)\approx x^{-1}\phi(x)$ as $x\to\infty$,[8] where $\Phi(.) and \phi(.) denote,$

respectively the cdf and pdf of a standard normal variate, we obtain from (2.6), $\mathrm{P}(N\geq n)=0\left(n^{-\frac{3}{2}}\right)$.

Thus, $P(N\geq n)=0$, $as\ n\to\infty$ and (2.2) follows. A similar proof holds when $\lambda_n=\frac{2\hat{\psi}_n^2}{q}$.

The basic inequality,ss

$$s+\left(\frac{a_N\hat{\lambda}_N}{d^2}\right)\leq N\leq s+\left(\frac{a_{N-1}\hat{\lambda}_{N-1}}{d^2}\right)+(m-1),$$

or,

$$\left(\frac{s}{n^{**}}\right)+\left(\frac{a_N}{2}\right)\left(\frac{\hat{\lambda}_N}{\lambda}\right)\leq\left(\frac{N}{n^{**}}\right)\leq\left(\frac{s}{n^{**}}\right)$$
$$+\left(\frac{a_{N-1}}{\mathrm{a}}\right)\left(\frac{\hat{\lambda}_{N-1}}{\lambda}\right)+\frac{m-1}{n\;**}$$

It follows from the definition of N that $\lim_{d\to 0} N=\infty$ a.s

and from (A_3) and strong law of large numbers [Bhat (1981 p.187)][2] that $\hat{\psi}_{\mathrm{N}} \overset{a.s.}{\to} \psi\,, a_N=\mathrm{a}$, as $N\to\infty$

$$\text{From}(A_3)\,, \hat{\psi}_n=\left\{\frac{\psi}{q(n-s)}\right\}\sum_{j=1}^{n-s} z_j^{(q)}$$

Hence, from Wieiner (1939) ergodic theorem [see, khan(1969,p.708)][6]

$$\sup_{n\geq s+1}\left\{\sum_{j=1}^{n-s}\frac{z_j^{(q)}}{q^{(n-s)}}\right\}$$

has its second moment finite. Since $\hat{\lambda}_n$ is either $\hat{\psi}_n$ or $\frac{2\hat{\psi}_n^2}{q}$. the expression dominating $(\frac{N}{n^{**}})$ in (2.7) is integrable. Result (2.4) now follows from (2.3) and dominated convergence theorem.

Finally, we have

$$P(\xi\in R_N)\quad\geq P\left(\underline{\xi}\in R_N^*\right)$$
$$=P\left[\left[\psi^{-1}\left(\hat{\theta}_N-\underline{\theta}\right)^{'}Q\left(\hat{\theta}_N-\underline{\theta}\right)+\left(\frac{q}{2\psi^2}\right)\left(\hat{\psi}_N-\psi\right)^2\leq\left(\frac{d^2}{\lambda}\right)\right.\right.$$

$$\geq P\left[N\psi^{-1}\left(\hat{\theta}_N-\underline{\theta}\right)'Q\left(\hat{\theta}_N-\underline{\theta}\right)+\frac{q\left(\hat{\psi}_N-\psi\right)^2}{2\psi^2(N-S)}\leq(N-S)\left(\left(\frac{d^2}{\lambda}\right)\right)\right]\text{.........}(2.8)$$

It follows from (2.3) and Theorem 1 of Anscombe [1](1952) that

$$N\psi^{-1}\left(\hat{\theta}_N-\underline{\theta}\right)'Q\left(\hat{\theta}_N-\underline{\theta}\right)+\frac{q\left(\hat{\psi}_N-\psi\right)^2}{2\psi^2(N-s)}$$

has limiting (as $d\to 0$) distribution $x^2_{(r+1)}$. Hence. utilizing $(2\cdot 3)$ and dominated convergence theorem, equation (2.8) gives

$$\lim_{d\to 0}P\left((\xi\in R_N)\right)\geq P\left[x^2_{(r+1)}\leq(n^{**}-s)\left(\frac{d^2}{\lambda}\right)\right]$$
$$=p\left[x^2_{(r+1)}\leq a\right]$$
$$=\alpha.$$

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