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A SHORT MATHEMATICAL BACKGROUD FOR GRAPH THEORY

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ABSTRACT

This paper summarizes aspects of language and mathematics that are not directly part of graph theory provide useful background for learning of graph theory in mathematics .In particularly Sets ,type of sets ,numbers ,venn diagram of sets ,sets of binary k-tuples , Quantifiers and proofs ,Induction and recurrence etc are term which are frequently used in graph theory .In particular this article explains Functions in details.

Key words: bounded ,integer ,target, absolute value ,finite ,image ,function , *Schematic representation ,* domain.

INTRODUCTION

FUNCTIONS

A function transforms elements of one set into elements of another.

Definition. A function f $from$ a set A *to* a set B assigns to each $a \in A$ a single element. $f(a)$ in B, called the image of a under f. For a function from A to B (written $f: A \to B$), the set A is the domain and the set B is the target. The image of a function f with domain A is

$$\{f \cdot (a): n \in A).$$

We take many elementary functions as familiar, such as the absolute value function and polynomials (both defined on $\mathbb{R}$). "Size" is a function whose domain is the set of finite sets and whose target is $\mathrm{NU}(0\}$.

Deflnition. For $x \in \mathrm{R}$ the floor $\lfloor x \rfloor$ is the greatest integer that is at most x. The ceiling $\lceil x \rceil$ is the smallest integer that is at least x. A sequence is a function f whose domain is $\mathbb{N}$.

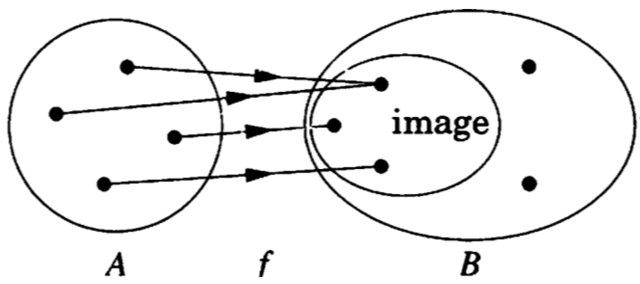
The floor function and ceiling function map $\mathbb{R}$ to $\mathbb{Z}$. When the target of a sequence is A, we have a sequence of elements in A, and we express the sequence as $a_1, a_2, a_3, ,$ where $a_n = f(n\rangle$. We have used induction to prove sequences of statements and to prove formulas specifying sequences of numbers.

We may want to know how fast a function from $\mathbb{R}$ to $\mathbb{R}$ grows, particularly when analyzing algorithms. For example, we say that the growth of a function g is (at most) quadratic if it is bounded by a quadratic polynomial for all sufficiently large inputs.

Remark. *Schematic representation*. A function $f: A \to B$ is defined on A and maps A into B. To visualize a function $f: A \to B$, we draw a region representing A and a region representing B, and from each $x \in A$ we draw an arrow to $f(x)$ in B. In digraph language, this produces an ońentation of a bipartite graph with partite sets A and B in which every element of A is the tail of exactly one edge.

The image of a function is contained in its target. Thus we draw the region for the image inside the region for the target.

domain target



To descńbe a function, we must specify $f(a)$ for each $a \in A$. We can list the pairs $(a, f(a))$, provide a formula for computing $f(a)$ from a, or describe the rule for obtaining $f(a)$ from a in words.

Definition. A function $f: A \rightarrow B$ is a bijection iffor every $b \in B$ there is exactly one $a \in A$ such that $f(a) = b$.

Under a bijection, each element of the target is the image of exactly one element of the domain. Thus when a bijection is represented as discussed above remark, every element of the target is the head of exactly one edge.

Example. *Pairing spouses*. Let M be the set ofmen at a party, and let W be the set of women. If the attendees consist entirely of married couples, then we can define a function $f: M \rightarrow W$ by letting $f(x)$ be the spouse ofx. For each woman $w \in W$, there is exactly one $x \in M$ such that $f(x) = w$. Hence f is a bijection from M to W.

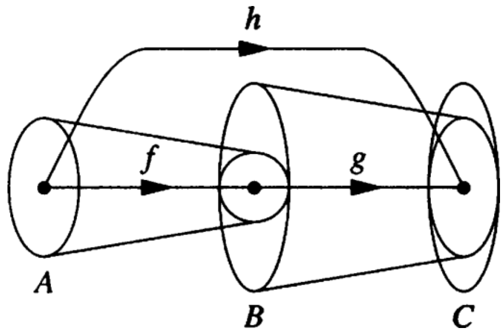
Bijections pair up elements from different sets. Thus we also describe a bijection from A to B as a **one-to-one correspondence** between A and B. Occasionally in the text we say informally that elèments ofone set correspond to elements of another; by this we meaii that there is a natural one-to-one correspondence between the two sets.

When A has n elements, listing them as $a_1, , a_n$ defines a bijection from $[n]$ to A. Viewing the correspondence in the other direction defines a bijection from A to $[n]$. All bijections can be $\text{inverted}'$.

Definition. If f is a bijection from A to B, then the inverse of f is the function $g: B \rightarrow A$ such that, for each $b \in B$, $g(b)$ is the unique element $x \in A$ such that $f(x) = b$. We wnte f^{-1} for the function g.

When the target of a function is the domain of a second function, we can create a new function by applying the first and then the second. This yields a function from the domain of the first function into the target of the second.

Definition. If $f: A \longrightarrow B$ and $g: B \rightarrow C$, then the composition of g with f is a function $h: A \longrightarrow C$ defined by $h(x) = g\big(f(x)\big)$ for $x \in A$. When h is the composition of g with f, we write $h = g \circ f$.



From the definitions, it is easy to verify that the composition of two bijections is a bijection. We can use this in Proposition in verifying for graphs that a composition of isomorphisms is an isomorphism.

RELATIONS

Given two objects s and t, not necessanly of the same type, we may ask whether they satisfy a given relationship. Let S denote the set of objects of the first type, and let T denote the set of objects of the second type. Some of the ordered pairs (s, t) may satisfy the relationship, and some may not. The next definition makes this notion precise.

Definition. When S and T are sets, a relation between S and T is a subset ofthe product $S \times T$. A relation on S is a subset of $S \times S$.

We usually specify a relation by a condition on pairs. In Section 1.1, we define several relations associated with a graph G. The *incidence relation* between $S = V(G)$ and $T =\cdot E(G)$ is the set of ordered pairs (v, e) such that $v \in V(G), e \in E(G)$, and v is an endpoint of edge e. The *adjacency relation* on the set $V(G)$ is the set of ordered pairs (x, y) ofvertices such that x and y are the endpoints of an edge.

Remark Let R be a relation defined on a set S. When discussing several items from S, we use the adjective pairwise to specify that each pair among these items satisfies R. Thus we can talk about a family of pairwise disjoint sets, or a family of pairwise isomorphic graphs. An independent set in a graph is a set of pairwise nonadjacent vertices. A set of distinct objects is a set of pairwise unequal objects.

We need the term “pairwise’ because the relation is defined for pairs. For the same reason, we don’t use “pairwise” when discussing only two objects. When two graphs are isomorphic, we don’t say they are pairwise isomorphic. Similarly, we say that the endpoints of an edge are adjacent, not pairwise adjacent; the adjacency relation is satisfied by certain pairs of vertices.

To specify a relation between S and T, we can list the ordered pairs satisfying it. Usually it is more convenient to let S index the rows and T the columns of a grid of positions called a matrix. We can then specify the relation by recording, in the position for row s and column t, a 1 if (s, t) satisfies the relation and a 0 if (s, t) does not satisfy the relation. Thus the adjacency and incidence matrices of a graph are the matrices recording the adjacency and incidence relations .

The condition “have the same parity” defines a relation on $\mathbb{Z}$. If x, y are both even or both odd, then (x, y) satisfies this relation; otherwise it does not. The key properties ofparity lead us to an important class of relations.

Definition. An equivalence relation on a set S is a relation R on S such that for all choices of distinct $x, y, z \in S$,

a) $(x, x) \in R$ (reflexive property).

b) $(x, y) \in R$ implies $(y, x) \in R$ (symmetric property).

c) $(x, y) \in R$ and $(y, z) \in R$ imply $(x, z) \in \dot{R}$ (transitive property).

For every set S, the equality relation $R = \{(x, x): x \in S\}$ is an equivalence relation on S. In Proposition 1.1we show that the isomorphism relation is an equivalence relation on graphs. The notation $G \cong H$ for this relation suggests “equal in some sense’.

Definition. Given an equivalence relation on S, the set of elements equivalent to $x \in S$ is the equivalence class containin αx.

THE PIGEONHOLE PRINCIPLE

The pigeonhole principle is a simple notion that leads to elegant proofs and can reduce case analysis. In every set of numbers, the average is between the minimum and the maximum. When dealing with integers, the pigeonhole principle allows us to take the ceiling or floor of the average in the desired direction.

Lemma. (Pigeonhole Principle) If a set consisting of more than kn objects is partitioned into n classes, then some class receives more than k objects.

Proof. The contrapositive states tha,t if every class receives at most k objects·' then in total there are at most kn objects.

The pigeonhole principle can reduce case analysis by allowing us to use additional information about an extreme element of a set. This simple idea can crop up unexpectedly, but its use can be quite effective. When we find that we need the pigeonhole pnnciple, there is no trouble applying it: we need a sufficiently big value in our set, and the pigeonhole principle provides it.

Proposition. If G is a simple n-vertex graph with $\delta(G) \geq (n-1)/2$, then G is connected.

Proof. Choose u, $v \in V(G)$. If $u * v$, then at least $n-1$ edgesjoin $\{u, v\}$ to the remaining vertices, since $\delta(G) \geq (n-1)/2$. There are $n-2$ other vertices, so the pigeonhole principle implies that one of them receives two of these edges. Since G is simple, this vertex is a common neighbor of u and v.

For every two vertices $u, v \in V(G)$, we have proved that u and v are adjacent or have a common neighbor. Thus G is connected.

The pigeonhole principle can also be useful in statements about trees, where the number of vertices is one more than the number of edges. If each vertex selects an edge in some way, then some edge must be selected twice. The idea is to design the selection so that when an edge is selected twice, the desired outcome occurs.

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