JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

TWO-DIMENSIONAL FLOW DUE TO THE STRETCHING OF THE SHEET OVER A NATURALLY PERMEABLE BRINKMAN BED

Dr.Kishan Singh Shekhawat

Associate Professor

S.K. Govt. Girls College,Sikar,Rajasthan

Abstract:

The study two-dimensional flow of a viscous incompressible fluid through a naturally permeable bed bounded by a stretching sheet has been investigated . The motion being caused solely by stretching of the sheet. The flow behavior due to the strerching of the sheet over a naturally permeable bed using Brinkman's equation, is examined. The results for , velocity distribution, stream function, pressure coefficient and coefficient of skin- friction at the stretching sheet have been obtained, drawn for different stretching and permeability parameters, and discussed.

1. Introduction

The fluid flow within a permeable material is represented by a macroscopic law known as Darcy's law. Brinklman(1) further suggested an impirical modification of heat which was later realized by Tam(2) that the proper way to derive this equation is through the Foldy's(3) approximation. Since then the Brinkman equation has become a major tool in the theiortical investigatiion of flow in porous media.Many researchers (4-9) also work on stretching sheet . Brinkman has proposed the plausible modification of Darcy's law within effective viscosity considerations. Most applications of the Brinkman equation Saffman (10),Happle(11) assume the effective viscosity and the pure fluid viscosity as same.

Flow in the boundary layer over moving solid surfaces was much investigated. Many researchers (12-17) studied the flow caused by stretching,too. However the flow caused by a stretching sheet in a porous bed does not seem to received much attention.

Flow and heat transfer effect through porous medium adjacent to a stretching sheet are disussed by Cortell(18), Liu(19) and others. Darcy Brinkmann model is used by Nazar(20) to examined the flow due to stretching verified sheet in a porous medium. Chauhan and Rastogi(21) studied unsteady flow through a porous medium past a stretching sheet and heat transfer in the presence of magnetic field. Kiwan and Alnimir (22) investigated similarly solutions for boundary layer flows.

In this paper the steady flow of viscous incompressible through a permeable bed caused by stretching a deformeable sheet over the porous matrix, is studied.The flow in the porous matrix is governed by the Brinkman equations.The effect of permeability on the velocity distribution stream function, pressure coefficient and coefficient of skin friction at the stretching sheet have been discussed.

2. FORMULATION OF THE PROBLEM

The two dimensional steady laminar flow of a viscous incompressible fluid through a porous bed is considered. Fluid motion is solely due to the stretching of the sheet over the upper surface of the porous bed filled with fluid. A Cartesian coordinate system is used with the origin at the top of the stretching sheet and y-axis normal to it. The stretching sheet is stretched by introducing two equal and opposite forces so that the position of the point $(0,0)$ remains unchanged.

The flow in the porous matrix $(0 \le y < -\infty)$ is governed by the Brinkman's equations:

$$\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-u\frac{\nu}{K}-\frac{1}{\rho}\frac{\partial p}{\partial x}=0, \tag{1}$$

$$\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-v\frac{\nu}{K}-\frac{1}{\rho}\frac{\partial p}{\partial y}=0, \tag{2}$$

And

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{3}$$

Where u, v are the fluid velocity components along the x and y directions in the porous matrix ; p the pressure ; ρ the density ; ν the kinematic viscosity; and K the permeability of the porous medium.

The boundary conditions are:

At $\quad y=0, u=Cx, v=0, \; as \; y \to -\infty, u \to 0,$ (4)

where $u = Cx$ represents the stretching velocity of the sheet with $C > 0$.

3. METHOD OF SOLUTION

Let

$$u = Cxf'(\eta)$$

$$v = -Chf(\eta),$$ and

$$\eta = \frac{y}{h} \tag{5}$$

where a prime denotes differentiation with respect to η and h is the average pore diameter. Substituting (5) into equation (1) to (4), we have

$$\frac{C^2x}{R}\left(\frac{1}{\beta}f^{'}(\eta)-f^{'''}(\eta)\right)=-\frac{1}{\rho}\frac{\partial p}{\partial x} \quad , \tag{6}$$

$$\frac{C^2h}{R}\left(f^{''}(\eta)-\frac{1}{\beta}f(\eta)\right)=-\frac{1}{\rho h}\frac{\partial p}{\partial \eta} \tag{7}$$

And the corresponding boundary conditions are

$$at\ \eta=0,\ f(0)=0,\ f^{'}(0)=1,$$

$$and\ \ as\ \ \eta\to-\infty,\ f^{'}(-\infty)=0 \tag{8}$$

Where $\mathrm{R}=\frac{Ch^2}{\nu}$ and $\beta=\frac{K}{h^2}$.

4. SOLUTIONS

The solutions are obtained on solving (6) and (7) under the boundary conditions (8). Thus, we have

$$f^{'}(\eta)=e^{\frac{\eta}{\sqrt{\beta}}} \tag{9}$$

And

$$f(\eta)=\sqrt{\beta}(e^{\frac{\eta}{\sqrt{\beta}}}-1)$$

(10)

The stream function for the porous region may be written as

$$\Psi=-\mathrm{R}\,\xi\, f(\eta),$$

(11)

where $\xi=\frac{x}{h}$

The shearing stress on the stretching sheet is

$$\tau=\mu\frac{\partial u}{\partial y}=\mu Cx\frac{1}{h}f^{''}(0)$$

Thus the coefficient of skin- friction is given by

$$(C_f)_{\eta=0}=\frac{\tau}{\mu(\nu/h)/h}=-R\xi f^{''}(0)$$

(12)

The pressure coefficient is given by

$$C_p = \frac{p}{\rho(\nu/h)^2} = -\frac{R}{\sqrt{\beta}}\eta$$

(13)

5. DISCUSSION

We have drawn velocity profiles and streamlines for the different permeability parameters in Fig 1 - 3. It is found that the fluid velocity components u and v increase with the increase in stretching parameter R and the permeability parameter β. The streamlines show that the fluid is thrownout near the sheet due to stretching and in order to fill the gap the fluid rushes from infinity causing inward flow in the permeable bed. Figures 4 and 5 show the variation of pressure coefficient and the coefficient of skin friction at the stretching sheet for different values of R and β. It is noted that both $(C_p \; and \; C_f)$ increase with the increase in R , while both decrease with the increase in β.

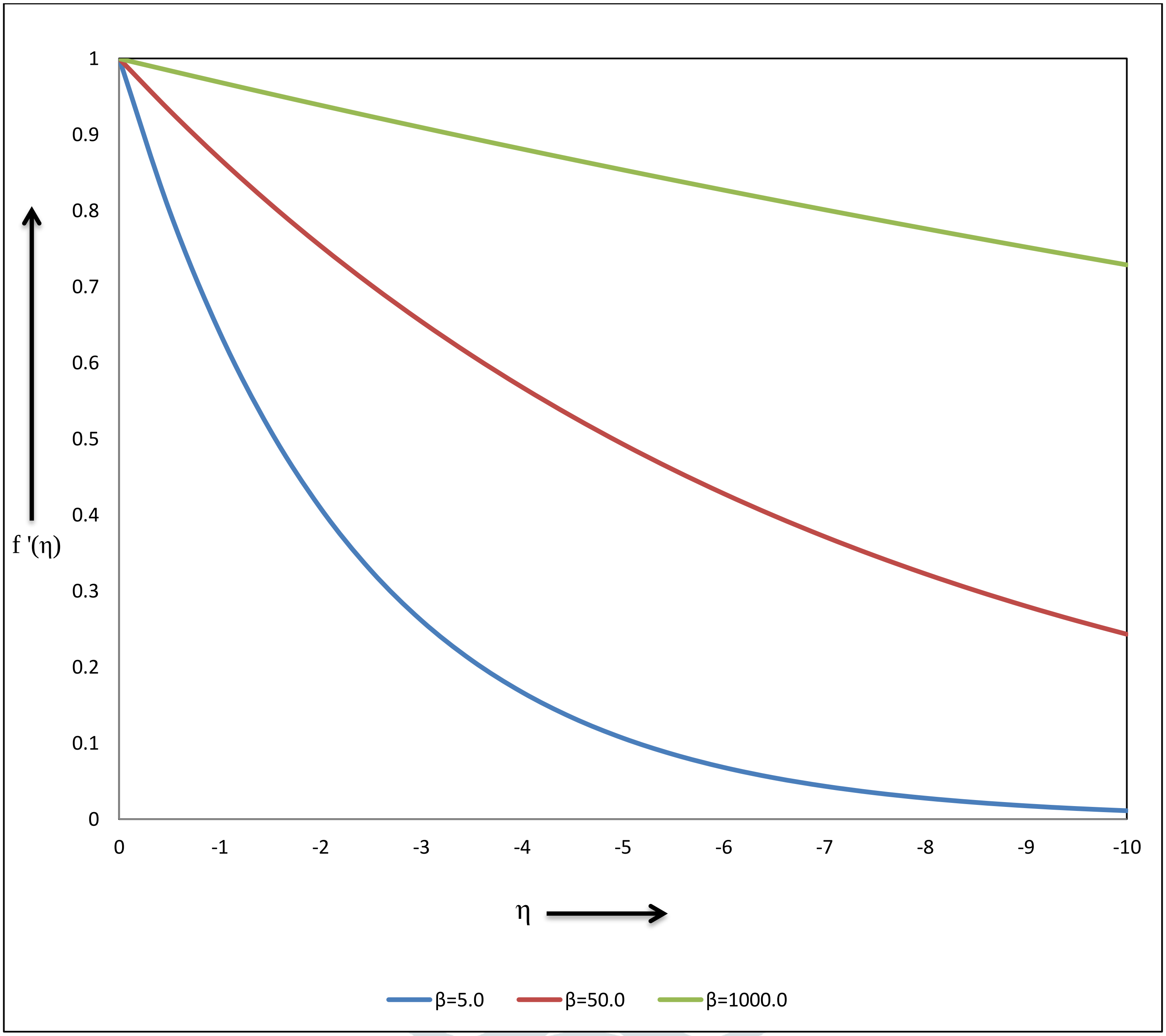

Fig. 1 $f^{'}(\eta)$ Vs η for $R=0.5$

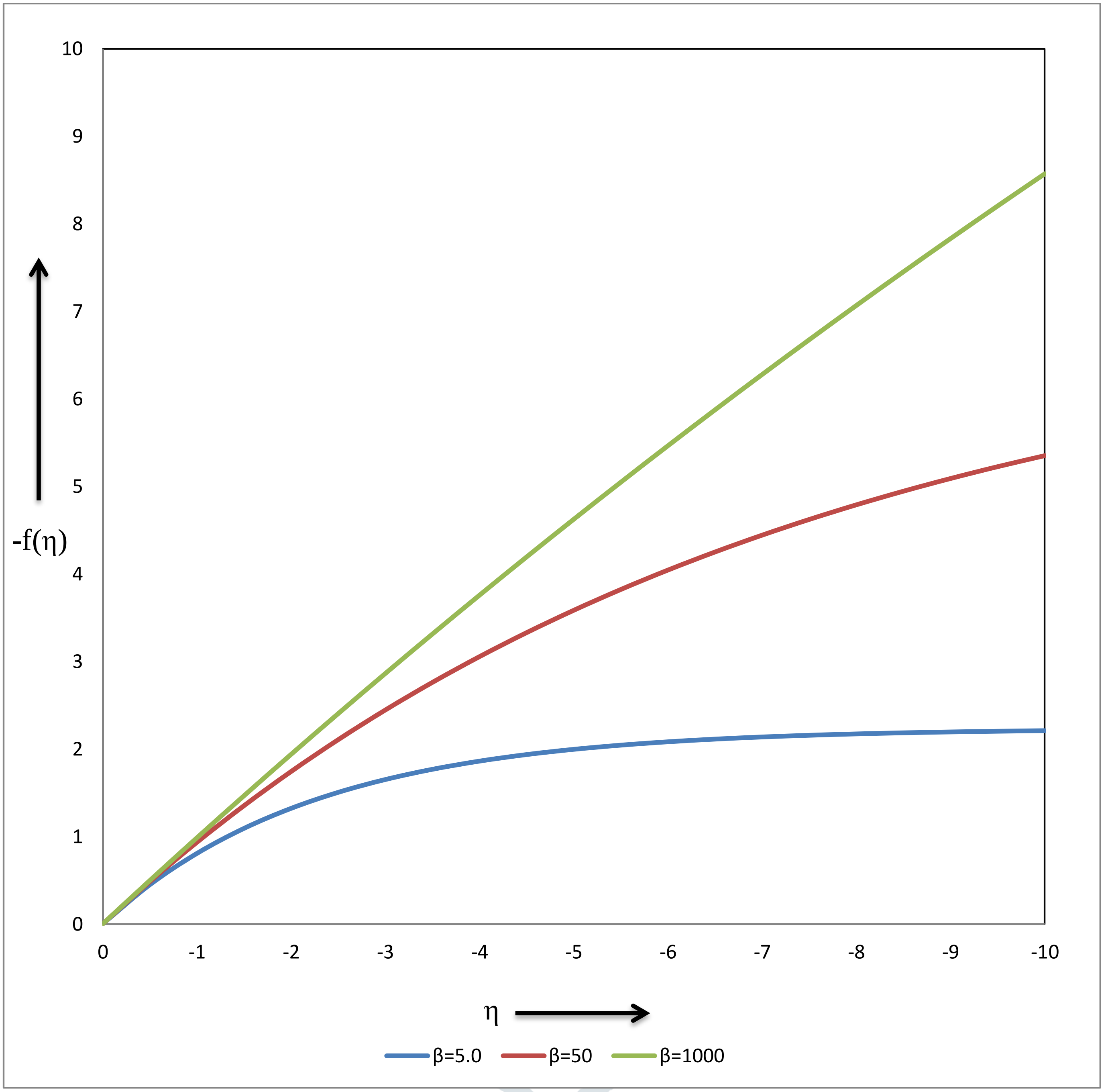

Fig. 2 $-f(\eta)$ Vs η for $R = 0.5$

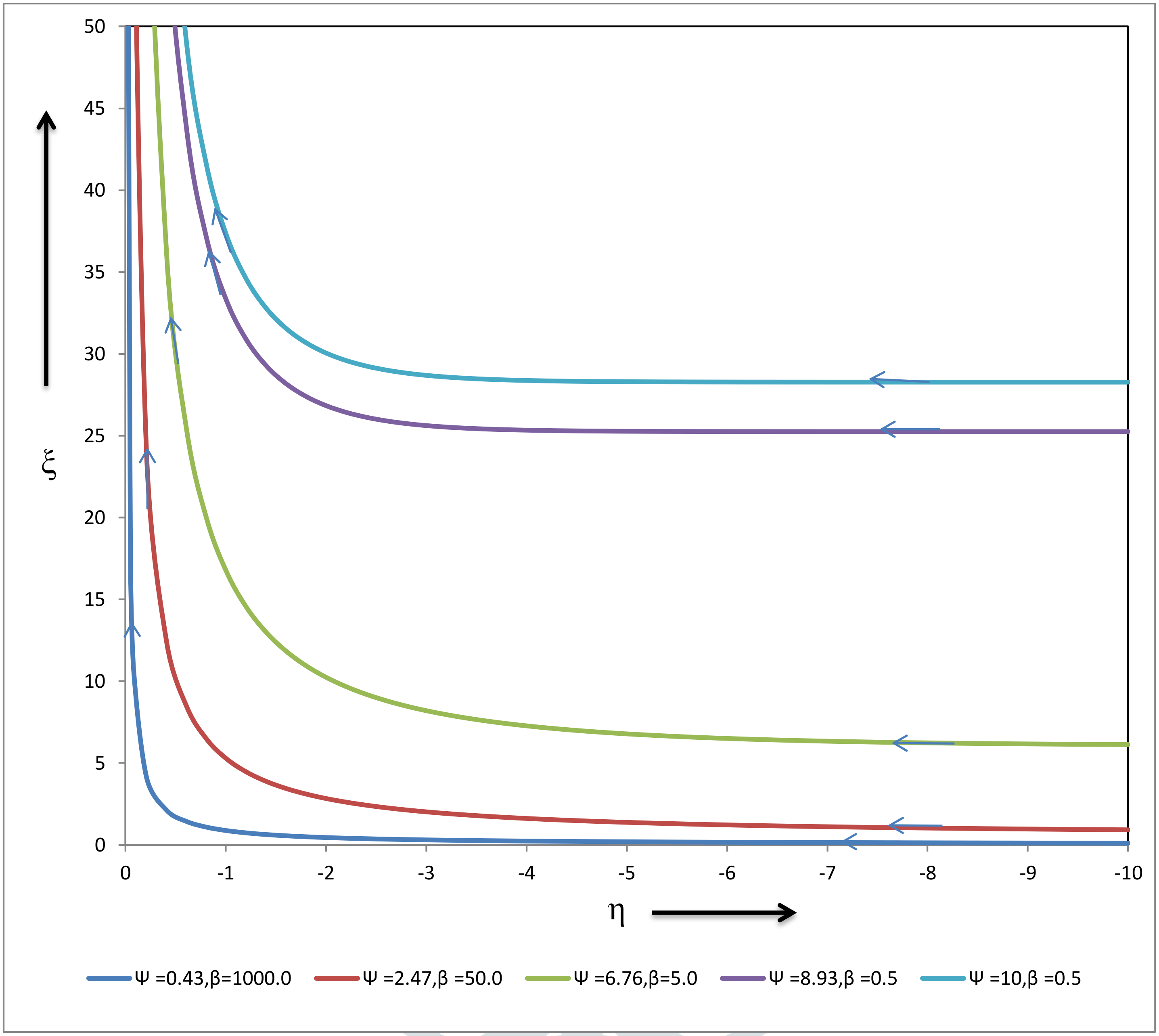

Fig.3 Streamlines Pattern for $R = 0.5$

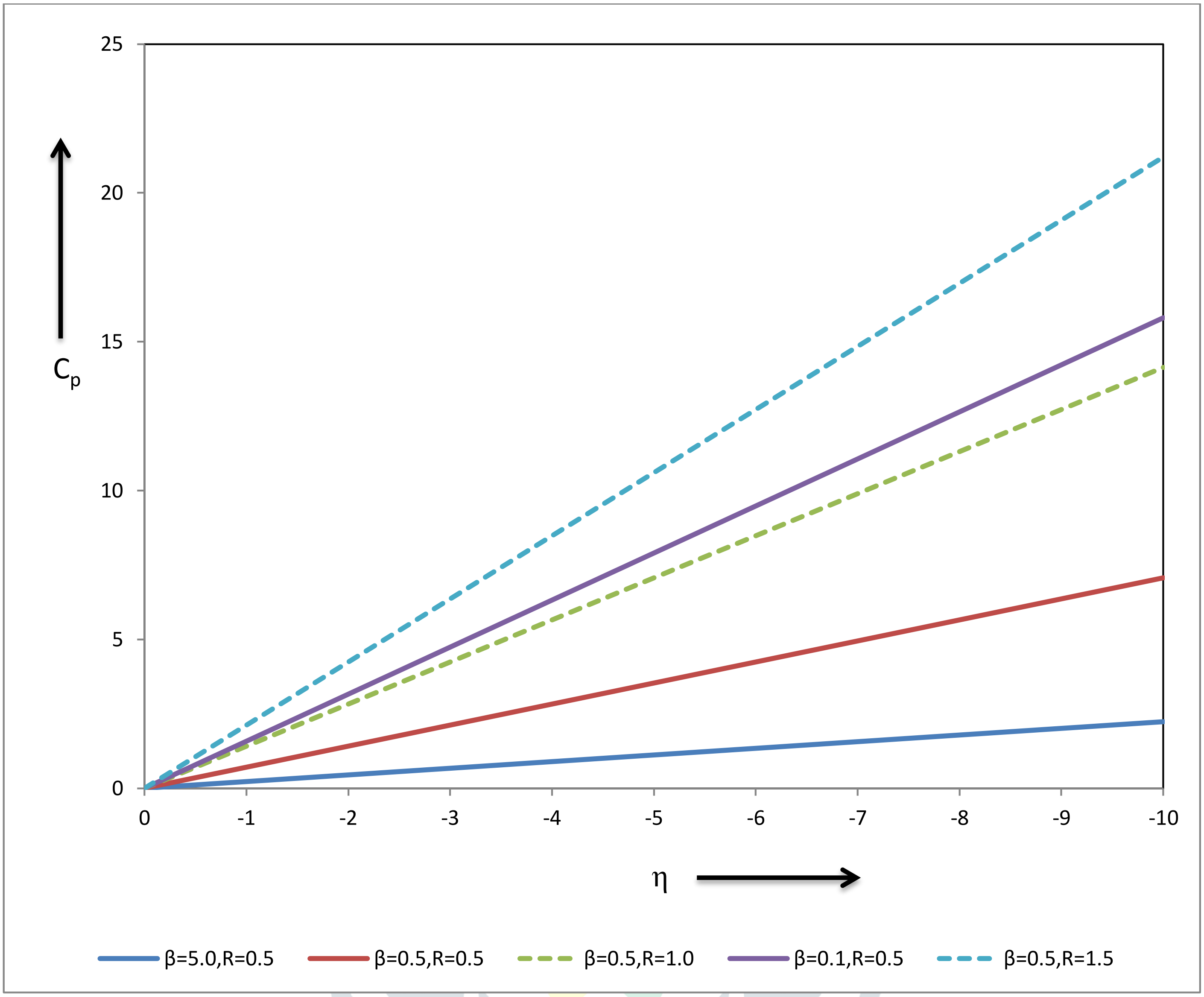

Fig.4 C_p Vs η

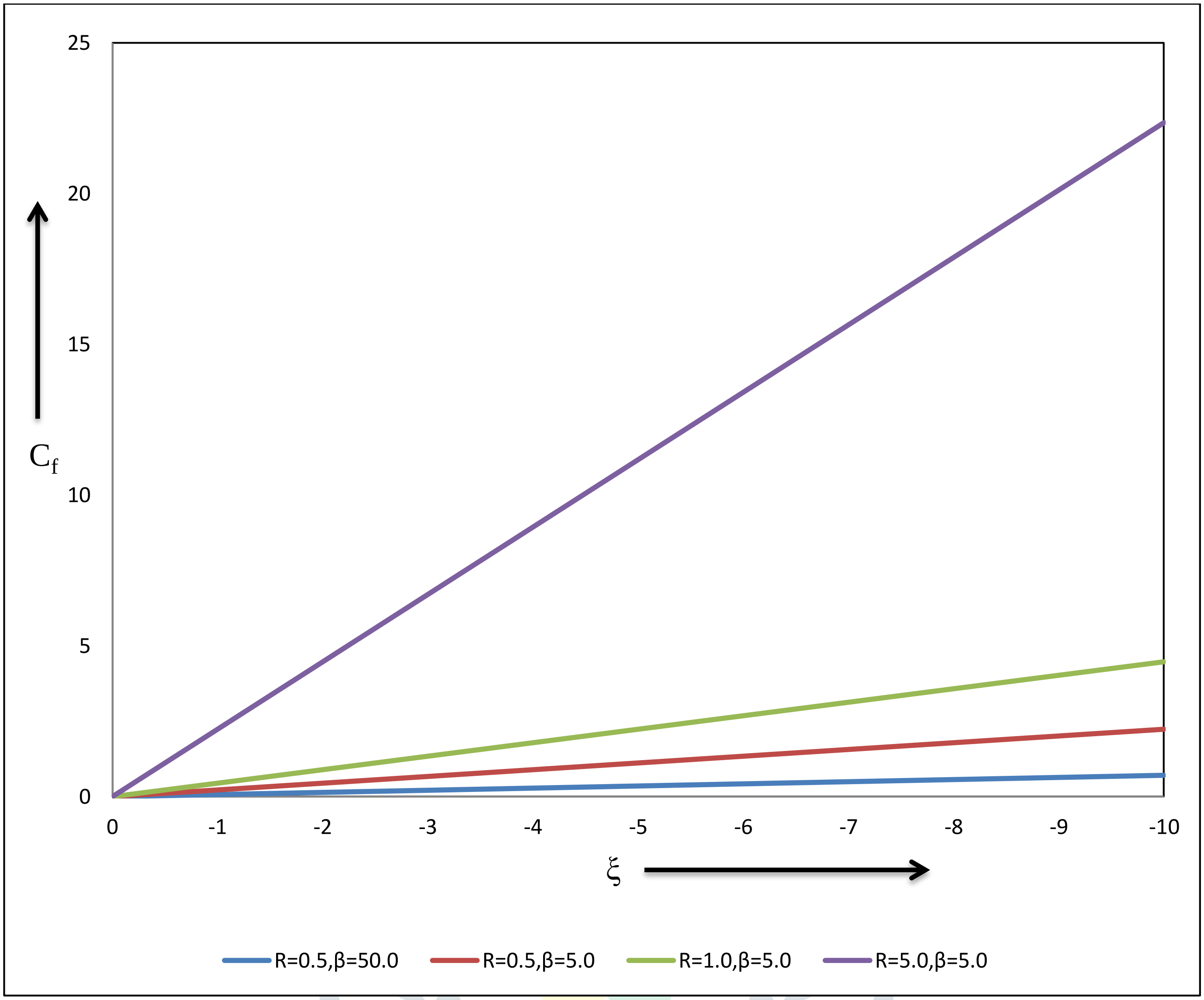

Fig. 5 C_f Vs ξ

REFERENCES:

[1] Brinkman, H. C. (1949). A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles. Flow, Turbulence and Combustion, 1, 27-34..

[2] Tam, C. K. (1969). The drag on a cloud of spherical particles in low Reynolds number flow. Journal of Fluid Mechanics, 38(3), 537-546.

[3] Foldy, L.L. (1945). The multiple scattering of waves, Phys. Rev., 67,107-119.

[4] Nield, D. A. (1983). The boundary correction for the Rayleigh-Darcy problem: limitations of the Brinkman equation. Journal of Fluid Mechanics, 128, 37-46.

[5] Kim, S., & Russel, W. B. (1985). Modelling of porous media by renormalization of the Stokes equations. Journal of Fluid Mechanics, 154, 269-286.

[6] Lundgren, T.S. (1972). Slow flow through a stationary random eds and suspensions of spheres,J.Fluid Mech.,51,273-299.

[7] Neale, G., & Nader, W. (1974). Practical significance of Brinkman's extension of Darcy's law: coupled parallel flows within a channel and a bounding porous medium. The Canadian Journal of Chemical Engineering, 52(4), 475-478.

[8] Howells, I. D. (1974). Drag due to the motion of a Newtonian fluid through a sparse random array of small fixed rigid objects. Journal of Fluid Mechanics, 64(3), 449-476.

[9] Childress,S. (1972). Viscous flow past a random array of spheres, J. Chem. Phys.56,2527-2539.

[10] Saffman, P. G. (1973). On the settling speed of free and fixed suspensions. Studies in Applied Mathematics, 52(2), 115-127.

[11] Happel, J., & Brenner, H. (2012). Low Reynolds number hydrodynamics: with special applications to particulate media (Vol. 1). Springer Science & Business Media..

[12] Crane, L. J. (1970). Flow past a stretching plate. Zeitschrift für angewandte Mathematik und Physik ZAMP, 21, 645-647

[13] MCCORMACK, P., & CRANE, L. (1973). Physical fluid dynamics(Book- Physical fluid dynamics.). New York, Academic Press, Inc., 1973. 506 p..

[14] Borkakoti, A. K., & Bharali, A. (1983). Hydromagnetic flow and heat transfer between two horizontal plates, the lower plate being a stretching sheet. Quarterly of Applied Mathematics, 40(4), 461-467.

[15] Wang, C. Y. (1984). The three-dimensional flow due to a stretching flat surface. The physics of fluids, 27(8), 1915-1917.

[16] Bhattacharyya, S.N. and Gupta,A.S.(1985). On the Stability of viscous flow over a stretching sheet., Quar.Appl.Math.,43,359.

[17] McLeod, J. B., & Rajagopal, K. R. (1987). On the uniqueness of flow of a Navier-Stokes fluid due to a stretching boundary. Archive for Rational Mechanics and Analysis, 98, 385-393.

[18] Cortell, R. (2005). Flow and heat transfer of a fluid through a porous medium over a stretching surface with internal heat generation/absorption and suction/blowing. Fluid Dynamics Research, 37(4), 231-245.

[19] Liu, I. C. (2006). Flow and heat transfer of viscous fluids saturated in porous media over a permeable non-isothermal stretching sheet. Transport in porous media, 64, 375-392.

[20] Nazar, R.(2012).Stretching vertical sheet in a porous medium using Darcy-Brinkman equation model.,Int.J.Appl.Mech. and Eng. 11,3,623-637.

[21] Chauhan, D.S. and Rastogi,P.(2012).Unsteady MHD flow and heat transfer through a porous medium past a non-isothermal stretching sheet with slip conditions., Int J. of Energy and Technology,4,17, 1-11.

[22] Kiwan,S. and Al-Nimir, M.A.(2010).Investigation into the similarly solution for boundary layer flows in Microsystems, ASME J.Heat Transfer,132,4,1-9.