JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

Kantowski -Sachs Cosmological Models with Magnetized Bulk Viscous Wet Dark Fluid In a Scalar Tensor Theory of Gravitation

Amrapali P. Wasnik
Bharatiya Mahavidyalaya, Amravati

Abstract : Spherically Symmetric wet dark fluid Cosmological models with bulk viscous magnetic field is obtained in the presence of a Scalar tensor theory of gravitation proposed by Saez Balllestor [1] To get a determination Solution of the field equations, we take a relation between metric potential $b = a^n$. Since physical & Kinematical properties of the models are also discussed .

Keywords : Bulk viscous, Magnetic field, Wet dark fluid, Scalar tensor theory,

Introduction :

Cosmological models with bulk – viscosity are important since bulk - viscosity has a greater rule in getting accelerated expansion of the universe popularly known as inflationary phase bulk viscous cosmological models in general relativity have been discussed by several authors Tripathy et. al [2], [3] have studied the Bianchi type cosmological models in the presence of cosmic strings & bulk viscosity . Bulk Viscous cosmological models have been discussed in Brans-Dicke theory of gravitation by many authors Rao et. al. [4] have discussed viscosity in a Scalar tensor theory proposed by Saez & Ballester [1]

In recent years, there has been lot of interest In Scalar tensor theories of gravitation proposed by Brans &Dicke [5] ,Nardvedt [6] and Saez & Ballester [1]. Among them Bans –Dicke and Saez – Ballester scalar –tensor theories are considered to be viable alternatives to general relativity Brans –Dicke theory include a long range Scalar field interacting equally with all forms of matter (with the exception of electromagnetism) while in Saez – Ballester Scalar-tensor theory metric is coupled with a dimensionless scalar field in a simple manner. A brief review of Saez Ballester theory is given by Reddy et al. [7] Electromagnetic field a property of space caused by the motion of an electric charge. A stationary change will produce only an electric field in the surrounding space. If the charge is moving, a magnetic field is also reduced. An electric field can be produced also by a changing magnetic field. The mutual interaction of electric and magnetic fields produces an electromagnetic field, which is considered as having its own existence in space apart from the changer currents (a stream of moving charges) with which it may be related under certain circumstances , this electromagnetic field can be described as a wave transporting electromagnetic energy. Recently the occurrence of magnetic fields with on galactic scales & their importance for a variety of astrophysical phenomena has been point art by several authors. Such as Yadav [8], Pradhan [9], Singh [10], Das [11], Kandalkar et al. [12]

Very recently, there has been considerable interest in cosmological models with dark energy in GR because of the fact that our universe is currently undergoing an accelerated expansion which has been confirmed by a hast of observations, such as type Ia supernova ([13],[14]). Several models have been proposed to explain dark energy ([15], [16],[17], [18], [19]).

The study of cosmological models in the frame of scalar tensor theories has been the active area of research for the last fewcade. In Particular Singh & Agrawal [20] Shri Ram & Tiwari [21] Reddy & Naidu [22] Rao et al [23] are some of the authors who have investigated several aspects of the cosmological models in Saez Ballester scalar-tensor theory .

Motivated by the above discussion in this paper, we have focused upon the problem of establishing a formative far studying a new integrability of magnetized bulk viscous Kantowski Sachs wet dark fluid cosmological model in a scalar tensor theory of relativity. The behavior of the models are also discussed.

The Metric and Field Equations:

We consider a spherically symmetric homogeneous anisotropic space time with metric

$$ds^2 = -dt^2 + a^2 dr^2 + b^2 d\Omega^2 \tag{1}$$

Where a, b are functions of t alone.

The field equations given by Saez and Ballester [1] for the combined scalar and tensor field are

$$R_{ij} - \frac{1}{2} g_{ij} R - \omega \phi^n \left(\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij} \phi_{,k}\phi^{,k} \right) = -T_{ij} \tag{2}$$

and the scalar field $\boldsymbol{\phi}$ satisfies the equation

$$2\,\phi^n \phi^{,i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} i = 0 \tag{3}$$

here $\boldsymbol{\omega}$ and $n$are constants, T_{ij} is the energy momentum tensor of the matter and comma and semicolon denote partial and covariant differentiation respectively.

The energy momentum for magnetized bulk viscous wet dark fluid is

$$T_{ij} = (\rho_{WDF} + P_{WDF}) u_i u_j + P_{WDF} g_{ij} - \xi \vartheta \left(u_i u_j + g_{ij} \right) + E_{ij} \tag{4}$$

Where $\boldsymbol{\rho}$ is the rest energy density of the system, $\vartheta = u^i_{;i}$ is scalar expansion, $\boldsymbol{\xi}$ the coefficient of bulk viscosity, the vector represent a u^idescribe the cloud four velocities and x^i represent a direction of anisotropy that is direction of string, satisfy the standard relation

$$u^i u_i = -x^i x_i = -1 \quad and \quad u^i x_i = 0 \tag{5}$$

E_{ij} is energy momentum for magnetic field given by

$$E_{ij} = \frac{1}{4\pi} \left(F_{i\alpha} F_{j\beta} g^{ij} - \frac{1}{4} g_{ij} F^{\alpha\beta} F_{\alpha\beta} \right) \tag{6}$$

where F_{ij} is the electromagnetic field tensor which satisfies the Maxwell equation

$$F_{[ij;\alpha]} = 0 \quad \left(F^{ij} \sqrt{-g} \right)_{;j} = 0 \tag{7}$$

In commoving co-ordinates, the incident magnetic field is taken along x-axis, with the help of Maxwell equation (7), the only non-vanishing component of F_{ij} is the equation

$$F_{23} = cons\,tan\,t = A \tag{8}$$

The non- vanishing components of the Einstein field equations are

$$\frac{1}{b^2} + 2\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} + \frac{\omega}{2}\phi^n\dot{\phi}^2 = 8\pi\rho_{WDF} + \frac{A^2}{\mu^2 b^4} \tag{9}$$

$$\frac{1}{b^2} + 2\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} - \frac{\omega}{2}\phi^n\dot{\phi}^2 = -8\pi p_{WDF} + \frac{A^2}{\mu^2 b^4} + \xi\vartheta \tag{10}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} = -8\pi p_{WDF} - \frac{A^2}{\mu^2 b^4} + \xi\vartheta \tag{11}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right) + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0 \tag{12}$$

The physical quantities that are of the importance in cosmology are proper volume$V$, expansion scalar θ and shear $\sigma^2$and have the following expression for the metric

$$V = ab^2 sin\vartheta \tag{13}$$

$$\theta = \frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} \tag{14}$$

$$\sigma^2 = \frac{2}{3}\left(\frac{\dot{b}}{b} - \frac{\dot{a}}{a}\right)^2 \tag{15}$$

where an overhead dot indicate ordinary time derivative.

Solutions of the Field Equations and the Model:

The field equations (9)-(12) are four independent equations in six unknown quantities $a, b, \rho_{WDF}, p_{WDF}$. Two additional conditions relating these unknown are required to obtain exact solutions of system.

Firstly, we assume the relation

$$b = a^n \tag{16}$$

From (10) and (11) , we obtain

$$\frac{\ddot{b}}{b} - \frac{\ddot{a}}{a} + \left(\frac{\dot{b}}{b}\right)^2 - \frac{\dot{a}\dot{b}}{ab} = 2\frac{A^2}{\mu^2 b^4} \tag{17}$$

Using equation (16) in (17), we obtain

$$\ddot{a} + 2n\frac{\dot{a}^2}{a} = 2\frac{A^2}{\mu^2(n-1)}a^{-4n+1} - \frac{1}{(n-1)}a^{-2n+1} \tag{18}$$

Let us consider

$$\dot{a} = f(a)$$

$$\ddot{a} = ff'wheref' = \frac{df}{da} \tag{19}$$

Using (19), (18) reduces to

$$\frac{d}{da}(f^2) + 4n\frac{f^2}{a} = \frac{4A^2}{\mu^2(n-1)}a^{-4n+1} - \frac{2}{(n-1)}a^{-2n+1} \tag{20}$$

After simplifying (20) , we obtain

$$f^2 = \frac{2A^2}{\mu^2(n-1)}a^{2(1-2n)} - \frac{1\text{`}}{(n^2-1)}a^{2(1-n)} + Sa^{-4n} \tag{21}$$

where S is integrating constant

Using (19), (21) becomes

$$\frac{da}{dt} = \left[\frac{2A^2}{(n-1)\mu^2}a^{2(1-2n)} - \frac{1}{n^2-1}a^{2(1-n)} + Sa^{-4n}\right]^{\frac{1}{2}} \tag{22}$$

Then the metric(1) becomes as

$$ds^2 = -\left[\frac{2A^2}{(n-1)\mu^2}a^{2(1-2n)} - \frac{1}{(n^2-1)}a^{2(1-n)} + Sa^{-4n}\right]^{-1} da^2 + a^2dr^2 + a^{2n}d\Omega^2 \tag{23}$$

Using suitable transformation

$$ds^2 = -\left[\frac{2A^2}{(n-1)\mu^2}T^{2(1-2n)} + \frac{1}{(n^2-1)}Ta^{2(1-n)} + ST^{-4n}\right]^{-1} dT^2 + T^2dr^2 + T^{2n}d\Omega^2 \tag{24}$$

For the model of equation (24), the other physical and geometrical parameters can be easily obtained.

Some Physical Properties of the Model

The scalar of expansion , the shear scalar , spatial volume , Pressure and density of WDF are given respectively by

$$\theta = (1+2n)\left[\frac{2A^2}{(n-1)\mu^2}T^{-4n} - \frac{1}{(n^2-1)}T^{-2n} + ST^{-(4n+2)}\right]^{\frac{1}{2}} \tag{25}$$

$$\sigma^2 = \frac{2}{3}(n-1)^2\left[\frac{2A^2}{(n-1)\mu^2}T^{-4n} - \frac{1}{(n^2-1)}T^{-2n} + ST^{-(4n+2)}\right] \tag{26}$$

The Special Volume in the model is given by

$$V = T^{2n+1}sin\boldsymbol{\vartheta} \tag{27}$$

The Scalar field in the model given by

$$\phi = \left\{\frac{n+2}{2}\left[-k_1\frac{T^{-2n}}{2n}+k_2\right]\right\}^{\frac{2}{n+2}} \tag{28}$$

where k_1 and k_2 are constant of integration

The Physical quantities that are important in cosmology :

Energy Density of WDF

$$\rho_{WDF} = \frac{1}{8\pi}\left\{\frac{1}{T^{2n}} + n(n+2)\left[\frac{2A^2}{(n-1)\mu^2}\frac{1}{T^{4n-1}} - \frac{1}{n^2-1}\frac{1}{T^{2n-1}} + \frac{S}{T^{4n+1}}\right] - \frac{A^2}{\mu^2}\frac{1}{T^{4n}} + \frac{\omega}{2}\left(\frac{n+2}{2}\left\{\frac{k_1}{2n}\frac{1}{T^{2n}} + k_2\right\}\right)^{\frac{2n}{n+2}} \cdot \frac{k_1^2}{n^2(n+2)^2}\left[\frac{n+2}{2}\left(\frac{-k_1}{2n}\frac{1}{T^{2n}} + k_2\right)\right]^{\frac{-2n}{n+2}}\frac{1}{T^{2(2n+1)}} \cdot \left[\frac{2A^2}{(n-1)\mu^2}\frac{1}{T^{2(2n-1)}} - \frac{1}{(n^2-1)}\frac{1}{T^{2(n-1)}} + \frac{S}{T^{4n}}\right]\right\} \tag{29}$$

The Pressure of WDF:

We further assume that the coefficient of bulk viscosity is inversely proportional to expansion i.e. $\xi\boldsymbol{\vartheta} = \boldsymbol{const\ M}$

$$\rho_{WDF} = \frac{1}{8\pi}\left\{\frac{1}{T^{2n}} + n(n-2)\left[\frac{2A^2}{(n-1)\mu^2}\frac{1}{T^{4n-1}} - \frac{1}{n^2-1}\frac{1}{T^{2n-1}} + \frac{S}{T^{4n+1}}\right] + n\left[\frac{2A^2(1-2n)}{(n-1)\mu^2}\frac{1}{T^{4n}} + \frac{1}{(n+1)}\frac{1}{T^{2n}} - \frac{2ns}{T^{2(2n+1)}}\right] + \frac{\omega}{2}\left(\frac{n+2}{2}\left\{\frac{k_1}{2n}\frac{1}{T^{2n}} + k_2\right\}\right)^{\frac{2n}{n+2}} \cdot \frac{k_1^2}{n^2(n+2)^2}\left[\frac{n+2}{2}\left(\frac{-k_1}{2n}\frac{1}{T^{2n}} + k_2\right)\right]^{\frac{-2n}{n+2}}\frac{1}{T^{2(2n+1)}} \cdot \left[\frac{2A^2}{(n-1)\mu^2}\frac{1}{T^{2(2n-1)}} - \frac{1}{(n^2-1)}\frac{1}{T^{2(n-1)}} + \frac{S}{T^{4n}}\right] - \frac{A^2}{\mu^2}\frac{1}{T^{4n}} - M\right\} \tag{30}$$

Hubble's Parameter $\boldsymbol{H}$:

$$H = \frac{1+2n}{3}\left[\frac{2A^2}{(n-1)\mu^2}\frac{1}{T^{4n-1}} - \frac{1}{(n^2-1)}\frac{1}{T^{2n-1}} + \frac{S}{T^{4n+1}}\right]^{\frac{1}{2}}$$

The coefficient of bulk viscosity:

$$\xi = \frac{M}{(1+2n)}\left[\frac{2A^2}{(n-1)\mu^2}\frac{1}{T^{4n}} - \frac{1}{(n^2-1)}\frac{1}{T^{2n}} + \frac{S}{T^{4n+2}}\right]^{-\frac{1}{2}}$$

The spatial volume V tend to zero when $T \to 0$ and increases as $T \to \infty$.

The expansion and shear scalar are infinite at $T = 0$and decreases with the increases in cosmic time. Thus, the universe starts evolving with the zero volume at the initial epoch with infinite rate of expansion which slows down for the later times of the universe.

The anisotropy parameter of the expansion $\lim_{T\to\infty}\left(\frac{\sigma}{\theta}\right) \neq 0$ is found to be constant. Thus, the model does not approach to isotropy for the future evolution of the universe.

It can be observed that the spatial volume increases as T increases while the scalar of expansionϑ and the shear scalar σ^2 decreases .At the initial epoch, the physical quantitiesϑ , σ^2 , $\rho_{WDF} P_{WDF}$diverge. As$T \longrightarrow \infty$ $\boldsymbol{\vartheta}$, σ^2, ρ_{WDF} ,P_{WDF} vanishes. The model (21) has no initial singularity. The scalar field in the model vanishes as $T \longrightarrow \infty$.

Conclusions :

In discussing the large scale structure and behavior of the universe, spatially homogeneous and anisotropic cosmological model play a vital rule in the frame work of general relativity. Also, it is well know that scalar field and magnetized bulk viscosity have significant role in getting an accelerated universe . Here we have studied Kantowski –Sachs magnetized bulk viscous wet dark fluid in scalar tensor theory. To be able to obtain a more general we assume that the coefficient of bulk is inversely proportional to the expansion i.e. $\xi\vartheta = Const\ M,$ and expansion is proportional to shear i.e. $\vartheta\ \alpha\ \sigma$

The presence of a magnetic field affect energy density, pressure (P) co-efficient of bulk viscosity $\boldsymbol{\xi}$, scalar field $\boldsymbol{\phi}$ and expansion as well as acceleration of universe. The bulk viscosity plays a greater role in the evolution of the properties of the model also helps us to get a universe with accelerated expansion. Hence the presence of bulk viscosity should be taken into account in the description of the universe.

References

1) Saez D., Ballester V.J.: “A simple coupling with cosmological implications”, Phys. Lett. A ,113, 9467-470 (1986)
2) Tripathy S.K. , Nayak S.K. , Routray T.R.:”Bulk viscous string cosmological models with electromagnetic field”, Astrophys. and Space sci. ,321, 247-252 (2009)
3) Tripathy S.K., Behera D. , Routray T.R.:” Anisotropic universe with cosmic strings and bulk viscosity”, Astrophys. and Space sci., 93, 325 (2010)
4) Rao V.U.M, Devi Kumar G.S., Sireesha K.U.S.:” Anisotropic universe with cosmic strings and bulk viscosity in a scalar- tensor theory of gravitation”, Astrophys. and Space sci., 332 (2). 635-638, (2011).
5) Brans G. and Dicke R.H.:” Mach’s Principle and a relativistic theory of gravitation”, Phys. Rev. ,124, 925 (1961).
6) Narvedt K. Jr.: “ Post- Newtonian metric for a general class of scalar-tensor gravitational observational consequences”: Astrophys. J. 161, 1069 (1970).
7) Reddy D.R.K, Rao M.V. :”Axially symmetric cosmic strings and domain walls in Lyra geometry” ,Astrophys. and Space sci.,302 (1), 157-160, (2006)
8) Yadav M.K , Anirudh Pradhan, Singh S.K. :” Some magnetized bulk viscous string cosmological models in general theory “, Astro . space Sci., 311, 423-429 (2007)
9) Anirudh Pradhan, Singh S.K. :” Some exact Bianchi type –V Cosmological models in scalar tensor theory : kinematic tes “, J. theory Phys 24, 407-424 (2010).

10) Singh C.P .:"Some exact solution of magnetized viscous models in string cosmology", India-Academy of Pramana journal of Science, 83(1), 63-81 (2014).

11) Das K., Nawsad Ali: "Magnetized Kantowski – Sachs bulk viscous string cosmological models with decaying vacuum energy density lambda (t)", Turkish Journal of physics 38(2) ,193-202(2014).

12) Kandalkar S.P. , Wasnik A.P., Gaikwad M.N. "Cosmic string in Bianchi type –III space time with bulk viscosity of magnetic field", Int. jour. of Astronomy. &Astrophy . 2(4)(2012).

13) A.G. Riess et. al.:" Observational Evidence from supernovae for an accelerating universe and cosmological constant ", Astronomical Journal, , 116(3), 1009 (1998).

14) Tegmark M. et. al. :" Cosmological parameters from SDSS and WMAP", Phys. Rev. D69, 103501 (2004)

15) Dewri M.:" A magnetized dark energy type R/W model with polytropic equation of state in Brans –Dicke theory ", Journal of scientific Research 12(3) ,251-257 (2020)

16) Anil Kumar yadav, Farook Rahman Saibal Ray, Goswami G.K.:"Magnetized dark energy & the late time acceleration ",General of physics 1204 . 0223(2012).

17) Axially magnetized dark energy cosmological model (val, 34, No.27, 1950217 (2019) B. Mishara, Pratik P . Ray , S.K. Tripathy and KazuhruBamba (Modern Physics lett .A)

18) Vinutha T., Rao VUM,. Mangesh M:" Anisotropic dark energy cosmological model with cosmic strings", Canadian Journal of Physics, 0306(2020).

19) Vidya Santhi M., Rao V U M , Aditya Y.:" Anisotropic magnetized holographic Ricci dark energy cosmological model", Canadian Journal of physics,0781 (2017).

20) Singh T. and Agrawal A. K.:" some Bianchi- Type cosmological models in a new scalar tensor theory", Astrophys. and space sci. ,182, 289-312(1991)

21) Shri Ram, Tiwari S.K., :" Inhomogeneous plane symmetric models in a scalar tensor theory", Astrophys. and space sci. 259, 91-97 (1998)

22) Reddy DRK, Naidu R.L.: " Five dimensional string cosmological models in a scalar tensor theory of gravitation", 307(4), 395-398 (2007).

23) Rao VUM, Vinutha T. Shanthi M.V. :" An exact Bianchi type- V cosmological model in Saez-Ballester theory of gravitation", 312(3), 189-191 (2007)