



Performance Analysis of the SFBC OFDM over Underwater Acoustic Communication Channels

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Abstract- An investigation on κ - μ shadowed fading model for space frequency block coding (SFBC) orthogonal frequency division multiplexing (OFDM) system are presented. The model is opted to reduce the losses that occurs in the signals, and improve the strength of the signals mainly in the aquatic environment. This model shows a higher efficiency in the underwater communication channels. In this paper, the exact and approximate expressions of average bit error rate (ABER) for uncoded and SFBC coded OFDM schemes are derived. The derived expressions are presented in the terms of integral form, closed form and infinite series form. Different multipath fading parameter, shadowing parameter, modulation level, transmitting and receiving antennas values are used to illustrate the graph. The graph is between average error bit rate and average SNR. The expressions of ABER for uncoded and SFBC coded OFDM over κ - μ shadowed fading channels are derived using MGF based method. For uncoded OFDM system, both exact and approximate ABER expressions are derived which is presented in infinite series and closed form. Further, the ABER results are compared with previous existing results, as special cases. In last the performance comparison for M-ary modulation schemes is illustrated for different values of modulation levels with κ - μ shadowed fading model parameters.

Keywords: OFDM, SFBC, Shadowed Fading, ABER, SNR

1. Introduction

Under water acoustic communication (UWAC) is the emerging area of communication technologies. The signals under water suffer many losses such as absorption attenuation loss, Doppler effect due to continuous flow of water and scattering loss [1-3]. The κ - μ shadowed is mostly used and acceptable fading model for UWAC system [4, 5]. In recent era multiple-input multiple-output (MIMO) OFDM system has achieved a significant improvement in signal reliability and data rates. Various types of block coding techniques have been used in implementation of the MIMO-OFDM system, such as space frequency block coding (SFBC), space time block coding (STBC) and may others. One of them widely used is SFBC, which is generally followed and accepted by researchers and scientist [6-8].

SFBC-OFDM system performance in the term of bit error rate (BER) over Rayleigh fading environment with and without receiving the information about the state of the channel was evaluated in [7]. The exact and approximate ABER analysis of uncoded and SFBC coded OFDM system for κ - μ /gamma and Fisher Snedecor shadowed fading model were presented in [8]. The MGF based ABER analysis over κ - μ and η - μ fading was presented in [9]. In [10], Discrete Fourier transform (DFT) precoded MIMO OFDM for underwater acoustic communication system was presented. The author investigated the transceiver techniques and performance evaluation with 16 QAM modulation [10]. The BER analysis of Fractional Fourier Transform (FrFT)-OFDM for frequency selective Rician fading channels over under water Acoustic communication systems was performed in [11]. In [12], the expressions of ABER for unzipped and SFBC OFDM system over TWDP fading channels were obtained. The Generalised MGF based error rate performance of SFBC OFDM system for Beaulieu- Xiefading model was evaluated in [13]. The analysis of BER using different fading models such as Rayleigh, Nakagami-q, Nakagami-n, Nakagami-m, TWDP, Beckmann, Generalised-K and η - λ - μ fading for SFBC-OFDM system were evaluated in [13].

The performance of SFBC OFDM for κ - μ shadowed fading model is not given in any open literature. The κ - μ shadowed fading model is simple and has closed form solution. The κ - μ shadowed fading model is the mixture of κ - μ and Nakagami-m distribution. In this, the mean of κ - μ distribution is randomly fluctuated by Nakagami-m shadowing. This shadowed fading model is the generalization of Rician shadowed fading model [4].

In this paper, the expressions of ABER for uncoded and SFBC coded OFDM over κ - μ shadowed fading channels are derived using MGF based method. For uncoded OFDM system, both exact and approximate ABER expressions are derived which is presented in infinite series and closed form, respectively. For SFBC OFDM system, the exact ABER expression is presented in integral form however, the approximate ABER expression is obtained in closed form. The outcome of ABER are presented for MQAM and MPSK

modulation schemes with various multipath fading parameter, shadowing parameter, and modulation levels. The proposed ABER results reduces to results of Rayleigh and Nakagami-m fading model as specific case.

2 K- μ Shadowed Fading Model

The K- μ shadowed fading model is the composition of κ - μ and Nakagami-m distribution. κ - μ is the multipath fading model where as Nakagami-m is characterised as shadowing. This model is adequate for underwater acoustic communication system [4].

$$f_{\gamma}(\gamma) = \frac{\mu^{\mu} m^m (1+\kappa)^{\mu}}{\Gamma(\mu) \bar{\gamma} (\mu\kappa + m)^m} \left(\frac{\gamma}{\bar{\gamma}}\right)^{\mu-1} \exp\left(-\frac{\mu(1+\kappa)\gamma}{\bar{\gamma}}\right) {}_1F_1\left(m, \mu, \frac{\mu^2 \kappa (1+\kappa) \gamma}{\mu\kappa + m \bar{\gamma}}\right) \quad (1)$$

where m and μ are the shadowing and multipath fading parameters. The κ parameter is defined as $\kappa = d^2/2\sigma^2\mu$ [4]. The expression of MGF in the term of SNR is formulated as [4]

$$M_{\gamma}(s) = \frac{(-\mu)^{\mu} m^m (1+\kappa)^{\mu}}{\bar{\gamma}^{\mu} (\mu\kappa + m)^m} \frac{\left(s - \frac{\mu(1+\kappa)}{\bar{\gamma}}\right)^{m-\mu}}{\left(s - \frac{\mu(1+\kappa)}{\bar{\gamma}} - \frac{m}{\mu\kappa + m}\right)^m} \quad (2)$$

3 Average Bit Error Rate

BER is an important parameter in analyzing the performance of any wireless system. during the process of transmission of any information or data over any telecommunication or wireless link from one end to another, the key parameter to optimize that how many error in the data appeared at the remote end is justified by BER.

3.1 ABER for Uncoded OFDM

Exact Analysis:

The ABER for uncoded OFDM system over any fading channel is given as [7]

$$P_e(E) = A \int_0^{\infty} \text{erfc}(\sqrt{B\gamma}) f_{\gamma}(\gamma) d\gamma \quad (3)$$

where, $A = (2/\log_2(M)) \cdot (1 - 1/\sqrt{M})$, $B = 1.5/(M-1)$ for MQAM and $A = 1/\log_2(M)$, $B = \sin^2(\pi/M)$ for MPSK.

$\gamma = |H[k]|^2 E_s / N_0$ is the instantaneous SNR, E_s is the symbol energy at the transmitter and N_0 is the variance of real/imaginary part of AWGN [7]. $\text{erfc}(\cdot)$ is the complementary error function [15] and M is the modulation level.

By putting (1) into (3), we get

$$P_e(E) = A \int_0^{\infty} \text{erfc}(\sqrt{B\gamma}) \frac{\mu^{\mu} m^m (1+\kappa)^{\mu}}{\Gamma(\mu) \bar{\gamma} (\mu\kappa + m)^m} \left(\frac{\gamma}{\bar{\gamma}}\right)^{\mu-1} \exp\left(-\frac{\mu(1+\kappa)\gamma}{\bar{\gamma}}\right) {}_1F_1\left(m, \mu, \frac{\mu^2 \kappa (1+\kappa) \gamma}{\mu\kappa + m \bar{\gamma}}\right) d\gamma \quad (4)$$

Using summation formula of exponential function [15] the above equation can be written as

$$P_e(E) = A \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(\mu) n!} \left(\frac{m}{\mu\kappa + m}\right)^m \left(\frac{\mu(1+\kappa)}{\bar{\gamma}}\right)^{\mu+n} \int_0^{\infty} \gamma^{\mu+n-1} \text{erfc}(\sqrt{B\gamma}) {}_1F_1\left(m, \mu, \frac{\mu^2 \kappa (1+\kappa) \gamma}{\mu\kappa + m \bar{\gamma}}\right) d\gamma \quad (5)$$

Using [15] and [16], after some mathematical calculations, we get the final expression as

$$P_e(E) = \frac{A}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(m) n!} \left(\frac{m}{\mu\kappa + m}\right)^m \left(\frac{\mu(1+\kappa)}{B\bar{\gamma}}\right)^{\mu+n} G_{3,3}^{1,3} \left(\begin{matrix} \frac{\mu^2 \kappa (1+\kappa)}{B(\mu\kappa + m)\bar{\gamma}} \\ 1-m, 1-\mu-n, \frac{1}{2}-\mu-n \\ 0, -\mu-n, 1-\mu \end{matrix} \right) \quad (6)$$

Approximate Analysis:

The approximate ABER expression over any fading channel is given as [7, 8]

$$P_e(E) = C \int_0^{\infty} \exp(-D\gamma) f_{\gamma}(\gamma) d\gamma \quad (7)$$

where, $C = 0.2$, $D = 1.6/(M-1)$ for MQAM, and $C = 0.2$ and $D = 7/(2^{1.91\log_2 M} + 1)$ for MPSK. Eq.(7) is the definition of MGF [27, Eq.(1.2)], therefore (7) can be expressed as

$$P_e(E) = C M_{\gamma}(D) \quad (8)$$

where $M(\cdot)$ is the MGF. The MGF of κ - μ shadowed fading model is given in (2). Thus by substituting (2) into (8), we can obtain the approximate closed form expression of ABER for uncoded OFDM system. It is noted that equation (6) is the exact expression of ABER for uncoded OFDM system which is presented in infinite series format.

3.2 ABER for SFBC OFDM

Exact Analysis:

Let's consider the SFBC-OFDM system having N_T transmit and N_R receive antennas. The normalized instantaneous SNR is given by [7, Eq. (30)]

$$\gamma = \frac{1}{N_T R_c} \sum_{j=1}^{N_R} \sum_{i=1}^{N_T} |H_{j,i}[k]|^2 E_s / N_0 = \frac{1}{N_T R_c} \sum_{j=1}^{N_R} \sum_{i=1}^{N_T} \gamma_{j,i} \quad (9)$$

where R_c is the code rate of SFBC-OFDM system, $H_{j,i}[k]$ is the k^{th} subchannel with i^{th} transmit and j^{th} receive antennas. Thus, the ABER for SFBC-OFDM system can be expressed as [7,Eq.(33)]

$$P_e(E) = A \int_0^\infty \dots \int_0^\infty \text{erfc} \left(\sqrt{\frac{B \sum_{j=1}^{N_R} \sum_{i=1}^{N_T} \gamma_{j,i}}{R_c N_T}} \right) f_{\gamma_{1,1}}(\gamma_{1,1}) \dots f_{\gamma_{N_R, N_T}}(\gamma_{N_R, N_T}) d\gamma_{1,1} \dots d\gamma_{N_R, N_T} \quad (10)$$

Complementary error function can be expressed in integral form as [27, Eq. (4A.6)]

$$\text{erfc}(x) = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{x^2}{2 \sin^2 \theta}\right) d\theta \quad (11)$$

By plugging (11) into (10), we obtain

$$P_e(E) = \frac{2A}{\pi} \int_0^\infty \dots \int_0^\infty \int_0^{\frac{\pi}{2}} \exp\left(-\frac{B \sum_{j=1}^{N_R} \sum_{i=1}^{N_T} \gamma_{j,i}}{2 R_c N_T \sin^2 \theta}\right) f_{\gamma_{1,1}}(\gamma_{1,1}) \dots f_{\gamma_{N_R, N_T}}(\gamma_{N_R, N_T}) d\gamma_{1,1} \dots d\gamma_{N_R, N_T} d\theta \quad (12)$$

On expanding the summation term inside the exponential function and by using the property of exponential function, (12) can be written as,

$$P_e(E) = \frac{2A}{\pi} \int_0^{\frac{\pi}{2}} \left[\int_0^\infty \exp\left(-\frac{B \gamma_{1,1}}{2 R_c N_T \sin^2 \theta}\right) f_{\gamma_{1,1}}(\gamma_{1,1}) d\gamma_{1,1} \dots \int_0^\infty \exp\left(-\frac{B \gamma_{N_R, N_T}}{2 R_c N_T \sin^2 \theta}\right) f_{\gamma_{N_R, N_T}}(\gamma_{N_R, N_T}) d\gamma_{N_R, N_T} \right] d\theta \quad (13)$$

In (13), the inner integral is the definition of MGF, therefore (13) can be express in MGF form as

$$P_e(E) = \frac{2A}{\pi} \int_0^{\frac{\pi}{2}} M_{\gamma_{1,1}}\left(-\frac{B}{2 R_c N_T \sin^2 \theta}\right) \dots M_{\gamma_{N_R, N_T}}\left(-\frac{B}{2 R_c N_T \sin^2 \theta}\right) d\theta \quad (14)$$

In a simplified way, (14) can be written as,

$$P_e(E) = \frac{2A}{\pi} \int_0^{\frac{\pi}{2}} \prod_{j=1}^{N_R} \prod_{i=1}^{N_T} M_{\gamma_{j,i}}\left(-\frac{B}{2 R_c N_T \sin^2 \theta}\right) d\theta \quad (15)$$

For the independent identically distributed fading channel, (15) becomes

$$P_e(E) = \frac{2A}{\pi} \int_0^{\frac{\pi}{2}} \left(M_\gamma\left(-\frac{B}{2 R_c N_T \sin^2 \theta}\right) \right)^{N_R N_T} d\theta \quad (16)$$

By substituting (2) into (16), we get

$$P_e(E) = \frac{2A}{\pi} \frac{(-\mu)^{\mu N_R N_T} m^{m N_R N_T} (1+\kappa)^{\mu N_R N_T}}{\bar{\gamma}^{\mu N_R N_T} (\mu\kappa + m)^{m N_R N_T}} \int_0^{\frac{\pi}{2}} \left(\frac{B}{2 R_c N_T \sin^2 \theta} - \frac{\mu(1+\kappa)}{\bar{\gamma}} \right)^{(m-\mu) N_R N_T} \left(\frac{B}{2 R_c N_T \sin^2 \theta} - \frac{\mu(1+\kappa)}{\bar{\gamma}} - \frac{m}{\mu\kappa + m} \right)^{m N_R N_T} d\theta \quad (17)$$

This is the exact ABER expression for SFBC-OFDM system presented in finite integral form. This expression can be solved using some mathematical simulation tools like Mathematica and Maple. Here $M_\gamma(\cdot)$ is the MGF of instantaneous SNR over any fading channel.

Approximate Analysis:

An approximate expression of ABER for SFBC-OFDM system over AWGN channel is given as [7]

$$P_e(E) = C \int_0^\infty \dots \int_0^\infty \exp\left(-\frac{D \sum_{j=1}^{N_R} \sum_{i=1}^{N_T} \gamma_{j,i}}{R_c N_T}\right) f_{\gamma_{1,1}}(\gamma_{1,1}) \dots f_{\gamma_{N_R, N_T}}(\gamma_{N_R, N_T}) d\gamma_{1,1} \dots d\gamma_{N_R, N_T} \quad (18)$$

By following the similar steps from (12) to (15), (18) can be written as,

$$P_e(E) = C \prod_{j=1}^{N_R} \prod_{i=1}^{N_T} M_{\gamma_{j,i}}\left(\frac{D}{R_c N_T}\right) \quad (19)$$

For the independent identically distributed channel

$$P_e(E) = C \left[M_\gamma \left(\frac{D}{R_c N_T} \right) \right]^{N_R N_T} \quad (20)$$

By putting (2) into (20), we get the closed form approximate ABER expression as

$$P_e(E) = \frac{C(-\mu)^{\mu N_R N_T} m^{m N_R N_T} (1+\kappa)^{\mu N_R N_T}}{\bar{\gamma}^{\mu N_R N_T} (\mu\kappa + m)^{m N_R N_T}} \frac{\left(\frac{D}{R_c N_T} - \frac{\mu(1+\kappa)}{\bar{\gamma}} \right)^{(m-\mu) N_R N_T}}{\left(\frac{D}{R_c N_T} - \frac{\mu(1+\kappa)}{\bar{\gamma}} - \frac{m}{\mu\kappa + m} \right)^{m N_R N_T}} \quad (21)$$

4. Numerical Results

In this Section the analysis of error probability are discussed for uncoded and SFBC OFDM system over κ - μ shadowed fading channels. Generally, MQAM and MPSK modulation schemes are used to analyse the BER. The outcomes are reduced to special cases by setting different fading and shadowing parameters.

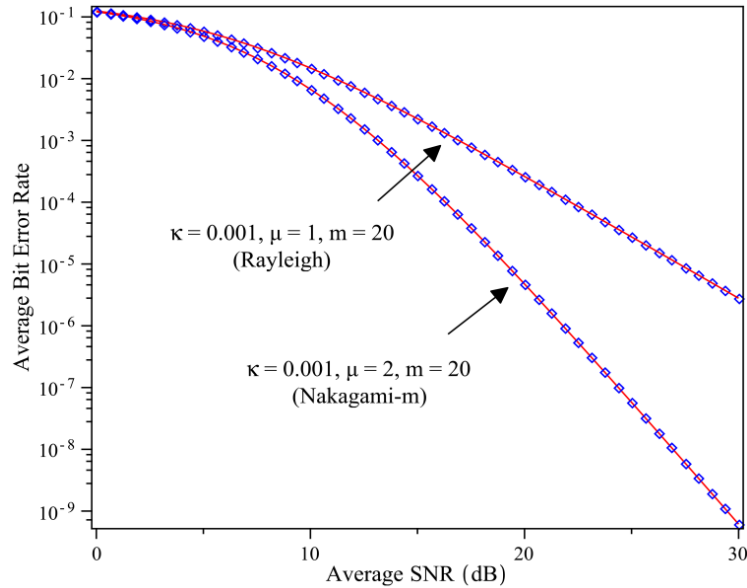


Fig. 1 ABER of 4 QAM SFBC OFDM for $\mu = 1, 2$ with $\kappa = 0.001$, $m = 20$, $N_T = 2$, $N_R = 1$.

Fig.1 shows the the plot of error rate versus average SNR for $\mu = 1, 2$ with $\kappa \rightarrow 0$, $m = 20$, $N_T = 2$, and $N_R = 1$. The results are compared with corresponding results of Rayleigh and Nakagami-m distribution. By putting $\mu = 1$, $\kappa = 0.001$, and $m = 20$, the ABER of κ - μ shadowed fading model reduced to ABER of Rayleigh distribution [7, Eq. 22]. In similar way, with $\mu = 2$, $\kappa = 0.001$, and $m = 20$, (21) reduces to ABER of Nakagami-m fading [14]. Moreover, on increasing μ (1 to 2), the error rate decreases because of decrease in fading severity. It is clear in Fig.1 that, a sharp reduction in error probability has been achieved by varying μ .

Fig.2 shows the plot of ABER for MQAM with $\kappa \rightarrow 0$, $\mu = 2$, and $m = 20$. Two transmitting and one receiving antenna (code rate $R_c = 1$) are used to plot the graph of error rate under SFBC OFDM system. It is clear in Fig. 2 that the ABER increases as M varying between 2 and 16.

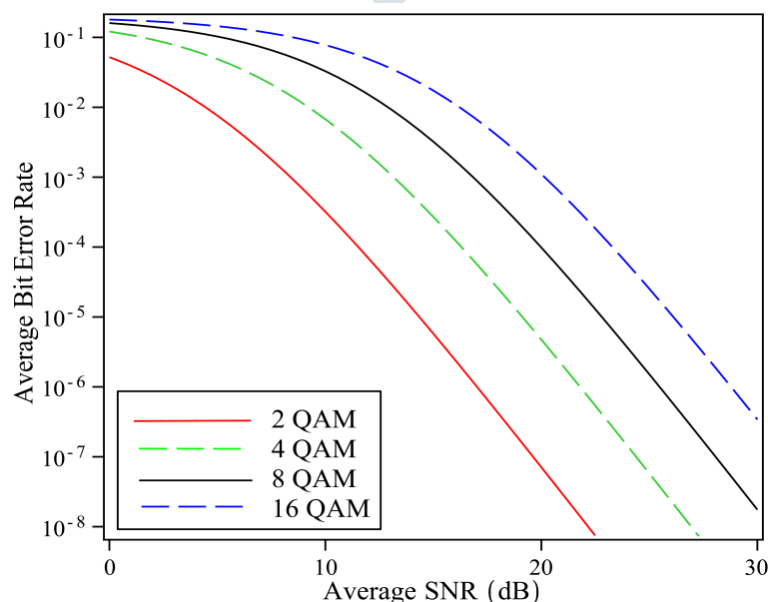


Fig.2 ABER of MQAM (M = 2, 4, 8, 16) for SFBC OFDM with $\kappa = 0.001$, $\mu = 2$, $m = 20$, $M_T = 2$, $M_R = 1$.

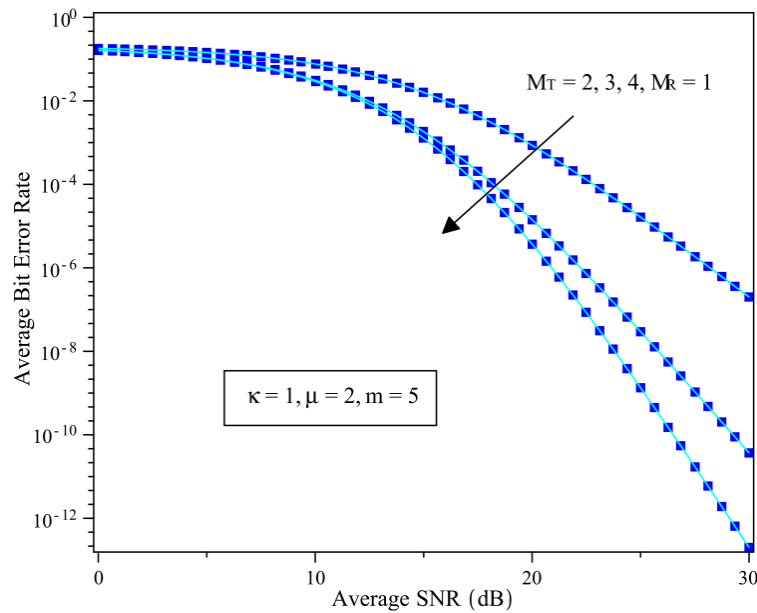


Fig. 3 ABER of 16 QAM-SFBC-OFDM for different antenna configuration ($M_T = 2, 3, 4, M_R = 1$) with fixed κ, μ, m .

Fig. 3 depicts the ABER for 16 QAM modulation scheme by varying the number of transmitting antenna M_T (2 to 4) with fixed number of receiving antenna $M_R = 1$, $\kappa = 1$, $\mu = 2$, and $m = 5$ under SFBC OFDM system. We have plot the BER curve for 2Tx-1Rx antennas ($R_c = 1$), 3Tx-1Rx antennas ($R_c = 0.5$) and 4Tx-1Rx antennas ($R_c = 1$), where R_c is the code rate. As observed in Fig.3 that the large number of transmitting antenna setup provides minimum error probability as compared to smaller one.

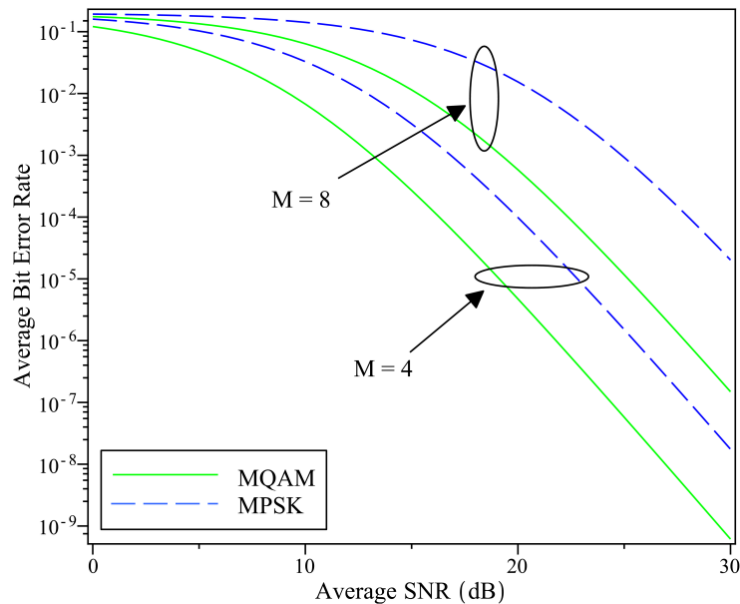


Fig. 4 ABER of MQAM and MPSK with ($M = 4, 8$) for $\kappa = 0.001, \mu = 2, m = 20, M_T = 2$ and $M_R = 1$ under SFBC OFDM.

Fig. 4 shows the comparison of curves between ABER versus average SNR of MQAM and MPSK ($M = 4, 8$) for SFBC OFDM system with $\kappa = 0.001, \mu = 2, m = 20, M_T = 2$ and $M_R = 1$. In Fig. 4, 4 QAM (or 4 PSK) modulation scheme shows better error rate performance in comparison with 8 QAM (or 8 PSK) modulation scheme. Moreover, it is also clear that the error rate for MQAM is lower as compared to MPSK scheme, as shown in Fig. 4.

5. Conclusions

We have derived the ABER expressions for MQAM/MPSK modulations schemes for SFBC OFDM system over κ - μ shadowed fading model. The exact and approximate analysis of the ABER shows the closeness of the improved system for high performance. We considered here the FFT method to formulate the closed form expression. We have observed in the reduction of parameter by the graphical representation. The proposed expressions have been represented in the terms of MGF. The new proposed numerical results have been compared with previous results, as special case. It has been observed that, in case of light shadowing, the proposed ABER results reduced to results of Rayleigh and Nakagami fading. Moreover, on increasing the number of multipath clusters, sharp improvement in error rate performance have been seen, which shows the improved efficiency, reliability of the system.

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