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g**β - CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract

Topology is an area of mathematics concerned with the properties of space that are preserved under continuous deformations including stretching and bending bur not tearing.

After the work of **Levine** in 1970, on generalized closed sets (briefly g-closed sets), several mathematicians turned attention to the various forms of generalized closed sets in Topology. Extensive research on generalizing closedness was done in recent years as notions of semi-generalized, generalized semi-closed , generalized **α** - closed, **δ** – generalized closed sets are investigated.

The aim of this paper is to introduce the concept of **g**β** – closed sets and investigate their basic properties. We also discuss their relationship with already existing concepts.

Keywords : **g**β** closed , **g*** closed, generalized semi- closed sets (briefly gs-closed), an generalized semi-pre closed set (briefly gsp-closed).

I . INTRODUCTION

Levine introduced the class of semi open sets in 1963 [11] and g-closed sets [12] in 1970. M.K.R.S.Veerakumar [1] introduced g* - closed sets in 1991. P.M.Helen [9] introduced g** - closed sets . H.Sujitha [6] introduced g*β- closed sets. We introduce a new class of sets called g**β – closed sets, which is properly placed in between the class of closed sets and the class of gs – closed sets.

II. PRELIMINARIES

Throughout this paper (X,τ) represent the non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl (A) and int (A) denote the closure and interior of A respectively.

A.Definition

A subset A of a topological space (X,τ) is called

1) a pre-open set [14] is A ⊆ (int cl (A)) and (cl (A)) and a pre –closed set if cl (int (A)) $\subseteq A$

2) a semi-open set [11] if A⊆ cl (int (A) and semi-closed set if int (cl (A)) ⊆ A
3) a semi –pre open set [10] if A⊆ cl (int (cl(A)) and a semi pre-closed set [10] if int (cl (int (A)) ⊆A
4) an X-open set [5] if A⊆ int (cl (int (A))) and an x-closed set if cl (int (cl (A))) ⊆ A
5) a regular-open set [4] if int (cl (A)) = A and regular –closed set if A= cl(int(A))

B. Definition

A subset A of a topological space (X, τ) is called

i) generalized closed set (briefly g-closed) [12] if cl(A) ⊆ U whenever A ⊆ U is open in (X,τ)
ii) g*- closed if [1] cl(A) ⊆ U whenever A ⊆ U and U is g* in (X, τ)
iii) genera;ozed semi-closed set (briefly gs –closed [13] is scl (A) ⊆ U whenever A ⊆ U and U is open in (X, τ)
iv) an generalized semi pre-closed set (briefly gsp-closed [3] if spcl (A) ⊆ U whenever A ⊆U and U is open in (X, τ)
v) regular generalized closed set (briefly rg-closed) [7] if cl (A) ⊆ U whenever A ⊆U and regular open in (X, τ)
vi) g** -closed [9] if cl(A) ⊆U whenever A ⊆U and U is open in (X, τ)
vii) generalized pre regular –closed sett (briefly gpr –closed) [4] if pcl (A) ⊆ U whenever A ⊆U and U is regular open in (X, τ)
viii) generalized pre –closed set (briefly gp-closed [5] if pcl(A) whenever A⊆U and U is open in (X, τ)
ix) g*β-closed [6] if βcl (A) ⊆ U whenever A ⊆U and U is g*-open in X
x) semi generalized closed (briefly sg-closed) [2] if scl(A) whenever A⊆U and U is semi-open in (X, τ)

C. Notations used:

i) gcl(A) – generalized closure of A

ii) scl (A)- semi closure of A

iii) pcl(A) – pre-closure of A

III. BASIC PROPERTIES OF G**β - CLOSED SETS

We now introduce the following definitions

Definition 3.1

A subset A of (X, τ) is said to be g**β - closed set if gcl (A)⊆U whenever A⊆U and U is g*β - open X. The complement of g**β - closed set is g**β - open. The collection of all g**β - closed and g**β - open sets are denoted G**β - C(X) and G**β - O(X).

Theorem 3.2

Every closed set is g**β-closed.

Proof :- Let A be a closed set in X. Let A⊆U and U is g*β - open .Since A is closed ,cl(A)⊆U

But gcl(A)⊆cl(A)⊆U⇒gcl(A)⊆ U . Hence A is g**β-closed.

Remark 3.3

Example 3.4 shows that the converse of the above theorem is not true.

Example 3.4

Let X={a,b,c} , τ = {Ø,X,{a}}, then A ={b} is g**β- closed but not closed set

Theorem 3.5

Every g* - closed set is g**β closed

Proof:- Let A be a g* - closed set X . Let A⊆U and U is g*β - open. But all g*β - open sets are g - open sets. Thus, A⊆U and U is g*β - open⇒ cl(A)⊆U (∵ A is g* closed set) But gcl(A)⊆ cl(A) ⊆U ⇒ gcl (A)⊆U . Hence A is g**β - closed.

Remark 3.6

Example 3.7 shows that the converse of the above theorem is not true

Example 3.7

Let X = {a, b, c} with τ = {Ø,X,{a, b}} then A {a, c} is g**β - closed but not a g* - closed

Proposition 3.8

Every α -closed set is g** β - closed

Proof follows from the definition

Proposition 3.9

Every g** - closed sett is g** β - closed

Proof follows from the definition

Proposition 3.10

Every g - closed set is g**β - closed

Proof follows from the definition

Proposition 3.11

Every β - closed set is g** - closed

Proof follows from the definition

Example 3.12

Let X = {a,b,c} ,τ = {Ø,X,{a, b}} then A = {b} is g**β - closed but not α - closed.

Example 3.13

Let X = {a, b, c,d}, τ = {Ø,X,{a},{b} {a,b} {a,b,c}} Let A = {c} is a g**β - closed set but not a g** - closed set . So the class of g**β - closed sets properly contains the class of closed sets and the class of g** - closed sets. Also {c} is not a g - closed set.

Example 3.14

Let X = {a, b, c}, τ = {Ø,X,{a}}. Then A = {a,b} is a g**β - closed set but not a β - closed .

Theorem 3.15

If A and B are g** - closed sets, then A∪B is also a g**β - closed in X.

Proof :-

Let A and B are g** β - closed set in X. Then A⊆U , U is g*β - open set and B⊆U, U is a g*β - open. Hence, gcl (A)⊆U and gcl (B)⊆U . Therefore, gcl(A∪B)⊆ gcl (A) ∪gcl (B)⊆U⇒gcl(A∪B)⊆U, U is a g*β - open set in X. Hence A∪B is also a g** β - closed in X.

Theorem 3.16

If A and B are g**β-closed sets then A∩B is also a g**β-closed in X.

Proof:- Let A and B are g**β-closed sets in X. Then A⊆U , U is a g*β - open set and B⊆U, U is a g*β – open set . Hence, gcl (A)⊆ U and gcl (B)⊆ U . Therefore, gcl(A∩B)⊆ gcl (A) ∩ gcl (B)⊆ U⇒gcl(A∩B)⊆ U, U is a g*β - open set in X. Hence A∩B is also a g** β - closed in X

Proposition 3.17

Every g**β-closed set is

(1)rg-closed

(2)gp-closed

(3)gpr-closed

(4)gsp-closed

(5)wg-closed

Proof follows from the definition

The converse of the above theorem need not be true .Refer below example

Example 3.18

Let X={a,b,c,d} , τ = {Ø,X,{a}, {a,b}, {c,d} ,{a,c,d }} , then A={b,c} is gpr – closed set and rg- closed set but not g*β- closed set

Example 3.19

Let X ={a,b,c,d}, τ ={ Ø,X,{a},{c},{d},{a,c},{c,d},{a,c,d},{a,d}} then A={a,b,c} is a gp-closed ,wg- closed and a gsp- closed set but a g**β-closed set of (X, τ)

Remark 3.20

The Above results and examples are summarized as follows A→B represent A implies B and A→B represent A does not implies B.

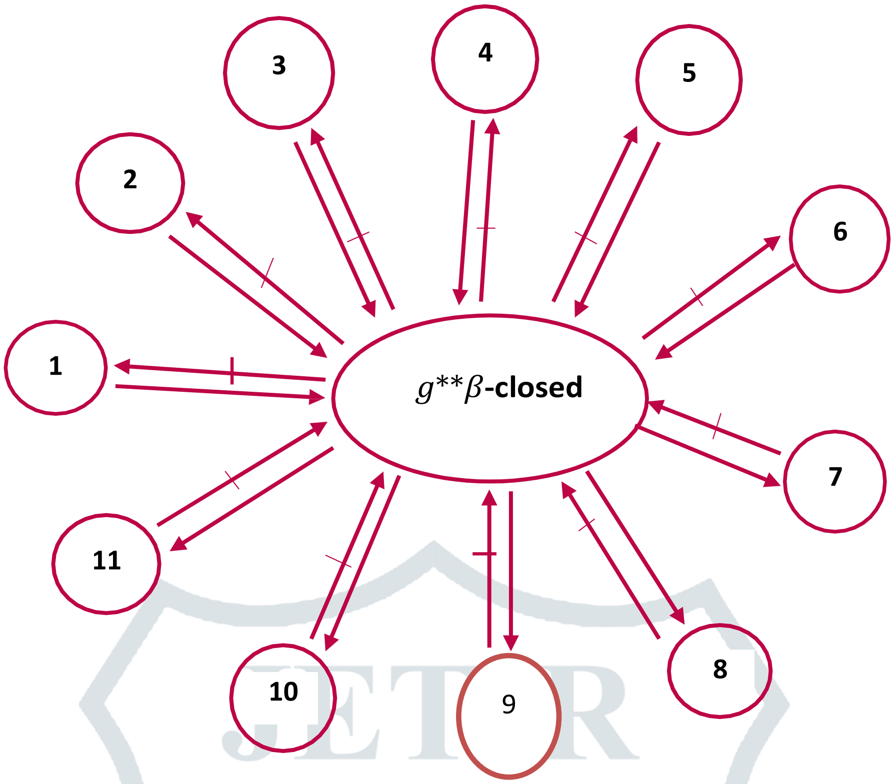

Figure .1.1 Relation between g** β - closed set with other known existing sets

1. Closed set 2. β- closed 3.α- closed 4.g- closed 5. g* - closed 6.g**- closed 7. rg - closed 8. gp - closed 9.wg-closed 10. gpr- closed 11.gsp - closed

Therorem 3.21

If A and B are two g**β-closed sets in a Topological space X such that either A⊆B or B⊆A then both intersection and union of two g**β-closed set is g**β-closed set.

Proof :- If A⊆B or B⊆A then A∪ B=B or A ∪ B=A respectively.Therefore A ∪ B is g**β-closed as A and B are g**β-closed sets. Similarly , A∩B is also a g**β-closed sets

Remark 3.22

Difference of two g**β-closed sets is not a g**β-closed set.

Let X ={a,b,c,d} , τ ={ Ø,X,{1},{2},{1,2},{1,2,3} }.Here A={1,2,4} and B={2,4} are g**β-closed sets but A-B= {a} is not.

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