JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

Visco-Elastic Flow Past an Infinite Flat Plate in Slip Flow Regime

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ABSTRACT

In this paper flow of an elastico-viscous fluid on a porous flat plate is considered with slip conditions of velocity and temperature given by street (14). To overcome the singularity, expressions for temperature distribution and Nusselt number are obtained at Prandtl number equal to 2 and not equal to 2 separately. It is observed that for prandtl number one, Nusselt number increases for increasing visco-elastic parameter while for prandtl number equal to two the process reverses. Furthermore, for Prandtl number one, Nusselt number increases as velocity slip parameter increases while it decreases for temperature slip parameter. In case for prandtl number two, the process reverses for slip parameter for velocitly but not for slip parameter for temperature.

Keywords- elastico-viscous fluid, Nusselt number, Prandtl number, Newtonian hypothesis, Non-Newtonian fluid.

INTRODUCTION

Many fluids such as blood oils, paints, polymer solutions and materials of industrial importance exhibit both viscous and elastic properties. Therefore, the study of such fluids attracted the interest of many research workers. These types of fluids which show a distinct deviation from Newtonian hypothesis are known as non-Newtonian fluids in literature. A certain class of non-Newtonian fluids in which memory of the fluid has been taken into account through the stress relaxation time and the rate of retardation time, are known as visco-elastic oldroyd type fluids. Oldroyd (1,2) studied a set of constitutive equations for visco-elastic fluids and explained the rheological behavior of the fluids. The rheological equations of visco-elastic or elastic-viscous fluids as obtained by walter (3) (In the case of fluids with shore memories i.e. short relaxation times) are

$$p_{ik} = -p\, g_{ik} + p'_{ik} \quad (1)$$

$$p'_{ik} = 2\eta_o \; e^{(1)ik} - 2k_o \frac{\delta}{\delta t} \; e^{(1)\,ik} \quad (2)$$

Where p_{ik} is stress tensor, p an arbitrary isotropic pressure, g_{ik} metric tensor of a fixed co-ordinate system x^i, $e^{(1)ik}$ $(=\frac{1}{2}(v^{1}_{,k} + v^{k}_{,1})$ is the rate of strain tensor, $\eta_o = \int_o^\infty N(\tau)\, d\tau$ is the limiting viscosity at small rate of shear, $k_o = \int_o^\infty \tau N(\tau)\, d\tau$ and $\frac{\delta}{\delta\iota}$ denotes the convected differentiation of tensor quantity, which for any contravariant tensor is given as

$$\frac{\delta b^{ik}}{\delta t} = \frac{\partial b^{ik}}{\partial t} + v^m \frac{\partial b^{ik}}{\partial x^m} - b^{im}\frac{\partial v^k}{\partial x^m} - b^{mk}\frac{\partial v^i}{\partial x^m} \qquad (3)$$

Where ϑ^i is the velocity vector and N(τ) is the distribution function of relaxation times τ as defined by walter(4). This idealized model is a valid approxiamation of a walter fluid (model B') talking very short memory into account so that terms involving

$$\int_o^\infty \tau^n \, N(\tau)\, d\tau \qquad n \geq 2$$

Has been neglected as discussed by Beard and walter(5).

In an attempt sachet (6) obtained solution of temperature distribution of an elastic-viscous (walter's liquid B') fluid on a parallel plate. Raptis and Tzivanidis (7) solved the problem for constant suction and constant heat flux, Soundalgekar and Mirky (8), Shri Ram and Singh (9), singh (10) and many others have discussed the problems of this type of fluid. Since blood is electrically cosducting Singh and Sharma (11) discussed MHD flow of blood through a porous channel. They assumed Newtonian behavior of the blood but the pressure as a function of time.

At high altitude flights the study of slip flow becomes very important. Keeping this in mind soundalgekar and Aranake (12) , Jain (13) considered the slip flow boundary conditions in their problems.

In the present paper visco-elastic flow past an infinite flat plate is considered taking slip conditions for velocity field and for temperature field. Expressions for velocity and temperature distributions are obtained. Graphs of temperature distribution and Nesselt number are plotted for different values of Pr (prandtl number), K (visco-elastic parameter), R (Slip parameter for velocity) and S (slip parameter for temperature).

It is being observed that θ decreases with the increase of K,S and pr, and for Nusselt number, Increase in K increase Nu for pr = 1 while for pr = 2 process reverses.

FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

A steady two dimensional flow of a visco-elastic fluid past an infinite porous flat plate is considered in slip flow regime. The x′-axis is taken along the plate in the direction of flow and y′-axis is taken normal to the plate. If u′, v′ be the velocity components along x′ and y′ directions then the flow equations of and elastic-viscous fluid (Walters liquid B) governed by equations in Introduction reduce to

$$\frac{dv'}{dy'} = 0 \qquad (4)$$

$$\vartheta' \frac{du'}{dy'} = -\frac{1}{\rho}\frac{\partial \rho}{\partial x'} + \nu \frac{d^2u'}{dy'^2} - \frac{k_o}{\rho}\left(v'\frac{d^3u'}{dy'^3} - 3\frac{du'}{dy'}\frac{d^2v'}{dy'^2} - 2\frac{dv'}{dy'} - \frac{d^2v'}{dy'^2}\right) \qquad (5)$$

$$\vartheta' \frac{dv'}{dy'} = -\frac{1}{\rho}\frac{\partial \rho}{\partial x'} + \nu \frac{d^2v'}{dy'^2} - \frac{2k_o}{\rho}\left(v'\frac{d^3v'}{dy^3} - 3\frac{dv'}{dy'}\frac{d^2v'}{dy'^2}\right) \qquad (6)$$

$$\rho\, c_p \left(v'\frac{dT}{dy'}\right) = k\frac{d^2T}{dy'^2} + \mu\left(\frac{du'}{dy}\right)^2 - k_o\; v'\frac{du'}{dy'}\frac{d^2u'}{dy'} \qquad (7)$$

And the boundary conditions are [Street (14)]

$$\left.\begin{aligned} u' &= \frac{2-f_1}{f_1}\, L\, \frac{du'}{dy'} = L_1 \frac{du'}{dy'} \\ T - T_w &= \frac{2-f_2}{f_2}\, \frac{2\gamma}{\gamma+1}\, \frac{L}{P}\, \frac{dT}{dy'} = L_2 \frac{dT}{dy'} \\ u' &\to u_\infty,\ T \to T_\infty \text{ as } y' \to \infty. \end{aligned}\right\} \text{ at y=0} \qquad (8)$$

ν Is kinematic viscosity, T is temperature, K is thermal diffusivity and k_o is a quantity depending on elastic property of the fluid.

For constant suction, equation (4) is integrated to

$$\vartheta' = -v_o \qquad (9)$$

If pressure is constant thorough out the motion, equations (5) to (8) reduce to the following equations

$$-v_o \frac{du'}{dy'} = \nu \frac{d^2u'}{dy'^2} + \frac{k_o}{\rho}\ v_o \frac{d^3u'}{dy'^3} \qquad (10)$$

$$-\quad c_p v_o \frac{dT}{dy'} = \frac{k}{\rho}\ \frac{d^2T}{dy'^2} + \nu \left(\frac{du'}{dy'}\right) + \frac{k_o}{\rho} \frac{du'}{dy'} \frac{d^2u'}{dy'^2} \qquad (11)$$

With the boundary conditions,

$$\left.\begin{aligned} u' &= \frac{2-f_1}{f_1}\, L\, \frac{du'}{dy'} = L_1 \frac{du'}{dy'} \\ T - T_w &= \frac{2-f_2}{f_2}\, .\ \frac{2\gamma}{\gamma+1}\, \frac{L}{P}\, \frac{dT}{dy'} = L_2 \frac{dT}{dy'} \\ u' &\to u_\infty,\ T \to T_\infty \text{ as } y' \to \infty \end{aligned}\right\} y'=0 \qquad (12)$$

Let us introduce the following non-dimensional quantities

$u = u_\infty$ f(y),

$y = \frac{v_o y'}{\nu}$,

$\theta = \frac{T-T}{T_w - T_\infty}$,

$E = \frac{u_\infty^2}{C_{p(T_w - T_w)}}$, (Eckert number)

$Pr = \frac{\mu C_p}{K}$ (prandtl number),

$K = \frac{k_o}{\rho}\frac{v_o^2}{\nu^2}$ visco-elastic parameter)

$R = \frac{L_1 v_o}{\nu}$ (slip parameter for velocity)

$S = \frac{L_2 V_o}{\nu}$ (slip parameter for Temperature) (13)

Equations (9)and(10) reduce to

$Kf''' + f'' + f' = 0$ (14)

$\theta'' + Pr\,\theta' = - Pr\,E\,f'^2 - K\,E\,pr\,f'\,f''$ (15)

With boundary conditions

$f = R\,f'\,,\ \theta = 1 + s\frac{d\theta}{dy}$: y=0

$f \rightarrow 1\,,\ \theta \rightarrow 0$: $y \rightarrow \infty$ (16)

Other symbols have their usual meanings.

SOLUTION OF THE PROBLEM

Following Gupta (15), the values of f is given by

$f = (1 - \frac{1}{1+R})\, e^{-y} + k\,(y + \frac{R}{1+R})\,\frac{e^{-y}}{1+R}$ (17)

Substituting the values of f from (17) in equation (15) and using the corresponding boundary conditions, we get

(For pr ≠ 2)

$$\theta = \frac{1}{(S\,pr+1)}\left[1 - \frac{(2S+1)}{4-2\,pr}\left\{ A + B\left(\frac{4-pr}{4-2pr}\right) + c\left(\frac{R^2}{(1+R)^2} + \frac{12-6pr+pr^2}{(4-2pr)^2}\right)\right\} + \frac{S}{4-2pr}(B + 2C)\right] e^{-y\,pr} + \frac{e^{-2y}}{(4-2pr)}\left[A + By + B\left(\frac{4-pr}{4-2pr}\right) + C\left(y^2 + 2y + \frac{R^2}{(1+R)^2} + \frac{12-6pr+pr^2}{(4-pr)^2}\right)\right]$$ (18)

(For pr = 2)

$$\theta = \frac{e^{-2y}}{(1+2S)}\left[1 - \frac{A}{2} - \frac{CR^2}{2(1+R)^2}\right] - \frac{1}{2}\left[A + \frac{By}{2} + \frac{Cy^2}{3} + \frac{CR^2}{(1+R)^2}\right].y\,e^{-2y}$$ (19)

Where

$A = \frac{pr\,E}{(1+R)^2}\,[\,(1+K)\,(2k^2 - 1) + K\,(2-3k^2)\,\frac{R}{1+R}]$ (20)

$B = \frac{pr\,E}{(1+R)^2}\,[\,K\,(2-3k^2) - 2k^2\,(1-k)\,\frac{R}{1+R}]$ (21)

$C = \frac{pr\,E}{(1+R)^2}\,K^2\,(k-1)$ (22)

NUSSELT NUMBER

Dimensionless heat transfer coefficient i.e. Nusslet number is given by

$$N = -\left(\frac{\partial\theta}{\partial y}\right)_{Y=0}$$

(for pr ≠ 2)

$$N = \frac{1}{2(S\,pr+1)}\left[A + B\left(\frac{4-pr}{4-2pr}\right) + C\left(\frac{R^2}{(1+R)^2} + \frac{12+6pr+pr^2}{(4-pr)^2}\right)\right] + \frac{4pr-2pr^2-B-2c}{(4-2pr)(1+S\,pr)}$$

(23)

(for pr = 2)

$$N = \frac{1}{1+2S}\left[2 - A - \frac{CR^2}{(1+R)^2}\right] + \frac{1}{2}\left[A + \frac{CR^2}{(1+R)^2}\right]$$

(24)

Where A , B and C are given by equations (20) to (22).

NUMERICAL DISUSSIONS

In figure-1, dimensionless temperature θ is plotted against the non-dimensional perpendicular distance y for different values of pr, S,R and k. We find that increase in K,S and pr decreases θ .

In figure-2, dimensionless heat transfer coefficient i.e. Nusselt number N is plotted against visco-elastic parameter K for several values of R and S, the slip parameter for velocity and temperature respectively taking Eckert number E fixed as 0.5. It is interesting to note that for pr = 1 , Nusselt number increases for increasing K while for pr = 2 It decreases with k. Furthermore, for pr = 1, Nusselt number increases as R increases while it decreases as S increases. In case of pr = 2, the process reverse for R but on for S.

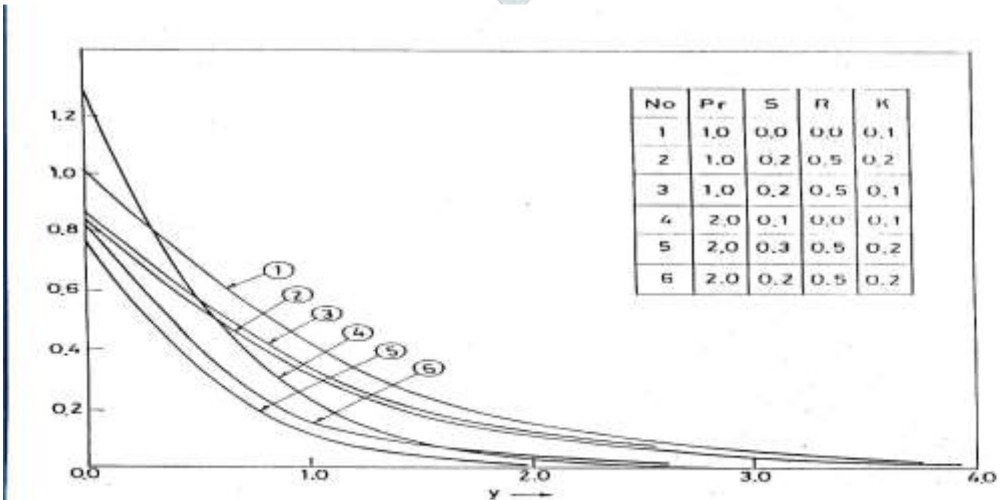

fig-1 Temperature θ plotted against y for different values of Pr, S, R and K

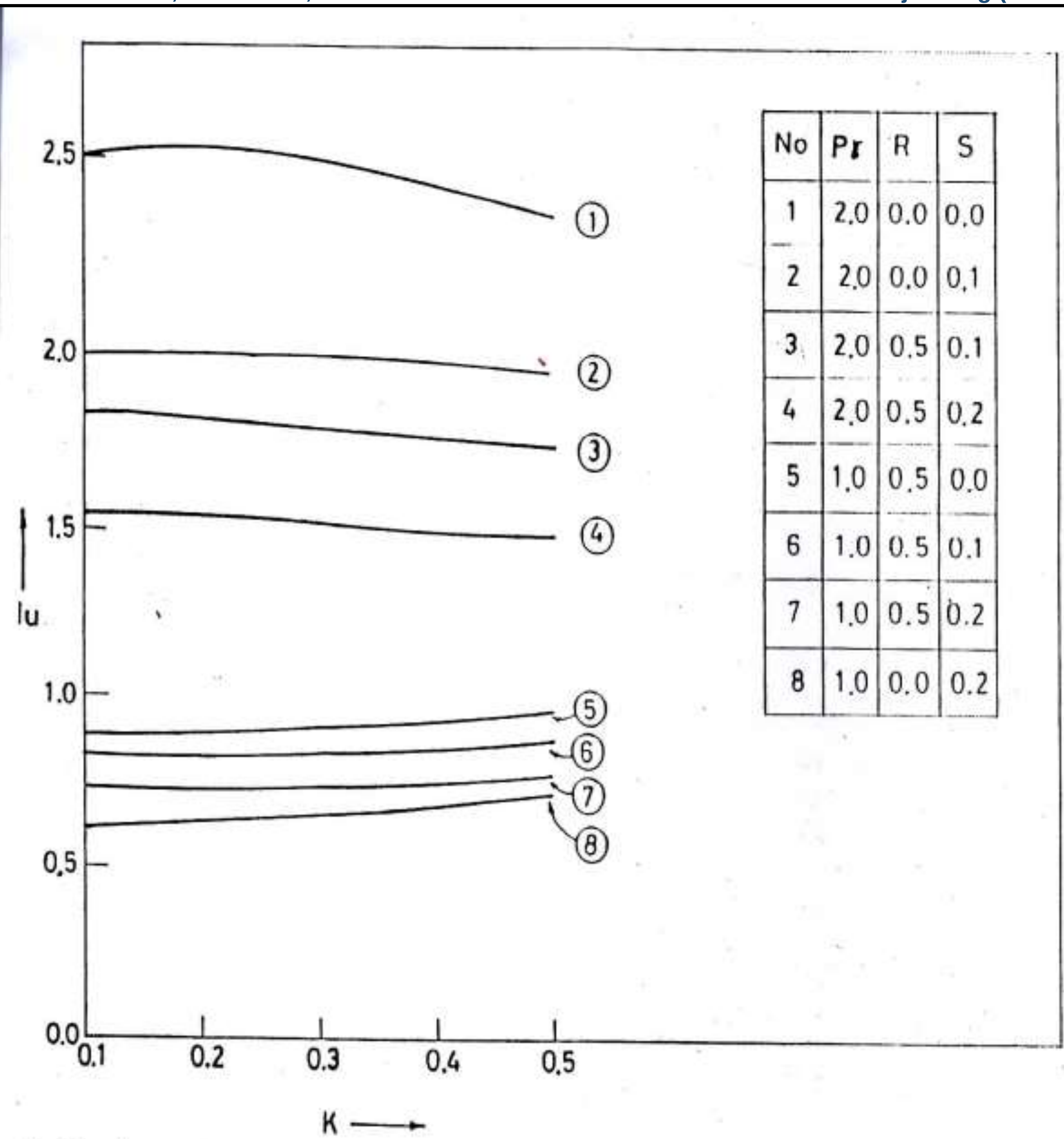

fig-2 Nusselt number Nu plotted against viscoelastic parameter K for different values of Pr, R and S.

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