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# ELECTRONIC ENERGY OF ELECTRON IN HYDROGEN ATOM BOHR-SOMMERFELD MODEL

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***Abstract:*** The article discusses the Bohr-Somerfield quantization theory and tries to explain why all earlier hypotheses about energy at the atomic level did not match the data that was observed. It also explains how this model resolved the mathematical inconsistencies and produced a model of atomic-scale energy exchange that is almost exact. In an extraordinarily successful attempt to define the nature of the orbits in which the electrons might revolve around the nucleus and to define the origin of spectral li, the author has attempted to demonstrate the historical connections, logical justifications, as well as mathematical expressions for the energy of electron in the hydrogen atom. He has also attempted to demonstrate how Bohr's celebrated theory of atomic structure is an application of Planck's theory of quanta to the Rutherford nuclear atom. Additionally, it demonstrates how Sommerfeld expanded the model to incorporate elliptical orbits. The hydrogen atom's electron receives no additional energy as a result of the elliptic orbits. We can now comprehend the significance of Bohr's atomic model because he also resolved the conundrum brought on by Rutherford's atomic model and, while acknowledging the failure of the classical theory, successfully applied the quantum theory to the Rutherford nuclear atom with rotating electron..

**keywords - Electromagnetic theory, Model, Electrical attraction, Energy, Atom.**

## I. INTRODUCTION

Faraday laid the first firm foundation for the present understanding of the atom when he observed that during electrolysis, every atom, regardless of the kind of element, released a constant amount of positive or negative charge. The earliest illustration of atom structure was provided by J.J. Thomson. In addition, he assumed that the positive charge was uniformly distributed in an atomic-scale sphere, a concept he believes lends itself best to mathematical analysis, and that the electrons were arranged inside the positive sphere in such a way that the forces of their mutual attraction and repulsion were perfectly balanced..

On the basis of his experiment, Rutherford was able to correctly describe the structure of the atom in 1911. He proposed that in an atom, the whole positive charge and virtually all of its mass are concentrated at its core in a tiny volume known as the nucleus, which is in planetary orbits at distances that are far greater than the nucleus's size. Despite having considerable experimental backing, he encountered several challenges. The electrons in motion are continually propelled in the direction of the nucleus. According to electromagnetic theory, these electrons would continuously emit energy in the form of electromagnetic waves. As a result, the atom would disintegrate as they quickly spiralled in and fell into the nucleus. In reality, atoms do not disintegrate. Additionally, Rutherford proposed that the electron may be considered to rotate about the nucleus at a speed similar to that of planets around the sun, allowing the mechanical centrifugal force to balance the subsequent excess of electrostatic attraction and therefore ensuring stability.

On the basis of Planck's quantum theory, Niel Bohr provided an explanation of the Rutherford atomic model. According to his theory, an electron can only go through orbits for which its angular momentum is a multiple of h, where h is Planck's constant. Despite its acceleration towards the orbit's centre, an electron travelling in any of the permissible orbits does not emit energy. As a result, it is claimed that the atom is in a stationary condition. When an electron jumps from one permissible orbit to another, radiation is either emitted or absorbed by the atom. One quantum of radiation is either emitted or absorbed, and its energy is equal to the energy difference between the electrons in the two involved orbits.

German physicist A. Sommerfeld expanded Bohr's theory in 1915 by including the concept of elliptical electronic orbits and accounting for the relativistic fluctuation of electron mass. The Sommerfeld relativistic atom model is the name given to the atom Sommerfeld suggested.

## II. THEORY

By introducing some quantum hypotheses into the Rutherford model, Niel Bohr fixed its flaws in 1913. On the basis of two key concepts, Bohr provided a general explanation for the origin of the line spectra. One is the idea of a photon, while the other is the idea of an atom's energy level.

Bohr combined Rutherford's purely mechanical model with the quantum notion to develop the three postulates that follow:

1. Despite the fact that mechanically an endless number of orbits are possible, only specific orbits known as stationary orbits allow electrons to revolve around a nucleus. Even though they are travelling at an accelerated pace, electrons do neither emit or absorb electromagnetic radiation while they are in the allowed orbits. The atom is stable as a result.
2. The electron orbits that are permitted are those whose angular momentum is a multiple of h, the Planck's constant, in integral form. The angular momentum of the electron is Iω = mvr.

Accordingly,

$$mvr = n\hbar \qquad (1)$$

3. They only emit energy when an electron jumps from an energy-rich higher authorised orbit, such as E2, to an energetic lower allowed orbit, such as E1. The transition's shift in energy is described by

$$E_2 - E_1 = h\nu \qquad (2)$$

where $\nu$ is the frequency of the emitted electron.

By assuming that an electron of mass m, charge e, and velocity v spins in a sphere with a radius of r around a nucleus of mass M and charge Ze, where Z is the element's atomic number, Bohr was able to determine the energies of the various states of the hydrogen atom. When Z=1 for an atom of hydrogen, E=e. The centripetal force that the electron experiences in a dynamically stable orbit is equal to the electrical force that attracts the electron to the nucleus. Thus,

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r}$$

or, $v^2 = \frac{e^2}{r\,m}$

Introducing the quantum condition for the orbit, (from postulate 2) we have

$P = I\omega = {}^{nh}$

(3)

$\varphi$ $2\pi$

But, $I\omega = mr^2\omega = mrv$

Therefore, $mrv = {}^{nh}$

$2\pi$

or, $v = {}^{nh}$

$2\pi mr$

Dividing equation (3) by equation (5), we get

(4)

(5)

From equation (5) we have,

$v =$ ${}^{1}\ 4\pi\varepsilon_0$

$2\pi e^2$

$nh$

(6)

$= \quad nh\ 2\pi mv$

Substituting the value of $v$ from equation (6) we get

$r = 4\pi\varepsilon_0\ n^2h^2$

$4\pi 2me2$

(7)

(8)

Thus, the radius r of the permitted orbit is directly proportional to $n^2$ since all other quantities are constant, where n is an integers, n= 1, 2, 3, ----.These integers are called the quantum numbers of the respective orbits.

The total energy W of the electronic system is equal to the sum of the kinetic and potential energies. That is W = kinetic energy + potential energy

The kinetic energy of electronic system = $\frac{1}{2}$

$= \quad me^4$

$8\varepsilon^2 n2h2$

$mv^2$

$= \frac{1}{2}$ m (

$\frac{1}{4\pi\varepsilon_0}$

$2\pi e^2\ ^2$

)

$nh$

[using equation (6)]

The potential energy of electronic system = $-\frac{e^2}{r}$

$= - \quad me^4 0$

[using equation (8)]

Therefore total energy of the electronic system is given by

$W_n$

$= \quad me^4$

$8\varepsilon^2 n2h2$

$- \quad me^4$

$0$

$= - \quad me^4 0$

1.

Where $W_n$ being the energy of the electron corresponding to the n^th orbit.

The orbital energy is inversely proportional to the square of the quantum number of the orbit, and all quantities in equation (9) other than n which are constants. Evidently, the energy is constant for every given orbit. Accordingly, contrary to the conventional electromagnetic theory, the electron cannot lose energy through radiation while it is in that orbit. It's crucial to understand how to interpret the negative sign in the orbital energy equation. The energy's absolute numerical value falls as n rises, but because of the negative sign, the real energy rises. This implies that the energy in the outer orbits is greater than in the inner ones. The negative sign also leads to another important conception that the electron is bound to the nucleus but attractive forces so that energy must be supplied to the electron in order to separate it completely from the nucleus.

Sommerfeld expanded on Bohr's theory by supposing that the electron has an elliptical orbit. He contended that the electron might also describe elliptical orbits since it is travelling around and being affected by a massive nucleus, much like a planet revolving around a massive sun. Now an electron moving in an elliptical orbit has two degree of freedom and its position at any instant can be fixed in terms of polar coordinates r and $\varphi$, where r is the radial distance of the electron from the nucleus at one of the foci of the ellipse and $\varphi$ the vectorial angle which the radius vector makes with the major axis of the ellipse as shown in the Figure 1. Sommerfeld postulated that each of these degree of freedom must be quantized separately.

According to the Wilson Sommerfeld quantization rule the angular and radial momenta $P_\theta$ and $P_r$ are given by,

$$\oint_0^{2\pi} P_\theta \, d\theta = k\,h \quad \text{and} \quad \oint P_r \, dr = n_r\, h \qquad (10)$$

k and $n_r$ where are integers called respectively azimuthal and radial quantum numbers. By integrating over a complete revolution, it can be proved that

$$1-\epsilon^2 = \frac{k^2}{(k+n_r)^2}$$

where $\epsilon$ is the eccentricity of the ellipse. Now we can put in the equation (11)

$$k + n_r = n \qquad (\text{Since } n=1,2,3 \quad ) \qquad (11)$$

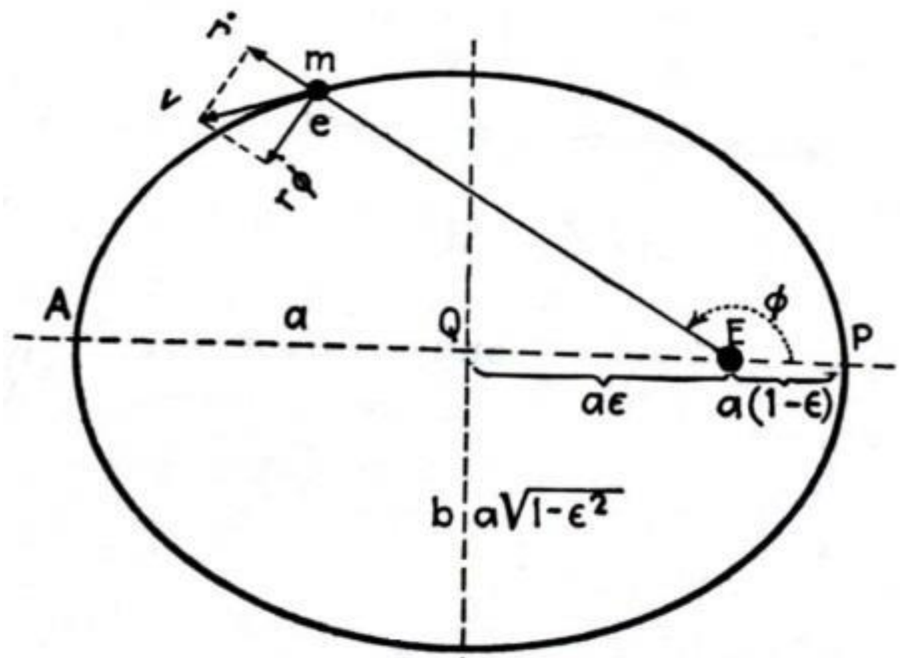


**Figure 1**

then we get ,

$$1-\epsilon^2 = \frac{k^2}{n^2} \qquad (13)$$

Where n is the total quantum number of the electron.

If a and b be the semi – major and semi-minor axes of the ellipse then we have

$$1-\epsilon^2 = \frac{b^2}{a^2} \qquad (14)$$

From equation (13) and equation (14) we have

$$\frac{b}{a} = \frac{k}{n} \qquad (15)$$

This is the condition of quantization for the orbits. Only those elliptic orbits are permitted for the electron for which the ratio of the major to the minor axes is the ratio of two integers.

The total energy E of an electron in a quantized elliptical orbit is the sum of the kinetic energy K and the potential energy U. That is,

E = K + U

$= \frac{1}{} m (\dot{r}^2 + r^2\dot{\theta}^2) - \frac{1}{}$

$e^2$

(16)

But $P_r = m\dot{r}$ and $P_\theta = mr^2\dot{\theta}$ with this substitution equation (16) becomes, 2 $4\pi e_0$

$E = \frac{1}{} m (P^2$

$r$

2

$+ \frac{}{}) -$

$1 \quad e^2$

(17)

Now $P$

$= m\dot{r} = m\frac{dr}{}$

$= m\, dr$

2

$d\theta = m\theta\, dr = m\, r^2\theta\, 1 \quad dr$

$r^2$

$= P\theta\, dr$

$4\pi e_0$

$r \quad dt$

$d\theta$

$dt$

$d\theta$

$r^2\, d\theta$

$r^2$

Putting the value of $P_r$ in the equation (17) we get,

$E = \frac{P^2}{}$

$1\, dr$

$) + 1] - \frac{1}{}$

$e^2$

(18)

Now the polar equation of an ellipse is

$2mr^2$

$r\, d\theta$

$4\pi g_0$

$r$

$1 = 1 \quad (1 - g\cos\theta)$

(19)

Differentiating equation (19), we get

$r \quad a \quad 1-g^2$

$1 = \quad e\sin\theta$

(20)

$r^2 \quad a\,(1-g)^2$

Dividing equation (20) by equation (19) we get,

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{\epsilon \sin\theta}{1+\epsilon\cos\theta} \quad (21)$$

Using equation (21) in equation (18) and solving them we get,

$$E = -\frac{me^4(1-e^2)}{(4\pi\epsilon_0)^2\, 2P_\theta^2} \quad (22)$$

For an isolated system, the angular momentum $P_\theta$ is constant. Then from equation (1) we get,

$$P_\theta = \frac{kh}{2\pi}, \quad \text{also } 1-\epsilon^2 = \frac{b^2}{a^2} = \frac{k^2}{n^2}$$

Substituting these values in equation (22) we get,

$$E = -\frac{m e^4 \left(\frac{k^2}{n^2}\right)}{(4\pi\epsilon_0)^2\, 2\left(\frac{kh}{2\pi}\right)^2}$$

$$E = -\frac{e^4}{8\epsilon_0^2 h^2}\left(\frac{1}{n^2}\right) \quad (23)$$

This is the exactly the same as the energy of electron in a circular Bohr's orbit because E is independent of k and depends upon n only which is clear from equation (23). Thus more introduction of elliptic orbit adds no new energy level.

**DISCUSSIONS**

In a remarkable attempt to define the nature of the orbits in which the electrons might revolve around the nucleus and to define the origin of the spectral lines of the elements, Bohr's celebrated theory of atomic structure applies Planck's theory of quanta to the Rutherford nuclear atom. The model was expanded by Sommerfeld to incorporate elliptical orbits. The hydrogen atom's electron receives no additional energy as a result of the elliptic orbits. Bohr also found a solution to the problem brought on by the Rutherford atomic model, and although acknowledging the shortcomings of the classical theory, he successfully applied the quantum theory to the Rutherford nuclear atom with rotating electron. This leads us to the consideration of the Bohr atom model.

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