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RADIO MEAN Gd-DISTANCE NUMBER OF SOME BASIC GRAPHS

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Abstract

A Radio Mean Gd-distance labeling of a connected graph G is an injective map f from the vertex set V(G) to $\mathbb{N}$ such that for two distinct vertices u and v of G, $d^{Gd}(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam^{Gd}(G)$, where $d^{Gd}(u,v)$ denotes the Gd-distance between u and v and $diam^{Gd}(G)$ denotes the Gd-diameter of G. The Radio Mean Gd-distance number of f, $rmn^{Gd}(f)$ is the maximum label assigned to any vertex of G. The Radio Mean Gd-distance number of G, $rmn^{Gd}(G)$ is the minimum value f of G. In this paper we find the radio mean Gd-distance number of some basic graphs.

Keywords: Gd-distance, Radio Mean Gd-distance, Radio mean Gd-distance number.

1. INTRODUCTION

By a graph G = (V(G), E(G)) we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively.

The Gd-distance was introduced by V. Maheswari and M. Joice Mabel. If u and v are vertices of a connected graph G, Gd-length of a u-v path is defined as $d^{Gd}(u,v) = d(u,v) + \deg(u) + \deg(v)$. The Gd-radius, denoted by $r^{Gd}(G) = min\{e^{Gd}(v): v \in V(G)\}$. Similarly the Gd-diameter $d^{Gd}(G) = max\{e^{Gd}(v): v \in V(G)\}$. We observe that for any two vertices u and v of G we have d(u, v) $\leq$ $d^{Gd}(u,v)$. The equality holds if and only if u, v are identical. If G is any connected graph, then the d^{Gd} distance is a metric on the set of vertices of G. We can check easily that for any non-trivial connected graph, $r^{Gd}(G) \leq d^{Gd}(G) \leq 2r^{Gd}(G)$.The lower bound is clear from the definition and the upper bound follows from the triangular inequality.

Radio mean labeling was introduced by R. Ponraj et al [17,18]. A radio mean labeling is a one to one mapping f from V(G) to $\mathbb{N}$ satisfying the condition, $d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam(G)$ for every u, v $\in$ V(G). The span of labeling f is the maximum integer that f maps to a vertex of G. The radio mean number of G, $rmn(G)$ is the lowest span taken over all radio mean labelings of the graph G.

In this paper, we introduced the concept of radio mean Gd-distance labeling of a graph G. Radio mean Gd-distance labeling is a function f from V(G) to $\mathbb{N}$ satisfying the condition

$d^{Gd}(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam^{Gd}(G)$, where $diam^{Gd}(G)$ is the Gd-distance diameter of G. A Gd-distance radio labeling number of G is the maximum label assigned to any vertex of G. It is denoted by $rmn^{Gd}(G)$.

Radio labeling can be regarded as an extension of distance-two labeling which is motivated by the channel assignment problem introduced by W. K. Hale [6]. G. Chartrand et al.[2] introduced the concept of radio labeling of graph. Also G. Chartrand et al.[3] gave the upper bound for the radio number of path. The exact value for the radio number of path and cycle was given by Liu and Zhu [9]. However G. Chartrand et al.[2] obtained different values for them. They found the lower and upper bound for the radio number of cycle. Liu [8] gave the lower bound for the radio number of Tree. M. M. Rivera et al.[22] gave the radio number of $C_n \times C_n$, the Cartesian product of C_n. In [4] C. Fernandez et al. found the radio number for Complete graph, Star graph, Complete Bipartite graph, Wheel graph and Gear graph. In this paper, we determine the radio mean Gd-distance number of some basic graphs.

2. MAIN RESULTS

Theorem 2.1.

The radio mean Gd-distance number of a complete graph K_n, $rmn^{Gd}(K_n) = n$

Proof.

For any complete graph K_n, $d^{Gd}(v_i, v_{i+1}) = 2n - 1 \; for \; 1 \le i \le n - 1$
so the $diam^{Gd}(K_n) = 2n - 1$.
The radio mean Gd-distance condition is $d^{Gd}(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \ge 1 + diam^{Gd}(G) = 2n$

Now, fix $f(v_1) = 1$

$$d^{Gd}(v_1, v_2) + \left\lceil \frac{1 + f(v_2)}{2} \right\rceil \ge 2n$$

$1 + f(v_2) \ge 0$, therefore $f(v_2) = 2$

$\therefore f(v_i) = i,\; 1 \le i \le n$

Hence, $rmn^{Gd}(K_n) = n, \forall\, n$

Theorem 2.2.

The radio mean Gd-distance number of a star graph, $rmn^{Gd}(K_{1,n}) \le 2\mathrm{n} - 3,\; \mathrm{n} \ge 4$

Proof.

Let $\mathrm{V}(K_{1,n}) = \{v_0, v_1, v_2, \dots, v_n\}$ be the vertex set, where v_0 be the central vertex and
$\mathrm{E}(K_{1,n}) = \{v_0 v_i\,; 1 \le i \le n\}$ be the edge set
Then, $d^{Gd}(v_0, v_i) = n + 2; 1 \le i \le n \; and \; d^{Gd}(v_i, v_j) = 4; 1 \le i, j \le n;\; i \ne j$
So, $diam^{Gd}(K_{1,n}) = n + 2$
Without loss of generality, $f(v_0) < f(v_1) < \cdots < f(v_n)$

We shall check the radio mean Gd-distance condition
$d^{Gd}(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \ge 1 + diam^{Gd}(G) = n + 3$

Fix $f(v_0) = n - 3$, for any pair $(v_0, v_i), 1 \le i \le n$

$$d^{Gd}(v_0, v_1) + \left\lceil \frac{f(v_0) + f(v_1)}{2} \right\rceil \ge \mathrm{n} + 2 + \left\lceil \frac{n - 3 + f(v_1)}{2} \right\rceil \ge n + 3$$

$n - 3 + f(v_1) \ge 0$, therefore, $f(v_1) = n - 2$

For any pair (v_i, v_j) $\quad 1 \le i, j \le n, i \ne j$

$$d^{Gd}(v_1, v_2) + \left\lceil \frac{f(v_1) + f(v_2)}{2} \right\rceil \ge 4 + \left\lceil \frac{n - 2 + f(v_2)}{2} \right\rceil \ge n + 3$$

$n - 2 + f(v_2) \ge 2n - 4$ therefore, $f(v_2) = n - 1$
$\therefore f(v_i) = n + i - 3,\; 0 \le i \le n$

Hence, $rmn^{Gd}(K_{1,n}) \le 2\mathrm{n} - 3, \mathrm{n} \ge 4$

Note. $rmn^{Gd}(K_{1,n}) = \mathrm{n} + 1$ if $1 \le n \le 3$

Theorem 2.3

The radio mean Gd-distance number of bistar graph $rmn^{Gd}(B_{n,n}) \le 4n - 2, n \ge 2$

Proof.

Let $\mathrm{V}(B_{n,n}) = \{v_1, v_2, \dots, v_n, x_1, x_2, u_1, u_2, \dots, u_n\}$ be the vertex set
and $\mathrm{E}(B_{n,n}) = \{x_1 u_i\,, x_2 v_i, x_1 x_2;\, 1 \le i \le n\}$ be the edge set
Then, $d^{Gd}(x_1, u_i) = d^{Gd}(x_2, v_i) = n + 3;\; 1 \le i \le n,\; d^{Gd}(x_1, x_2) = 2\mathrm{n} + 3,\; d^{Gd}(u_i, v_j) = 5; 1 \le i, j \le n, i \ne j,$
$d^{Gd}(u_i, u_j) = d^{Gd}(v_i, v_j) = 4;\; 1 \le i, j \le n, i \ne j,\; d^{Gd}(x_1, v_i) = d^{Gd}(x_2, u_i) = n + 4;\; 1 \le i \le n$
It is clear that $diam^{Gd}(B_{n,n}) = 2n + 3$
Without loss of generality $f(x_1) < f(x_2) < f(v_1) < \dots < f(v_n) < f(u_1) < \cdots < f(u_n)$

We shall check the radio mean Gd-distance condition
$d^{Gd}(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \ge 1 + diam^{Gd}(G) = 2n + 4$ for every pair of vertices (u,v); $u \ne v$

Case 1. Fix, $f(x_1) = 2n - 3$, For (x_1, x_2)

$$d^{Gd}(x_1, x_2) + \left\lceil \frac{f(x_1) + f(x_2)}{2} \right\rceil \ge 2\mathrm{n} + 3 + \left\lceil \frac{2n - 3 + f(x_2)}{2} \right\rceil \ge 2n + 4$$

$2n - 3 + f(x_2) \ge 0$, therefore $f(x_2) = 2n - 2$

For (x_2, v_i) $1 \le i \le n$

$$d^{Gd}(x_2, v_1) + \left\lceil \frac{f(x_2) + f(v_1)}{2} \right\rceil \ge n + 3 + \left\lceil \frac{2n - 2 + f(v_1)}{2} \right\rceil \ge 2n + 4$$

$2n - 2 + f(v_1) \ge 2n$, therefore $f(v_1) = 2n - 1$

For non adjacent vertices (v_i, v_j) $1 \le i, j \le n, i \ne j$

$$d^{Gd}(v_1, v_2) + \left\lceil \frac{f(v_1) + f(v_2)}{2} \right\rceil \ge 4 + \left\lceil \frac{2n - 1 + f(v_2)}{2} \right\rceil \ge 2n + 4$$

$2n - 1 + f(v_2) \ge 4n - 2$, therefore $f(v_2) = 2n$

Therefore $f(v_i) = 2n + i - 2,\ 1 \le i \le n$

Case 2. For (x_1, u_i) $1 \le i \le n$

$$d^{Gd}(x_1, u_1) + \left\lceil \frac{f(x_1) + f(u_1)}{2} \right\rceil \ge n + 3 + \left\lceil \frac{2n - 3 + f(u_1)}{2} \right\rceil \ge 2n + 4$$

$2n - 3 + f(u_1) \ge 2n$, therefore $f(u_1) = 3n - 1$

For non adjacent vertices (u_i, u_j) $1 \le i, j \le n, i \ne j$

$$d^{Gd}(u_1, u_2) + \left\lceil \frac{f(u_1) + f(u_2)}{2} \right\rceil \ge 4 + \left\lceil \frac{3n - 1 + f(u_2)}{2} \right\rceil \ge 2n + 4$$

$3n - 1 + f(u_2) \ge 4n - 2$, therefore $f(u_2) = 3n$

$\therefore f(u_i) = 3n + i - 2, \quad 1 \le i \le n$

Hence $rmn^{Gd}(B_{n,n}) \le 4n - 2, n \ge 2$

*The subdivision of a star $K_{1,n}$ denoted by S($K_{1,n}$) is a graph obtained from $K_{1,n}$ by inserting a vertex on each edge of $K_{1,n}$

Theorem 2.4

The radio mean Gd-distance number of a subdivision of a star,

$rmn^{Gd}S(K_{1,n}) \le 3n - 4, n \ge 5$

Proof.

Let $V(S(K_{1,n})) = \{v_0, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$ be the vertex set, where v_0 is the central vertex and $E(S(K_{1,n})) = \{v_0u_i, v_iu_i;\ 1 \le i \le n\}$ be the edge set

Then, $d^{Gd}(v_0, v_i) = d^{Gd}(v_0, u_i) = n + 3; 1 \le i \le n, d^{Gd}(v_i, v_{i+1}) = d^{Gd}(u_i, u_{i+1}) = 6; 1 \le i \le n$,

$d^{Gd}(v_i, u_i) = 4;\ 1 \le i \le n$

It is clear that $diam^{Gd}(S(K_{1,n})) = n + 3$

Without loss of generality $f(v_0) < f(u_1) < \cdots < f(u_n) < f(v_1) < \cdots < f(v_n)$

We shall check the radio mean Gd-distance condition

$d^{Gd}(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \ge 1 + diam^{Gd}(G) = n + 4$ for every pair of vertices (u, v); $u \ne v$

Fix $f(v_0) = n - 4$, for (v_0, u_i) $1 \le i \le n$

$$d^{Gd}(v_0, u_1) + \left\lceil \frac{f(v_0) + f(u_1)}{2} \right\rceil \ge n + 3 + \left\lceil \frac{n - 4 + f(u_1)}{2} \right\rceil \ge n + 4$$

$n - 3 + f(u_1) \ge 0$, therefore $f(u_1) = n - 3$

For a non adjacent vertices (u_i, u_{i+1}) $1 \le i \le n - 1$

$$d^{Gd}(u_1, u_2) + \left\lceil \frac{f(u_1) + f(u_2)}{2} \right\rceil \ge 6 + \left\lceil \frac{n - 3 + f(u_2)}{2} \right\rceil \ge n + 4$$

$n - 3 + f(u_2) \ge 2n - 6$, therefore $f(u_2) = n - 2$

$\therefore f(u_i) = n + i - 4; \quad 1 \le i \le n$

For any adjacent vertices (u_i, v_i) $1 \le i \le n$

$$d^{Gd}(u_1, v_1) + \left\lceil \frac{f(u_1) + f(v_1)}{2} \right\rceil \ge 4 + \left\lceil \frac{n - 3 + f(v_1)}{2} \right\rceil \ge n + 4$$

$n - 3 + f(v_1) \ge 2n - 2$, therefore $f(v_1) = 2n - 3$

For a non adjacent vertices (v_i, v_{i+1}) $1 \le i \le n - 1$

$$d^{Gd}(v_1, v_2) + \left\lceil \frac{f(v_1) + f(v_2)}{2} \right\rceil \ge 6 + \left\lceil \frac{2n - 3 + f(v_2)}{2} \right\rceil \ge n + 4$$

$2n - 3 + f(v_2) \ge 2n - 6$, therefore $f(v_2) = 2n - 2$

$\therefore\ f(v_i) = 2n + i - 4;\quad 1 \le i \le n$

Hence, $rmn^{Gd}S(K_{1,n}) \le 3n - 4, n \ge 5$

Note. $rmn^{Gd}S(K_{1,n}) \le 2n + 1$ if $2 \le n \le 4$

Theorem 2.5

The radio mean Gd-distance number of a path $rmn^{Gd}(P_n) \le 2n - 4, n \ge 5$

Proof.

Let $V(P_n) = \{v_1, v_2, \dots, v_n\}$ be the vertex set and $E(P_n) = \{v_i v_{i+1}; 1 \le i \le n-1\}$ be the edge set

Then, $d^{Gd}(v_1, v_n) = d^{Gd}(v_2, v_n) = n + 1, d^{Gd}(v_1, v_2) = d^{Gd}(v_{n-1}, v_n) = 4,$

$d^{Gd}(v_i, v_{i+1}) = 5;\ 2 \le i \le n-2$

It is clear that $diam^{Gd}(P_n) = n + 1$

Without loss of generality $f(v_1) < f(v_2) < \cdots < f(v_n)$

We shall check the radio mean Gd-distance condition

$d^{Gd}(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \ge 1 + diam^{Gd}(G) = n + 2$ for every pair of vertices (u, v); $u \ne v$

Fix $f(v_1) = n - 3$, for (v_1, v_2)

$$d^{Gd}(v_1, v_2) + \left\lceil \frac{f(v_1) + f(v_2)}{2} \right\rceil \ge 4 + \left\lceil \frac{n - 3 + f(v_2)}{2} \right\rceil \ge n + 2$$

$n - 3 + f(v_2) \ge 2n - 6$, therefore $f(v_2) = n - 2$

If both (v_i, v_j), $2 \le i, j \le n - 1, |i - j| = 1$ are intermediate adjacent vertices

$$d^{Gd}(v_2, v_3) + \left\lceil \frac{f(v_2) + f(v_3)}{2} \right\rceil \ge 5 + \left\lceil \frac{n - 2 + f(v_3)}{2} \right\rceil \ge n + 2$$

$n - 2 + f(v_3) \ge 2n - 8$, therefore $f(v_3) = n - 1$

$\therefore f(v_i) = n + i - 4,\ 1 \le i \le n$

Hence, $rmn^{Gd}(P_n) \le 2n - 4,\ n \ge 5$

Note. $rmn^{Gd}(P_n) = n,\ 2 \le n \le 4$

REFERENCES

[1] F. Buckley and F. Harary, Distance in Graphs,Addition- Wesley, Redwood City, CA, 1990.

[2] G. Chartrand, D. Erwinn, F. Harary,and P. Zhang, “Radio labeling of graphs,” Bulletin of the Institute of Combinatories and Its Applications, vol. 33,pp. 77-85, 2001.

[3] G. Chartrand, D. Erwinn,and P. Zhang, Graph labeling problem suggested by FM channel restrictions, Bull. Inst. Combin. Appl., 43, 43-57(2005).

[4] C. Fernandaz, A.Flores, M.Tomova, andC.Wyels, “ The Radio Number of Gear graphs,” arXiv:0809. 2623, September 15, (2008).

[5] J.A. Gallian, A dynamic survey of graph labeling,Electron. J.Combin. 19(2012)”£Ds6.

[6] W.K. Hale, Frequency assignment: Theory and applications, Proc. IEEE 68 (1980), pp. 1497-1514.

[7] F.Harary, Graph Theory, Addition Wesley,New Delhi(1969).

[8] D. Liu, X. Zhu, “Radio number for trees”, Discrete Math.308(7)(2008) 1153-1164.

[9] D. Liu,.X.Zhu, Multilevel distance labeling for paths and cycles, SIAM J. Discrete Math. 19(3)(2005) 610-621

[10] V. Maheswari and M. Joice Mabel “ Gd- Distance in Graphs”, International Journal of Mathematical Archive-9(2), 2018, 1-5 ISSN 2229-5046

[11] T. Nicholas, K. John Bosco, Radio D-distance Number of some graphs,IJESR , vol.5 Issue 2, Feb.2017.

[12] T. Nicholas, K. John Bosco, M. Antony, V. Viola, Radio mean D-distance Number of BananaTree, Thorn Star and Cone Graph ,IJARIIT, Vol.5 Issue 6, Feb.2017, ISSN:2456 – 132X

[13] T. Nicholas, K. John Bosco, M. Antony, V. Viola, On Radio Mean D-distance Number of DegreeSplitting Graphs, IJARSET, Vol.4 Issue 12, Dec2017, ISSN: 2350 - 0328.

[14] T. Nicholas, K. John Bosco, V. Viola, On Radio mean D-distance Number family of Snake Graph,IJIRS, Vol.8 Issue VI, June 2018, ISSN:2319 - 9725.

[15] T. Nicholas, K. John Bosco, V. Viola, On Radio Mean D-distance Number of Graph Obtained from Graph Operation IJMTT, Vol.58 Issue 2, June 2018, ISSN: 2231 - 5373.

[16] T. Nicholas, V. Viola., A Study on Radio Mean Dd- distance number of some basic graphs, International Journal of Applied Engineering Research ISSN 0973-4562 volume 14, number 8, 2019

[17] R. Ponraj, S. Sathish Narayanan and R. Kala, Radio mean labeling of graphs, AKCE International Journal of Graphs and Combinatories 12 (2015) 224- 228

[18] R. Ponraj, S. Sathish Narayanan and R. Kala, On Radio mean Number of Some Graphs, International J. Math. Combin. Vol.3(2014), 41-48

[19] Reddy Babu,D., Varma, P.L.N., "D-distance in graphs", Golden Research Thoughts, 2(2013),53-58.

[20] Reddy Babu,D., Varma, P.L.N., "Average D-distance Between Vertices of a graph", ItalianJournal of pure and applied mathematics-N. 33; 2014(293;298).

[21] Reddy Babu,D., Varma, P.L.N., "Average D- distance Between Edges of a graph" Indian Journal of Science and Technology, Vol 8(2), 152-156, January 2015.

[22] M.M. Rivera, M.Tomova, C. Wyels, and A.Yeager, "The Radio Number of C_n _ C_n, resubmitted to ArsCombinatoria, 2009.