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Porous Medium's Impact on Blood Flow Oscillations in an Atherosclerotic Blood Vessel

Prabal Pratap Singh[1], Janaki Prasad[2], Seema Raghav[3]

[1] Professor & Head, Department of Mathematics, J.S. University, Shikohabad, Uttar Pradesh, India
[2]Research Scholar, Department of Mathematics, J.S. University, Shikohabad, Uttar Pradesh, India
[3]Department of Basic Education, Firozabad, Uttar Pradesh, India

Abstract: To better understand the blood flow dynamics flow in an atherosclerotic blood vessel, a mathematical model has been constructed. Newtonian fluid theory is applied to the blood. Analytical solutions to the equations of motion have yielded formulae for axial velocity, VFR, resistive impedance, and WSS of the vessel. These idioms portray the amazing shifts in blood flow's features that occur as a result of narrowing. The porosity parameter's impact is accounted for, which eliminates the need for suction. Several factors, including the porosity constant, have been studied in relation to how they affect blood flow in a narrowed artery. The analytical results are verified by doing a numerical experiment. The present study's findings are consistent with previous research conclusions to a reasonable degree.

Keywords: Resistance to flow, volumetric flow rate, WSS, porous medium

I. INTRODUCTION

A severe narrowing of an artery is known to significantly alter blood flow. Stenosis in the coronary artery is the primary cause of this problem. Most often, atherosclerotic plaque builds up in the coronary arteries, limiting blood flow and causing this insufficiency. A heart attack becomes more likely when arteries become blocked. Thus, it is reasonable important to investigate arterial blood flow. Many studies have examined arterial blood flow by treating blood as a Newtonian or non-Newtonian fluid. Since most biological flow problems are related to porous medium, research into this area is of great theoretical and experimental importance.

A universal equation of motion for the flow of a viscous fluid through a porous material was discovered by Ahmadi and Manvi (1971). A computational model was provided by Shukla et al. (1980) to examine the effect of peripheral layer viscosity on the physiologic distinction of blood stream during a constricted arterial segment. Perkkio and Keskinen (1983) analyzed the consequences of the consideration profile dependency on blood flow through a narrowed arterial tube. Haldar (1985) investigated the issue of blood flow by treating blood as a non-Newtonian fluid. The impact of stenosis's geometric form on blood flow was another topic he covered. Misra and Verma (2007) investigated the impact of the porosity parameter and stenosis elevation on the WSS.In a uniform transverse MF, Mustapha et al. (2009) examined blood flow across numerous stenosed arteries of varying shapes and sizes. The effects of a composite stenosis on blood flow parameters in an arterial segment were investigated by Bhatnagar et al. (2015).

The divergent and convergent blood flow oscillations were studied by Okuyade et al. (2016). The system of non-linear equations is solved using the standard perturbation series solutions technique. Blood flow via a mildly stenotic artery was investigated using Newtonian mechanics by Kakati et al. (2017). Blood's Casson fluid behavior during a porous material with slip velocity was analyzed by Bhatnagar et al. (2018), who created a mathematical model to do so. Researchers Hussain et al. (2019) considered cartesian coordinates for fluid the channel's flow taking a cosine-shaped constriction as they studied blood flow through a stenosed channel. The blood flow is modeled using the Eyring-Powell fluid equations. Blood flow via a cosine-shaped constriction in an arterial duct was mathematically simulated by Hisham et al. (2020). helped clarify how this particular aspect of blood flow occurs in the body.

We have made an effort in this work to see how a porous medium affects the blood's pulsating flow via an obstructed artery. In some biological situations, the buildup of cholesterol or fatty material and blood clots in the arterial lumen can be treated as if they were a porous medium, therefore it is thought that the artery is similarly permeable. Here, the stenosis axisymmetric cosine-shaped pattern is taken into account.

II. STATEMENT OF THE ISSUE

Let us think about blood flowing axisymmetrically through a stenosed axisymmetric cosine-shaped artery segment. The assumed length of the artery section, L, makes it a hard circular tube. The presumed stenosis geometry, as seen in the lumen of the narrowed artery, is characterized as

$$\frac{\bar{R}(z)}{R_0} = \begin{cases} 1-\frac{\delta}{2R_0}\left[1+\cos\frac{2\pi}{L_0}\left(z-d-\frac{L_0}{2}\right)\right]; & d<z<d+L_0 \\ 1 ; & otherwise \end{cases} \quad (1)$$

L_0 indicates the greatest height of the stenosis, *d* the location of the stenosis relative to the normal artery's radius R_0, R the radius of the stenotic artery, and the stenosis's length.

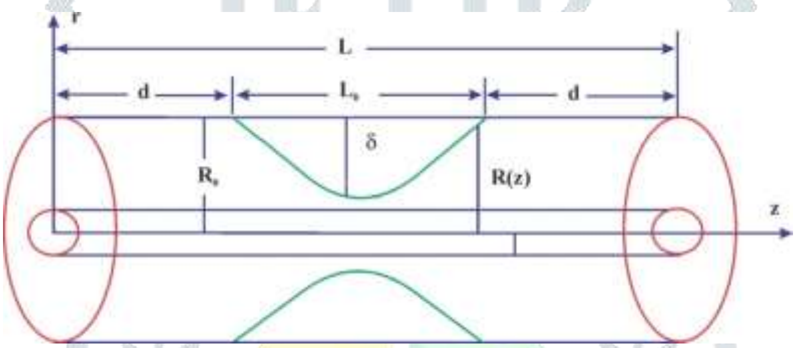

Figure 1 Cosine shaped geometry in the arterial lumen

$$-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)+\frac{\mu}{k}u=\rho\frac{\partial u}{\partial t} \quad (2)$$

$$\frac{\partial u}{\partial z} \quad (3)$$

$$\frac{\partial p}{\partial \bar{r}} \quad (4)$$

$$\frac{1}{\bar{r}}\frac{\partial p}{\partial \theta} \quad (5)$$

where p signifies the fluid's pressure, k stands for the porous parameter, u stands for the fluid's velocity in the axial direction, μ stands for the fluid's viscosity, and ρ stands for the fluid's density.

The following are the boundary conditions:

$$u=0 \text{ at } \bar{r}=\bar{R}(z) \quad (6)$$

$$\frac{\partial u}{\partial \bar{r}}=0 \text{ at } \bar{r}=0 \quad (7)$$

III. RESOLVING THE ISSUE

Now, we introduce a dimensionless variable $r = \bar{r}/R_0$ that changes the governing equations of motion to look like:

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{R_0^2}{k}u - \frac{\rho R_0^2}{\mu}\frac{\partial u}{\partial t} = \frac{R_0^2}{\mu}\frac{\partial p}{\partial z} \quad (8)$$

$$\frac{\partial p}{\partial r} \quad (9)$$

$$\frac{1}{r}\frac{\partial p}{\partial \theta} \quad (10)$$

Here are the revised boundary conditions:

$$u = 0 \text{ at } r = R(z) = \bar{R}(z)/R_0 \quad (11)$$

$$\frac{\partial u}{\partial r} = 0 \text{ at } r = 0 \quad (12)$$

Both equation (9) and (10) show that p does not depend on r and θ, equation (9) shows that $\bar{u}$ does not depend on z. (3). Thus, the following modifications make sense to make:

$$u(r,t) = \bar{u}(r)e^{i\alpha t} \quad (13)$$

$$\frac{\partial p}{\partial z} = -Pe^{i\alpha t} \quad (14)$$

The solution to (8) can be rewritten using Eqs. (13) and (14), yielding

$$\frac{d^2\bar{u}}{dr^2} + \frac{1}{r}\frac{d\bar{u}}{dr} - \left(\frac{i\alpha\rho R_0^2}{\mu} - \frac{R_0^2}{k}\right)\bar{u} = -\frac{PR_0^2}{\mu} \quad (15)$$

When boundary conditions (11) and (12) are applied to the solution of equation (15), we get:

$$\bar{u}(r) = \frac{Pk}{i\alpha\rho k - \mu}\left[1 - \frac{J_0(i\beta r)}{J_0(i\beta R)}\right] \quad (16)$$

where J_0 and $\beta^2 = R_0^2\left(\frac{i\alpha\rho}{\mu} - \frac{1}{k}\right)$ is a complex argument of Bessel's function of zeroth order.]

Blood axial velocity may now be calculated using Eqs. (13) and (16), which

$$u(r,t) = \frac{Pk}{i\alpha\rho k - \mu}\left[1 - \frac{J_0(i\beta r)}{J_0(i\beta R)}\right]e^{i\alpha t} \quad (17)$$

Here's a formula for calculating the volumetric flow rate:

$$Q = 2\pi\int_0^R r u dr \quad (18)$$

Using the value of u from equation (18) and integrating the previous equation, we get:

$$Q = \frac{\pi PRk}{i\alpha\rho k - \mu}\left[R - \frac{2J_1(i\beta R)}{i\beta J_0(i\beta R)}\right]e^{i\alpha t} \quad (19)$$

Where J_1 is a complex argument of the first order Bessel's function.

Using Eq. (19), we can calculate the pressure gradient as:

$$-\frac{\partial p}{\partial z} = Pe^{i\alpha t} = \frac{i\alpha\rho k - \mu}{\pi k}\left[\frac{i\beta J_0(i\beta R)}{i\beta R^2 J_0(i\beta R) - 2RJ_1(i\beta R)}\right]Q \quad (20)$$

Formula for resistive impedance

$$\lambda = \frac{1}{Q}\int_{0}^{L}\left(-\frac{\partial p}{\partial z}\right)dz \tag{21}$$

This, when applied to Eq. (20), yields

$$\lambda = \frac{i\alpha\rho k-\mu}{\pi k}\left[\frac{i\beta J_0(i\beta)}{i\beta J_0(i\beta)-2RJ_1(i\beta)}(L-L_0)+\int_{d}^{d+L_0}\left(\frac{i\beta J_0(i\beta R)}{i\beta R^2 J_0(i\beta R)-2RJ_1(i\beta R)}\right)dz\right] \tag{22}$$

Wall shear stress is calculated as follows:

$$\tau = -\left(\mu\frac{\partial u}{\partial r}\right)_{r=R} \tag{23}$$

When Equation (17) is substituted into Equation (23), the resulting

$$\tau = \frac{i\mu\beta Pk}{\mu-i\alpha\rho k}\left[\frac{J_1(i\beta r)}{J_0(i\beta R)}\right]e^{i\alpha t} \tag{24}$$

IV. RESULTS AND DISCUSSION

Here, we present the numerical results for quantifying the effects of the different factors on the VFR, resistive impedance, axial blood velocity, WSS, etc. MATLAB is used to create the computer code and to graphically represent the numerical findings.

Blood axial velocity varies radially along the artery, as seen in Figure 2. Similar to the real-world biological situation, where the velocity profile shows a parabolic curve movement, the axial velocity is greatest at the tube's axis, then slows to zero near its end $(r/R=1)$.It also demonstrates how the porosity parameter affects the blood flow rate. The porosity parameter is shown to be responsible for the observed drop in velocity profile (Figure 2). Figure 3 depicts the relationship between stenosis height and blood axial velocity, showing that as the stenosis height rises, the blood axial velocity drops. Figure 4 shows that the WSS increases with the severity of the stenosis; the stress does not increase noticeably in the case of mild stenosis $\delta/R_0 = 0.2$, but it increases noticeably for moderate stenosis ($\delta/R_0 = 0.4\ or\ 0.6$), and it increases exponentially in the case of severe stenosis ($\delta/R_0 = 0.8$). The wall's porosity has a positive correlation with WSS. Volumetric flow rate as a function of porosity limitation and stenosis height is shown in Figure 5. In Figure 5, we see that the volumetric flow rate decreases with increasing stenosis size, reaches a minimum at the stenosis neck (where the elevation of the stenosis is greatest), and then continues climbing to an identical maximum value at the two extremities of the limitation. The porosity characteristic decreases as it increases.

Figure 6 depicts the relationship between stenosis size and resistive impedance, showing that for severe stenosis, the impedance is higher than for moderate and mild stenosis. From the data, we may infer that both porosity and stenosis length have a multiplicative effect on the effects of impedance.

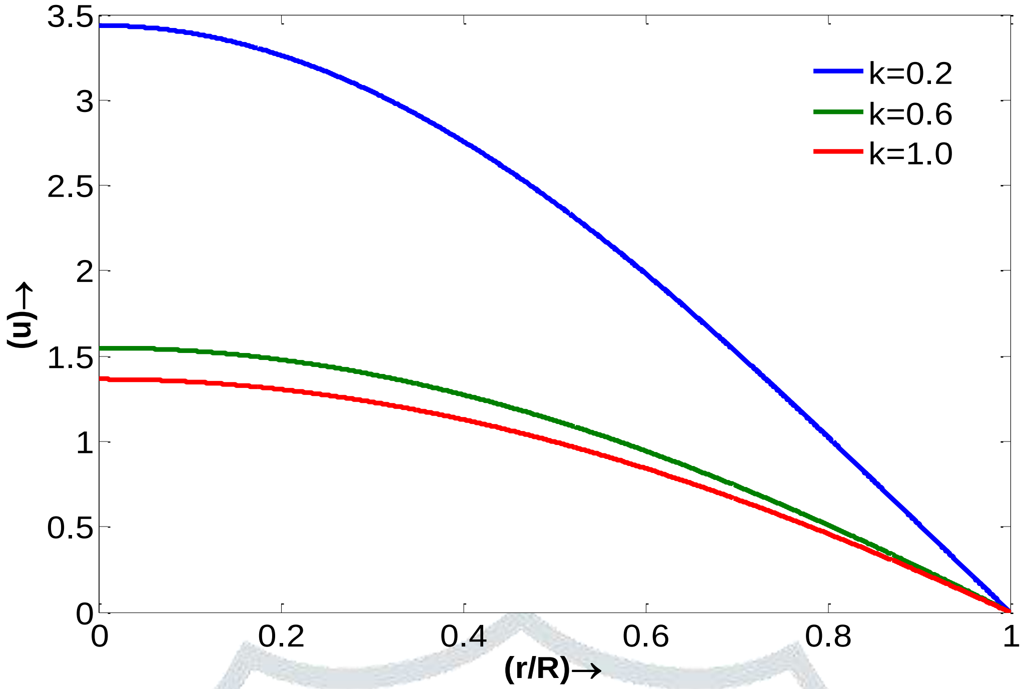

Figure 2 Axial velocity versus radial distance with different values of porous parameter (k)

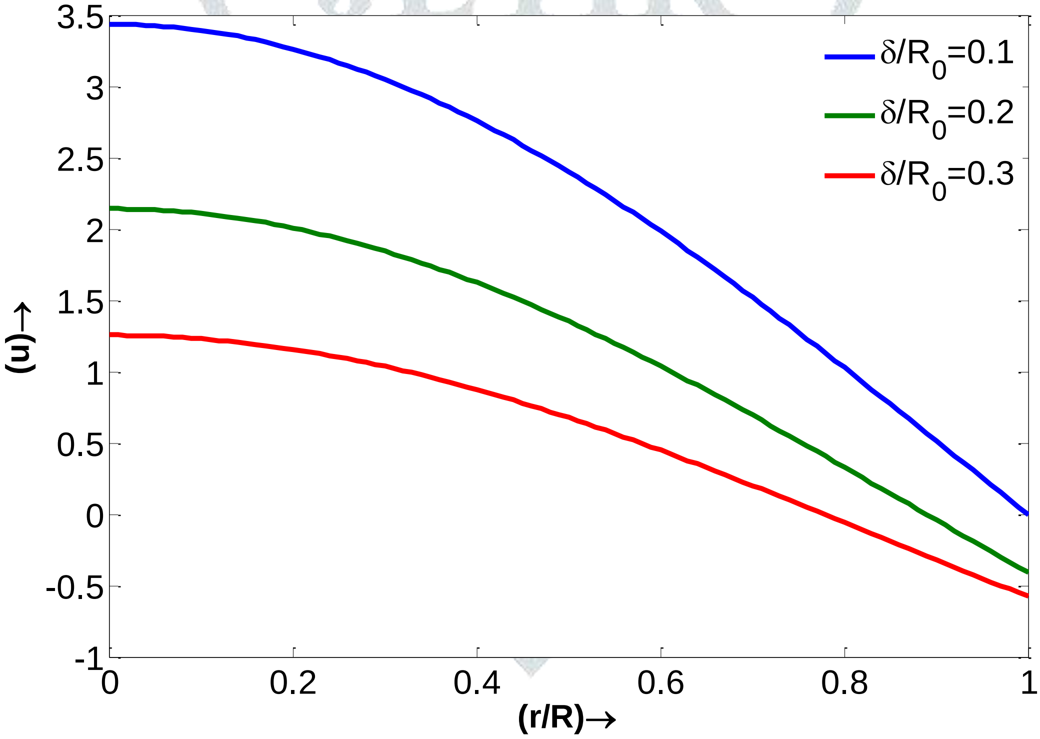

Figure 3 Axial velocity versus stenosis height with different values of porous parameter (k)

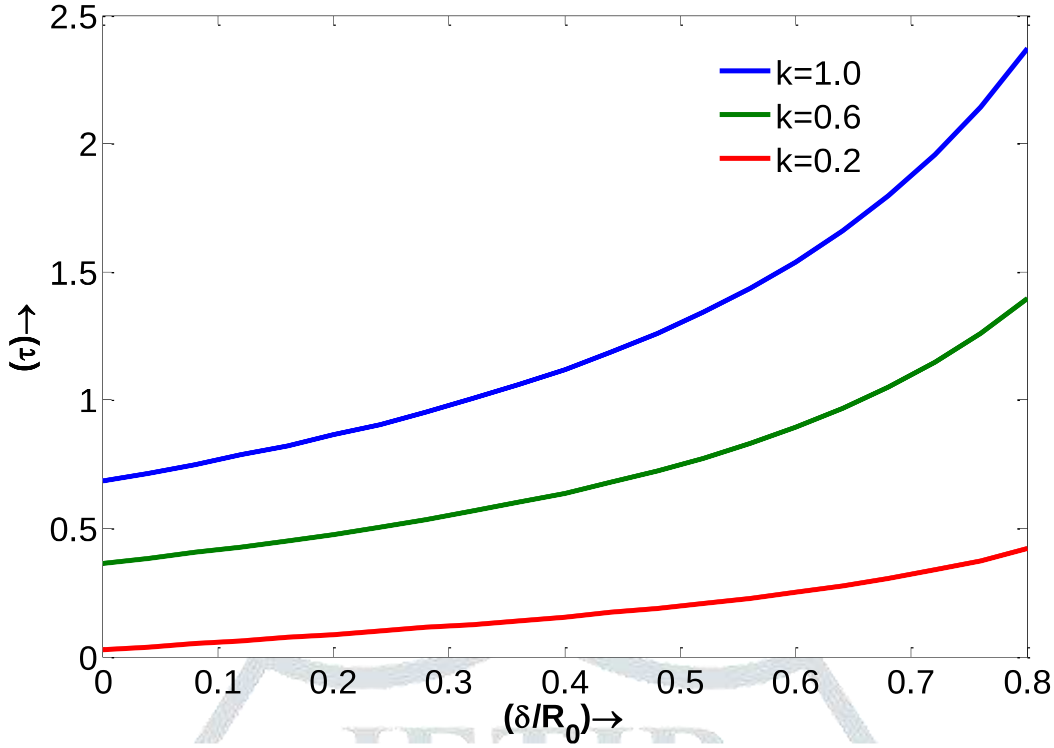

Figure 4 Wall shear stress versus stenosis height for various values of porous parameter (k)

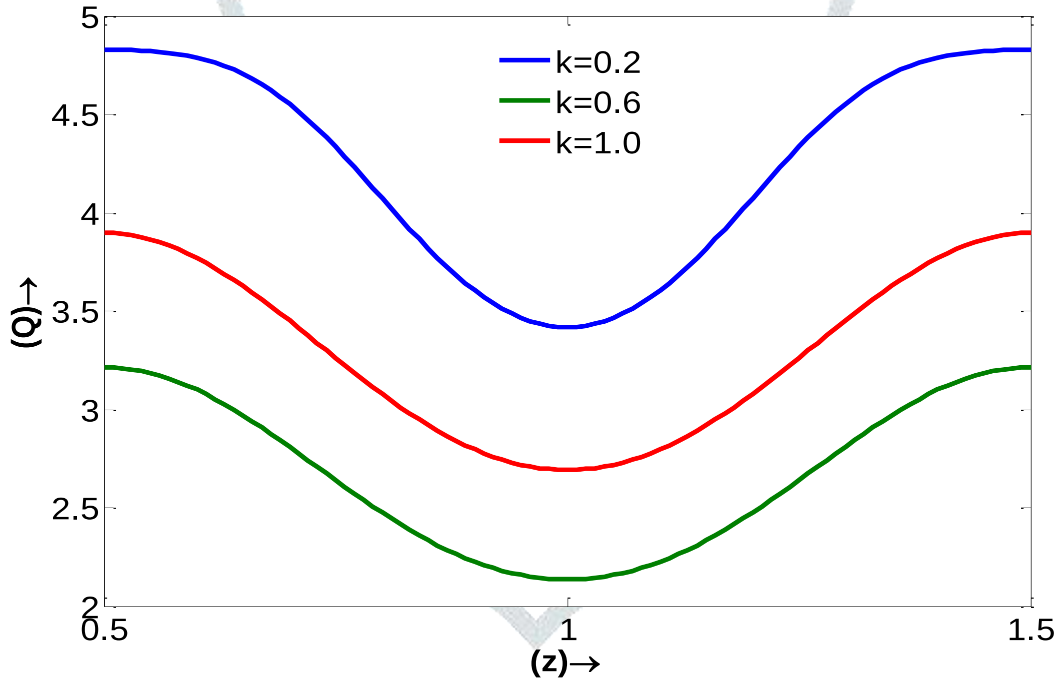

Figure 5 Volumetric flow rate versus axial length of artery for various values of porous parameter (k)

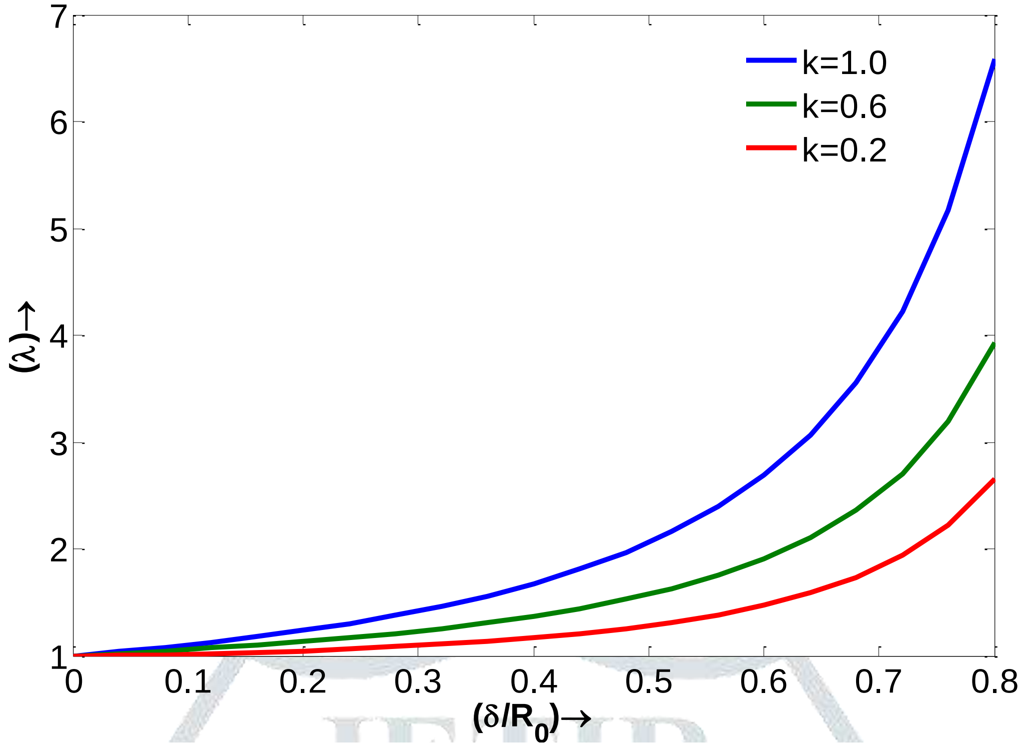

Figure 6 Resistive impedance versus stenosis height for various values of porous parameter (k)

V. CONCLUDING REMARKS

This study analyzes how changing the porosity parameter and the stenosis height affects the blood flow characteristics through an axisymmetric cosine stenosis. Our study's findings enable for the mathematical estimation of shearing stress at vessel walls, volumetric flow rate, axial blood velocity, and resistive impedance. Blood volume flow rate and axial velocity are both reduced when porosity and stenosis height are present. Both of these parameters enhance resistive resistance and shearing stress at the vessel walls. The length of the stenosis has an effect on the resistive impedance, which in turn affects blood flow. The Newtonian properties of blood are revealed, providing hope that it can aid in the circulation of injured and diseased blood vessels.

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