JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

ON OPTIMAL OPERATING POLICIES OF EPQ MODEL FOR DETERIORATING ITEMS WITH TIME DEPENDENT PRODUCTION DEMAND AND DETERIORATION

[1] S.ACHYUTA[1]
Researcher
Visakhapatnam-530 003

[2]K.SRINIVAS RAO[2]
Professor
Department of Statistics
Andhra University
Visakhapatnam-530 003

Abstract : Economic Production Quantity models provide the basic framework for scheduling production systems. In this paper, we develop and analyze an EPQ model with the assumption that the rate of replenishment is time dependent. It is also further assumed that demand is linearly dependent on time. Using difference-differential equations the instantaneous state of inventory is derived. With suitable cost considerations, the total cost function and Profit rate function is obtained. By maximizing the total cost function, the optimal operating policies are derived. Through numerical illustration, the solution procedure of the model is demonstrated. The sensitivity analysis of the model revealed that the replenishment parameters and deterioration parameters have a significant influence on the operating policies of the model.

Keywords: EPQ Model, Time Dependent Demand, Time-Dependent Replenishment, Time-Dependent Deterioration.

1 INTRODUCTION

In the previous years, much work has been reported regarding inventory models for deteriorating items with time-dependent demand.

In some papers, the authors(Nahmias 1982), reviewed the relevant literature on the problem of determining the ordering policies for both fixed life perishable inventory and inventory subject to continuous exponential decay. He considered both deterministic and stochastic demand for single and multiple products. Both optimal and sub-optimal order policies were discussed. The review of the application of the models to blood bank management was also included.

(Aggarwal and Goyal 1982), developed an order level inventory model for deteriorating items having power pattern demand where a constant fraction of on-hand inventory deteriorates per unit of time. Both deterministic, as well as probabilistic cases of demand with and without shortages, were discussed. (Raafat 1991) presented a review of inventory literature for deteriorating (decaying) inventory items. The effect of deterioration as a function of on-hand inventory level was discussed. Basic features, extensions and a classification scheme are also presented. , (Goyal and Giri 2001), presented a review of deteriorating inventory literature by classifying by the shelf-life characteristics of the inventoried goods. They have further sub-classified the models based on demand variation and various other conditions or constraints like the price discount, permissible delay in payments, instantaneous and time value of money.(Srinivasa Rao, *et al.* 2005**),** developed and analyzed an inventory model for deteriorating items with the assumption that the lifetime of the commodity is random and follows a generalized Pareto distribution. They obtained the optimal ordering and pricing policies of the model variations on the demand rate when it depends on time and selling price. Again (Srinivasa Rao, *et al.* 2006), pursued an Inventory model with hypo exponential lifetime having demand as a function of selling price and time and (Uma Maheswara Rao, *et al.* 2010) have assumed the replenishment (production) is either instantaneous or finite rate. (Sunil Kawale, Pravin Bansode 2012), In this paper, they developed the inventory model when the deterioration rate follows three parameters Weibull distribution, with demand and production rate as constant. Holding cost was considered as a time-varying function and shortages were not allowed. The numerical example supported the developed model.(Vinod Kumar Mishra, Lal Sahab Singh and Rakesh Kumar2013) considered a deterministic inventory model with time-dependent demand and time-varying holding cost where deterioration is time proportional. The model considered here allows for shortages, and the demand is partially backlogged. The model is solved analytically by minimizing the total inventory cost. The result was illustrated with the numerical example for the model. The model can be applied to optimize the total inventory cost for the business enterprises where both the

holding cost and deterioration rate are time dependent(Juanjuan Qin2015), This paper investigates an EPQ model with the increasing demand and demand dependent production rate involving the trade credit financing policy, which is seldom reported in the literature. The model considers the manufacturer is offered by the supplier a delayed payment time. It was assumed that the demand is a linearly increasing function of the time and the production rate is proportional to the demand. That is, the production rate is also a linear function of time. This study attempts to offer the best policy for the replenishment cycle and the order quantity for the manufacturer to maximize its profit per cycle. First, the inventory model was developed under the above situation. Second, some useful theoretical results were derived to characterize the optimal solutions for the inventory system. The Algorithm was proposed to obtain the optimal solutions of the manufacturer. Finally, the numerical examples were carried out to illustrate the theorems, and the sensitivity analysis of the optimal solutions concerning the parameters of the inventory system was performed. Some important management insights were obtained based on the analysis. (Srinivasa.Rao et. al,2016), In this paper an economic production quantity model is developed and analyzed for deteriorating items. Here it is assumed that the production rate is dependent on stock on-hand having a Pareto rate of decay. It is further assumed that the demand follows a power pattern with an index parameter. The model behaviour is analyzed by deriving the instantaneous state of inventory at time t, stock loss due to deterioration and production quantity. By minimizing the total cost of producing the optimal values of production downtime (time point at which production stops), production uptime (time point at which production resumes) and optimal production quantity were derived and sensitivity analysis was also performed. (Sharmila, Uthayakumar 2016), here, an inventory model with three different rates of production rate and stock dependent demand was considered. In this paper, a two-parameter Weibull distribution was used to represent the deterioration rate. The objective was to determine the optimal total cost and the optimal schedule of the plan for the proposed model numerical examples were carried out to illustrate the developed model. (Ardak, Borade 2017). in the model, proposed an EPQ model exclusively for deteriorating items with varying rates. The mix demand pattern along with time dependent holding cost is used as a reference. Optimal solutions of the model were derived which includes a numerical example and related sensitivity analysis. From the analysis, they observed that the production uptime and total cost per unit time are highly sensitive to change in the rate of deterioration and demand rate. An increase in the rate of deterioration increases total cost and total production cycle time. (Aruna Kumari 2017), developed and analyzed an inventory model with the assumption that the demand is a powerful pattern and follows an exponential distribution. However, in some other inventory systems, the demand is dependent on production quantity for example in oil exploration industries, manufacturing industries; the demand is a function of production quantity. To analyze this sort of situation an economic production quantity (EPQ) model for the deteriorating item was developed with an assumption that the production rate is time dependent and demand is dependent on the production quantity. The instantaneous level of inventory at any given time was derived through differential equations. With suitable cost considerations, the optimal ordering policies were obtained. (Ardak, Borade, Renata Stasiak Betlejewska 2017), This work proposed the EPQ model exclusively for deteriorating items. The mix demand pattern was used as a reference. In this model, the demand rate in production up time was considered as inventory dependent and kept constant during downtime. A differential calculus method was employed to get the optimum solution of the model. The effect of inventory dependent consumption was also discussed in this model. The proposed EPQ model competently proved the convexity of the total cost function. The study includes numerical examples and related sensitivity analyses. It showed that the total cost is a convex function of production uptime. Sensitivity analysis was also performed, it was observed that production uptime and total cost per unit time are highly sensitive to change in production and demand rate. Demand consumption parameter reduces the total inventory and hence holding cost. (Deepa Khurana, Shilpy Tayal and Singh 2018). This is an economic production quantity model for deteriorating items. To fulfil the market demands and expectations, the production rate was taken as a function of the demand rate. The demand and deterioration of the products were time dependent functions. Shortages were allowed and partially backlogged. The backlogging phenomenon in the literature is often modelled using backorder and lost sale costs. The backlogging option gets used only when it is economic to do so. The inventory policy proposed here considers the optimal production run time, production quantity and shortage period such that the total average cost can be minimised. Numerical examples are provided to illustrate and sensitivity analyses of optimal solutions were given for the proposed inventory model.
(Ali Akbar Shaikh, Amalesh Kumar Manna 2020)
Here, they build up the work for an EPQ model for deteriorating item under partial trade credit policy considering inflation, they studied the effect of reliability factor of system, and the demand depending on the price of a product where selling price is optimized. The production inventory model was formulated as a nonlinearly constrained optimization problem, analyzing different cases. Senstativity analysis was carried out using a numerical problem

(Chayanika Rout, Debjani Chakraborty and Adrijit Goswami 2021).The authors investigated a production inventory model under the classical EPQ framework with an assumption that the customer demand during the stock-out period is affected by the accumulated back-orders. and backlog rate was not fixed; instead, the demand during stock-out was assumed to decrease proportionally to the existing backlog which is thereby approximated by a piecewise constant function. Deteriorating items are taken into consideration in this work. numerical examples were presented which were then compared to the results obtained by considering an exact (non-approximated) backlogging rate.

(Malumfashi, Mohd Tahir Ismail,Majid Khan Majahar Ali2022). Here they considered two phase production periods, exponential demand rate and linearly increasing function of time, holding cost were proposed to solve production problem for the EPQ model for deteriorating items. They adopted differential calculus and the necessary and sufficient conditions for optimality of this model were characterized in without shortages. The best replenishment cycle length corresponding to the optimal total variable cost and production quantity of imperfect production industry were determined in this paper. The numerical illustrations and sensitivity analysis were carried out.

(KATARIYA, Dharmesh K; SHUKLA, Kunal Tarunkumar. 2023). The author is saying Consumers use green, fresh, perishable products because of their freshness, healthfulness, and sustainability. In the paper, they developed the continuous production inventory model for the producer who produces and sells fresh perishable products with the input of green efforts. The two different kinds of product decay were considered. products whose physical condition slowly deteriorates over time at a constant rate, and products looses freshness quality declines with time. Demand of that will be influenced by the selling price of the product, its freshness level, and its greening efforts. to increase the sales of inventory and the profit from clearing stocks at the end of their life, they adopted the markdown policy. Due to freshness degradation, a markdown strategy is adopted after a period of product deterioration to boost demand. Here their aim was to find out the optimal period for replenishment cycle time, the optimal value of greening efforts, and the optimal markdown percentage such that the producer's total profit is maximum. Numerical analysis and sensitivity were studied

But in many practical cases, the replenishment is variable and dependent on time due to various factors such as procurement, transportation, material availability, variation in the supply of raw material, variation in order quantity, storage capacity. Very little work has been reported in the literature regarding inventory models for deteriorating items with time dependent replenishment time and demand. Demand pattern is highly influenced by time. The time dependent time is usually characterized by power pattern demand. Here, it is considered that the demand rate is of the form $\lambda(t)=\frac{rt^{\frac{1}{n}}}{nT^{\frac{1}{n}}}$, where 'n' is the indexing parameter, r is the total demand during the cycle length 'T'. For different values of parameter 'n', the demand rate may be increasing /decreasing/constant. The power pattern demand includes a wide variety of demand functions for different types of commodities.

In addition to demand and replenishment, ageing influences inventory. The lifetime of the commodity for some items is finite and dependent on time. The time dependent deterioration is represented by a linear function is of the form $h(t)=\alpha+\beta t$. In this paper, an inventory model for deteriorating items is developed and analyzed with the assumptions that replenishment, demand and deterioration are all dependent on time.

The instantaneous state of inventory at any given time t, at a cycle length T, is derived through differential equations. The total cost function is of the time with suitable cost considerations on the cost of a unit, set up cost, penalty cost etc. By minimizing the total cost function, the optimal time for replenishment (production start-up and production downtime) are derived along with order quantity are derived. The sensitivity of the model, for variation in parameters and costs, is started through numerical illustrations. This model also includes the model without shortages as a limiting case. This model is useful for analyzing the situations in food processing, agricultural production units where the production, demand and deterioration are all time dependent demand.

2 ASSUMPTIONS

The following assumptions are made for developing the model.

i) The lifetime of a commodity is finite and dependent on time. The instantaneous rate of deterioration is $h(t) = \alpha + \beta t$ - (1),

where α and β are constants. If $\beta > 0$ then, this is an increasing rate of deterioration. If $\beta < 0$ then, it includes a decreasing rate of deterioration. If $\beta = 0$ then, this is a constant rate of deterioration.

ii) The demand rate $\lambda(s)$ is a linear function of unit selling price and it is of the form $\lambda(s)=(d-f\,s)$, where, d and f are positive constants.

iii) The rate of production is time dependent and is of the form $R(t)=a+bt$ such that $R(t)\geq 0$, where 'a' and 'b' are constants, $a>0, b>0$. This production rate includes increasing/decreasing/constant rates of production for b > 0, b < 0 and b=0 respectively.

iv) Lead time is zero.

v) Cycle length, T is known and fixed.

vi) There is no repair or replacement of deteriorated item which occurs during the production cycle and the deteriorated item is thrown as scrap.

Notations

The following notations are used for developing the model.

- A: Ordering cost
- C: Cost per unit
- h: Inventory holding cost per unit time
- π: Shortage cost per unit time
- Q: Total quantity of items produced in one cycle
- s: The selling price of a unit
- $\lambda(s)$: demand rate
- I (t): On hand inventory at time t, $0 \leq t \leq T$.
- t_1: Time at which replenishment stops.
- t_2: Time at which shortages start.
- t_3: Time at which replenishment is restarted.

we further assume that,

Demand is a power function of time. Let the size of demand over the prescribed period (0, T) be 'r' and let 'n' be the demand pattern index. For different values of 'n', this demand rate includes different types of demand. Then the rate of demand at any time 't ' is defined as

$$\lambda(t) = \frac{r t^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}}, \tag{2}$$

Here, we have considered, λ (t) as demand rate and 'n' is the index parameter. If n=1 then the demand rate reduces to a constant rate of demand.

3 INVENTORY MODEL WITH SHORTAGES

In this section, we consider the production level inventory model, in which, shortages are allowed. The production starts at time t=0 and the inventory level gradually increases with time due to production after fulfilling demand and deterioration. The stock level reaches a maximum at time t_1. The inventory decreases partly due to demand and partly due to the deterioration of items during the time interval (t_1, t_2). The shortages that occurred during the time interval (t_2, t_3) are fully backlogged. The production recommences at time t=t_3 and backlogged demand is cleared during the interval (t_3, T) and the cycle is continuous thereafter.

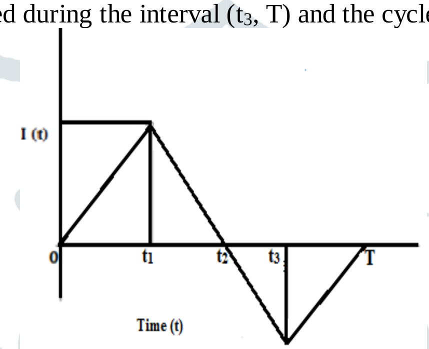


Figure 3.1 Instantaneous state of inventory for model

The schematic diagram representing the instantaneous state of inventory is given in figure 1.

Let I (t) denote the inventory level of the system at time t, $0 \le t \le T$.

The differential equations describing the instantaneous states of I (t) in the interval (0, T) are:

$$\frac{d}{dt}I(t)+(\alpha+\beta t)I(t) = (a+bt)-\frac{r t^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} \quad ; \quad 0\le t\le t_1 \tag{3.1}$$

$$\frac{d}{dt}I(t)+(\alpha+\beta t)\, I(t) = -\frac{r t^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} \quad ; \quad t_1\le t\le t_2 \tag{3.2}$$

$$\frac{d}{dt}I(t) = -\frac{r t^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} \quad ; \quad t_2\le t\le t_3 \qquad (3.3$$

$$\frac{d}{dt}I(t) = (a+bt)-\frac{r t^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} \quad ; \quad t_3\le t\le T \tag{3.4}$$

With the initial conditions $I(0)=0,\ I(t_2)=0,\ I(T)=0$.

Solving the differential equations (3) to (6) using the boundary conditions, the instantaneous state of inventory at any time t, during the interval (0, T) is obtained as

$$I(t)= e^{-\left(\alpha t+\frac{\beta t^2}{2}\right)} \int_0^t \left[(a+bt)-\frac{r t^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} e^{\left(\alpha t+\frac{\beta t^2}{2}\right)}\right] dt \quad ; \quad 0\le t\le t_1 \tag{3.5}$$

$$I(t)= e^{-\left(\alpha t+\frac{\beta t^2}{2}\right)} \int_t^{t_2} \frac{r t^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} e^{\left(\alpha t+\frac{\beta t^2}{2}\right)} dt \quad ; \quad t_1\le t\le t_2 \tag{3.6}$$

$$I(t) = -\int_{t_2}^{t} \frac{r\,t^{\frac{1}{n}-1}}{n\,T^{\frac{1}{n}}}\,dt \qquad ; \qquad t_2 \le t \le t_3 \tag{3.7}$$

$$\frac{d}{dt}I(t) = (a+bt) - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} ; \qquad t_3 \le t \le T \tag{3.8}$$

The stock loss due to deterioration in the interval (0, T) is

$$L(T) = at_1 + b\,\frac{t_1^2}{2} - \left(\frac{r}{(n+1)(T)^{\frac{1}{n}}}\right) t_2^{\frac{1}{n}} \tag{3.9}$$

The total production in the cycle of length T is

$$Q = a\,(t_1 - t_3 + T) + \frac{b}{2}\,(t_1^2 + T^2 - t_3^2) \tag{3.10}$$

From equations (3.9) and (3.10), we get

$$I(t) = -\frac{r}{T^{\frac{1}{n}}}\left(t^{\frac{1}{n}} - [t_2]^{\frac{1}{n}}\right) \tag{3.11}$$

$$I(t) = -\left[a(T-t) + \frac{b}{2}(T^2 - t^2) - r\left\{1 - \left(\frac{t}{T}\right)^{1/n}\right\}\right] \tag{3.12}$$

When t $= t_3$, the equations (3.13) and (3.14) becomes

$$I(t_3) = -\frac{r}{T^{\frac{1}{n}}}\left(t_3^{\frac{1}{n}} - t_2^{\frac{1}{n}}\right) \tag{3.13}$$

$$I(t_3) = -\left\{a(T-t_3) + \frac{b}{2}(T^2 - t_3^2) - r\left(1 - \left(\frac{t_3}{T}\right)^{1/n}\right)\right\} \tag{3.14}$$

On equating these equations (3.15) and (3.16) and on simplification, we get t_2 in terms of t_3 as,

$$t_2 = \left[t_3^{1/n} - \frac{T^{1/n}}{r}\left[a(T-t_3) + \frac{b}{2}(T^2 - t_3^2) - r\left(1 - \left(\frac{t_3}{T}\right)^{1/n}\right)\right]\right]^n = y_1 \text{ say} \tag{3,15}$$

Let K (t_1, t_2, t_3) be the total cost per unit. Since the total cost is the sum of the setup cost per unit time, purchasing cost per unit time, holding cost per unit time and shortage cost per unit time. Then K (t_1, t_2, t_3) becomes

$$K(t_1,t_2,t_3) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left[\int_0^{t_1} I(t)\,dt + \int_{t_1}^{t_2} I(t)\,dt\right] + \frac{\pi}{T}\left[-\int_{t_2}^{t_3} I(t)\,dt - \int_{t_3}^{T} I(t)\,dt\right] \tag{3.16}$$

Substituting the values of I (t) and Q, given in

equations, from (3.7) to (3.10), (3.12) and writing t_2 in terms of t_3 and the values in equation (3.16) we have

$$\begin{aligned}K(t_1,t_3) = {} & \frac{A}{T} + \frac{C}{T}\left\{a(t_1 - t_3 + T) + \frac{b}{2}(t_1^2 - t_3^2 + T^2)\right\} \\ & + \frac{h}{T}\left\{\int_0^{t_1} e^{-(\alpha t + \frac{\beta t^2}{2})}\left(\int_0^{t}\left((a+bu) - \frac{r\,u^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right) e^{(\alpha u + \frac{\beta u^2}{2})}\,du\right)dt \right. \\ & \left. + \int_{t_1}^{y_1(t)} e^{-(\alpha t + \frac{\beta t^2}{2})}\left(\int_t^{y_1(t)} \frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\, e^{(\alpha u + \frac{\beta u^2}{2})}\,du\right)dt\right\} \\ & + \frac{\pi}{T}\left\{\int_{y_1(t)}^{t_3}\left(\int_{y_1(t)}^{t} \frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\,du\right)dt + \int_{t_3}^{T}\left(\int_t^{T}\left((a+bu) - \frac{ru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)du\right)dt\right\}\end{aligned} \tag{3.17}$$

On integrating and simplifying the equation (17), we get

$$
\begin{aligned}
K(t_1, t_3) = {} & \frac{A}{T} + \frac{C}{T}\left[a\left(t_1 - t_3 + T\right) + \frac{b}{2}\left(t_1^2 - t_3^2 + T^2\right)\right] \\
& + \frac{h}{T}\left\{ a\int_0^{t_1} e^{-(\alpha t+\beta\frac{t^2}{2})}\left(\int_0^{t} e^{(\alpha u+\beta\frac{u^2}{2})}du\right)dt \right. \\
& + b\int_0^{t_1} e^{-(\alpha t+\beta\frac{t^2}{2})}\left(\int_0^{t} u\, e^{(\alpha u+\beta\frac{u^2}{2})}du\right)dt \\
& - \left(\frac{r}{n\,T^{\frac{1}{n}}}\right)\int_0^{t_1} e^{-(\alpha t+\beta\frac{t^2}{2})}\left(\int_0^{t} u^{\frac{1}{n}-1} e^{(\alpha u+\beta\frac{u^2}{2})}du\right)dt \\
& \left. + \frac{r}{n\,T^{\frac{1}{n}}}\int_{t_1}^{y_1(t)} e^{-(\alpha t+\beta\frac{t^2}{2})}\left(\int_{t}^{y_1(t)} u^{\frac{1}{n}-1} e^{(\alpha u+\beta\frac{u^2}{2})}du\right)dt \right\} \qquad (3,18) \\
& + \frac{\pi}{T}\left\{ \frac{r}{n\,T^{\frac{1}{n}}}\left(\frac{n}{n+1}\right)\left[\left(\left(t_3 - y_1(t)\right)\left(1-\left(y_1(t)\right)\right)^{\frac{1}{n}}\right]\right] + \frac{a}{2}\left(T^2 - Tt_3 + t_3^2\right) \right. \\
& \left. + \frac{b}{2}\left(T^2 - T\,t_3 + \frac{t_3^3}{3}\right) - \frac{r\,T^2}{2\,T^{\frac{1}{n}}} + \frac{r\,t_3}{T^{\frac{1}{n}}}\left(1 - \frac{t_3}{2}\right)\right\}
\end{aligned}
$$

4 OPTIMAL PRICING AND ORDERING POLICIES OF THE MODEL

In this section, we obtain the optimal policies of the inventory system under study. To find the optimal values of t_1, t_3, we obtain the first-order partial derivatives of K (t_1, t_3) given in equation (20) concerning t_1, t_3 and equate them to zero. The condition for minimization of K (t_1, t_2, t_3) is

$$
D = \begin{vmatrix} \dfrac{\partial^2 K(t_1,t_3)}{\partial t_1^{\,2}} & \dfrac{\partial^2 K(t_1,t_3)}{\partial t_1 \partial t_3} \\[2ex] \dfrac{\partial^2 K(t_1,t_3)}{\partial t_3 \partial t_1} & \dfrac{\partial^2 K(t_1,t_3)}{\partial t_3^{\,2}} \end{vmatrix} > 0
$$

where D is the determinant of the Hessian matrix.

Differentiating $K(t_1, t_3)$ concerning t_1 and equating it to zero, we get

$$
\begin{aligned}
& \frac{C}{T}\left[a + b\,t_1\right] + \frac{h}{T}\left\{ a\,e^{-(\alpha t_1+\beta\frac{t_1^2}{2})}\left(\int_0^{t_1} e^{(\alpha u+\beta\frac{u^2}{2})}du\right) + b\,e^{-(\alpha t_1+\beta\frac{t_1^2}{2})}\left(\int_0^{t_1} u\,e^{(\alpha u+\beta\frac{u^2}{2})}du\right)\right. \\
& - \left[\frac{r}{nT^{\frac{1}{n}}} e^{-(\alpha t_1+\beta\frac{t_1^2}{2})}\left(\int_0^{t_1} u^{\frac{1}{n}-1} e^{(\alpha u+\beta\frac{u^2}{2})}du\right)\right] \\
& \left. - \left[\frac{r}{nT^{\frac{1}{n}}}\left(e^{-(\alpha t_1+\beta\frac{t_1^2}{2})}\int_t^{t_1} u^{\frac{1}{n}-1} e^{(\alpha u+\beta\frac{u^2}{2})}du\right)\right]\right\} = 0
\end{aligned}
\qquad (4.1)
$$

Differentiate $K(t_1, t_3)$ concerning t_3 and equating it to zero, we get

$$
\begin{aligned}
&-\frac{C}{T}\big[a+bt_3\big]+\frac{h}{T}\Bigg\{e^{(\alpha y_1+\beta y_1^2/2)}\int_{t_1}^{y_1} e^{(\alpha yt+\beta t^2/2)}dt+\frac{rt}{T^{1/n}}\Big[t_3^{1/n}-\frac{T^{1/n}}{r}\Big[a(T-t_3)\\
&\qquad +\frac{b}{2}(T^2-t_3^2)-r(1-(t_3/T)^{1/n})\Big]^{n-1}\Big[\frac{1}{n}t_3^{-1+1/n}+\frac{T^{1/n}}{r}+bt_3T^{1/n}-t_3^{-1+1/n}\Big]\Big]\Bigg\}\\
&\qquad\quad +\frac{\pi}{T}\Bigg\{\frac{r}{T^{1/n}(n+1)}\Bigg[\Big[1+\frac{T^{1/n}}{r}a+b\frac{t_3T^{1/n}}{r}-\frac{r}{n}\Big(\frac{t_3}{T}\Big)^{(-1+1/n)}\frac{1}{T}\Big]\\
&\qquad\qquad n\Big[t_3^{1/n}-\frac{T^{1/n}}{r}\Big[a(T-t_3)+\frac{b}{2}(T^2-t_3^2)\ -r\Big(1-\frac{t_3}{T}\Big)^{(-1+1/n)}\Big]\Big]^{(n-1)}\\
&\qquad\qquad -\frac{1}{n}t_3^{-1+1/n}+\Big[\frac{T^{1/n}}{r}a+b\frac{t_3T^{1/n}}{r}-\frac{r}{n}\Big(\frac{t_3}{T}\Big)^{(-1+1/n)}\frac{1}{T}\Big]^{(-1+1/n)}\Bigg]\\
&\qquad\qquad\qquad +\frac{a}{2}(2t_3-T)-\frac{b}{2}(t_3-T)-\frac{r(T^2+2-t_3)}{2T^{1/n}}\Bigg\}
\end{aligned}
\tag{4.2}
$$

Solving the equations (21) and (22) simultaneously, we obtain the optimal time at which the replenishment should be stopped i.e., t_1^* of t_1, and optimal time t_3^* of t_3 at which replenishment is restarted after the accumulation of backorders. The optimum ordering quantity Q^* of Q in the cycle of length T is obtained by substituting the optimal values of t_1 and t_3 in (12) as

$$Q^* = a\ (t_1^*-t_3^*+T)+\ \frac{b}{2}\ (t_1^{*2}+T^2-t_3^{*2}) \tag{4.3}$$

5 NUMERICAL ILLUSTRATION

In this section, we discuss the solution procedure of the model through a numerical illustration by obtaining the replenishment (production) time, replenishment (production) downtime, optimal selling price optimal quantity and profit of an inventory system. Here, it is assumed that the commodity is deteriorating and shortages are allowed and fully backlogged. For demonstrating the solution procedure of the model the deteriorating parameter 'α' is considered to vary between 0.01, 0.02 and 0.03, the values of the other parameters and costs associated with the model are:

A= 100,200, 300; C = 9, 10, 11; a = 2, 3, 4; b = 1.5, 2, 2.5; β= 0.01, 0.02, 0.03; π = 0.3, 0.4, 0.5; h = 0.1, 0.2, 0.3; n = 1.5, 1.75, 2; r = 1100, 1200, 1300; T =12 months.

Substituting these values of optimal ordering quantity Q*, replenishment time, time at which the shortage starts, an optimal value of time and total cost are computed and presented in Table 1.

As the ordering cost 'A' increases from 100 to 300, all the values remain constant. When the cost per unit C increases from 9 to 11, there is a decrease in optimal ordering quantity Q*, it decreases from 176.472 to 170.970, the optimal value of t_1^* decreases from 4.724 to 3.795, the optimal replenishment time t_3^* decreases from 5.001 to 4.588 and the total cost increases from 87.514 to 116.432.

When the Production parameter 'a' increases from 2 to 4, there is an increase in optimal ordering quantity Q*, from 164.050 to 182.308, the optimal value of t_1^* decreases from 4.672 to 3.786, the optimal replenishment time t_3^* decreases from 5.010 to 4.570 and the total cost increases from 91.465 to 112.550. When 'b' is increasing from 1.5 to 2.5, there is an increase in the optimal ordering quantity Q*, from 142.192 to 206.163, the optimal values of t_1^* decreases from 5.393 to 3.365, the optimal replenishment time t_3^* decreases from 5.555 to 4.158 and the total cost increases from. 68.904 to 135.984.

As the deteriorating parameters 'α' increases from 0.01 to 0.03, there is a decrease in optimal ordering quantity Q*, from 176.445 to 173.407, the optimal value of t_1^* decreases from 4.433 to 4.237, the optimal replenishment time t_3^* increases from 4.725 to 4.785 and the total cost increases from 101.063 to 102.087. When 'β' is increasing from 0.01 to 0.03, there is a decrease in the optimal ordering quantity Q*, from 177.662 to170.332, the optimal value of t_1^* increases from 4.511 to 4.036, the optimal replenishment time t_3^* increases from 4.702 to 4.849 and the total cost increases from 101.128 to 102.853.

As the shortage cost per unit time 'π' increases from 0.3 to 0.5, there is a decrease in optimal ordering quantity Q*, from 179.686 to 167.667, the optimal value of t_1^* increases from 3.897 to 4.502, the optimal replenishment time t_3^* increases from 3.926 to 5.454 and the total cost decreases from 110.651 to 95.450. When inventory holding cost per unit time 'h' increases from 0.1 to 0.3, there is an increase in the optimal ordering quantity Q*, from 148.697 to 198.099, the optimal

As the ordering cost 'A' increases from 100 to 300, all the values remain constant. When the cost per unit C increases from 9 to 11, there is a decrease in optimal ordering quantity Q*, it decreases from 176.472 to 170.970, the optimal value of t_1^* decreases from 4.724 to 3.795, the optimal replenishment time t_3^* decreases from 5.001 to 4.588 and the total cost increases from 87.514 to 116.432.

When the Production parameter 'a' increases from 2 to 4, there is an increase in optimal ordering quantity Q*, from 164.050 to 182.308, the optimal value of t_1^* decreases from 4.672 to 3.786, the optimal replenishment time t_3^* decreases from 5.010 to 4.570 and the total cost increases from 91.465 to 112.550. When 'b' is increasing from 1.5 to 2.5, there is an increase in the optimal ordering quantity Q*, from 142.192 to 206.163, the optimal values of t_1^* decreases from 5.393 to 3.365, the optimal replenishment time t_3^* decreases from 5.555 to 4.158 and the total cost increases from. 68.904 to 135.984.

As the deteriorating parameters 'α' increases from 0.01 to 0.03, there is a decrease in optimal ordering quantity Q*, from 176.445 to 173.407, the optimal value of t_1^* decreases from 4.433 to 4.237, the optimal replenishment time t_3^* increases from 4.725 to 4.785 and the total cost increases from 101.063 to 102.087. When 'β' is increasing from 0.01 to 0.03, there is a decrease in the optimal ordering

quantity Q*, from 177.662 to170.332, the optimal value of t_1^* increases from 4.511 to 4.036, the optimal replenishment time t_3^* increases from 4.702 to 4.849 and the total cost increases from 101.128 to 102.853.

As the shortage cost per unit time ‘π’ increases from 0.3 to 0.5, there is a decrease in optimal ordering quantity Q*, from 179.686 to 167.667, the optimal value of t_1^* increases from 3.897 to 4.502, the optimal replenishment time t_3^* increases from 3.926 to 5.454 and the total cost decreases from 110.651 to 95.450. When inventory holding cost per unit time ‘h’ increases from 0.1 to 0.3, there is an increase in the optimal ordering quantity Q*, from 148.697 to 198.099, the optimal value of t_1^* increases from 2.048 to 5.665, the optimal replenishment time t_3^* decreases from 5.125 to 4.266, and the total cost decreases from 111.555 to 84.057. As the indexing parameter ‘n’ increases from 1.5 to 2, there is an increase in the optimal ordering quantity Q*, from 156.897 to 173.407, the optimal value of t_1^* increases from 4.209 to 4.237, the optimal replenishment time t_3^* decreases from 5.963 to 4.785 and the total cost increases from 97.824 to 102.087. As the demand parameter ‘r’ increases from 1100 to 1300, there is an increase in optimal ordering quantity Q*, from 171.983 to 174.967, the optimal value of t_1^* increases from 3.774 to 4.662, the optimal replenishment time t_3^* increases from 4.486 to 5.058, and the total cost decreases from 108.682 to 95.486.

TABLE 1
OPTIMAL VALUES OF t_1^*, t_3^*, Q^* and K^* FOR DIFFERENT VALUES OF PARAMETERS WITH SHORTAGES

A	C	a	b	α	β	π	h	n	r	T	t_1^*	t_3^*	K^*	Q*
200	10	3	2	0.03	0.02	.4	.2	2	1200	12				
100											4.237	4.785	93.754	173.407
200											4.237	4.785	102.087	173.407
300											4.237	4.785	110.421	173.407
	9										4.724	5.001	87.514	176.472
	10										4.237	4.785	102.087	173.407
	11										3.795	4.588	116.432	170.970
		2									4.672	5.010	91.465	164.050
		3									4.237	4.785	102.087	173.407
		4									3.786	4.570	112.550	182.308
			1.5								5.393	5.555	68.904	142.192
			2								4.237	4.785	102.087	173.407
			2.5								3.365	4.158	135.489	206.163
				0.01							4.433	4.725	101.063	176.445
				0.02							4.331	4.756	101.595	174.860
				0.03							4.237	4.785	102.087	173.407
					0.01						4.511	4.702	101.128	177.662
					0.02						4.237	4.785	102.087	173.407
					0.03						4.036	4.849	102.853	170.332
						0.3					3.897	3.926	110.651	179.686
						0.4					4.237	4.785	102.087	173.407
						0.5					4.502	5.454	95.450	167.667
							0.1				2.048	5.125	111.555	148.697
							0.2				4.237	4.785	102.087	173.407
							0.3				5.665	4.266	84.057	198.099
								1.5			4.209	5.963	97.824	156.897
								1.75			4.217	5.325	99.926	166.107
								2			4.237	4.785	102.087	173.407
									1100		3.774	4.486	108.682	171.983
									1200		4.237	4.785	102.087	173.407
									1300		4.662	5.058	95.486	174.967

6 SENSITIVITY ANALYSIS OF THE MODEL

Sensitivity analysis is carried out to explore the effect of changes in model parameters and costs on the optimal policies, by varying each
parameter (-15%,-10%,-5%, 0%, 5%, 10%, 15%) at a time for the model under study. The results are presented in Table 2.

As the ordering cost ‘A’ decreases, the optimal value of t_1^*, the optimal replenishment time t_3^*, the optimal ordering quantity Q* are constant and total cost is decreasing and if ‘A’ increases, the optimal value of t_1^*, the optimal replenishment time t_3^*, the optimal ordering quantity Q* are constant and total cost is increasing. When the cost per unit ‘C’ is decreasing, the optimal value of t_1^*, the optimal replenishment time t_3^* and the optimal ordering quantity Q* are increasing and the total cost is decreasing. If ‘C’ is increasing, the optimal value of t_1^*, the optimal replenishment time t_3^*, the optimal ordering quantity Q* are decreasing and the total cost is increasing.

As the Production parameters ‘a’ decreases, the optimal value of t_1^*, the optimal replenishment time t_3^* are increasing, the total cost and optimal parameter (-15%,-10%,-5%, 0%, 5%, 10%, 15%) at a time for the model under study. The results are presented in Table 2.

As the ordering cost ‘A’ decreases, the optimal value of t_1^*, the optimal replenishment time t_3^*, the optimal ordering quantity Q* are constant and total cost is decreasing and if ‘A’ increases, the optimal value of t_1^*, the optimal replenishment time t_3^*, the optimal ordering quantity Q* are constant and total cost is increasing. When the cost per unit ‘C’ is decreasing, the optimal value of t_1^*, the optimal replenishment time t_3^* and the optimal ordering quantity Q* are increasing and the total cost is decreasing. If 'C' is increasing, the optimal value of t_1^*, the optimal replenishment time t_3^*, the optimal ordering quantity Q* are decreasing and the total cost is increasing.

As the Production parameters 'a' decreases, the optimal value of t_1^*, the optimal replenishment time t_3^* are increasing, the total cost and optimal ordering quantity Q* are decreasing. If 'a' increases, the optimal value of t_1^*, the optimal replenishment time t_3^* are decreasing, total cost and the optimal ordering quantity Q* are increasing. If 'b' decreases, the optimal value of t_1^*, the optimal replenishment time t_3^* are increasing, total cost and the optimal ordering quantity Q* are decreasing. If 'b' increases, the optimal value of t_1^*, the optimal replenishment time $t_3^*$are decreasing, total cost and the optimal ordering quantity Q* are increasing.

As the deteriorating parameters 'α' decreases, the optimal value of t_1^*, the optimal ordering quantity Q* are increasing, the optimal replenishment time t_3^* and the total cost is decreasing. If 'α' increases, the optimal value of t_1^* and the optimal ordering quantity Q* are decreasing, the optimal replenishment time t_3^* and total cost are increasing. If 'β' decreases, the optimal value of t_1^*, the optimal ordering quantity Q* are increasing. The optimal replenishment time t_3^* and total cost are decreasing. If 'β' increases, the optimal value of t_1^* and the optimal ordering quantity Q* are decreasing, the optimal replenishment time t_3^* and total cost are increasing

When the shortage cost per unit 'π' decreases, the optimal value of t_1^*, the optimal replenishment time t_3^* are decreasing, the optimal ordering quantity Q* and the total cost is increasing.
If 'π' increases, the optimal value of t_1^*, the optimal replenishment time t_3^* is increasing and the optimal ordering quantity Q* and total cost are decreasing.

As the holding cost per unit 'h' decreases, the optimal value of t_1^* and the optimal ordering quantity Q* are decreasing, the optimal replenishment time t_3^*, and total cost are increasing. If 'h' increases, the optimal value of t_1^*, the optimal ordering quantity Q* is increasing, the optimal replenishment time t_3^* and total cost are decreasing. When the indexing parameter 'n' decreases, the optimal value of t_1^*, the optimal ordering quantity Q* and total cost are decreasing and the optimal replenishment time t_3^* is increasing. If 'n' increases, the optimal value of t_1^*, the optimal ordering quantity Q*, and total cost are increasing and the optimal replenishment time t_3^* is decreasing. quantity Q*and total cost is increasing and the optimal replenishment time t_3^* is decreasing. value of t_1^*, the optimal replenishment time t_3^* and the optimal ordering quantity Q* are decreasing and the total cost is increasing.

As the demand parameter 'r' decreases, the optimal value of t_1^*, the optimal replenishment time t_3^* and the optimal ordering quantity Q* are decreasing and the total cost is increasing. If 'r' increases, the optimal value of t_1^*, the optimal replenishment time t_3^* and the optimal ordering quantity Q* are increasing and the total cost is decreasing

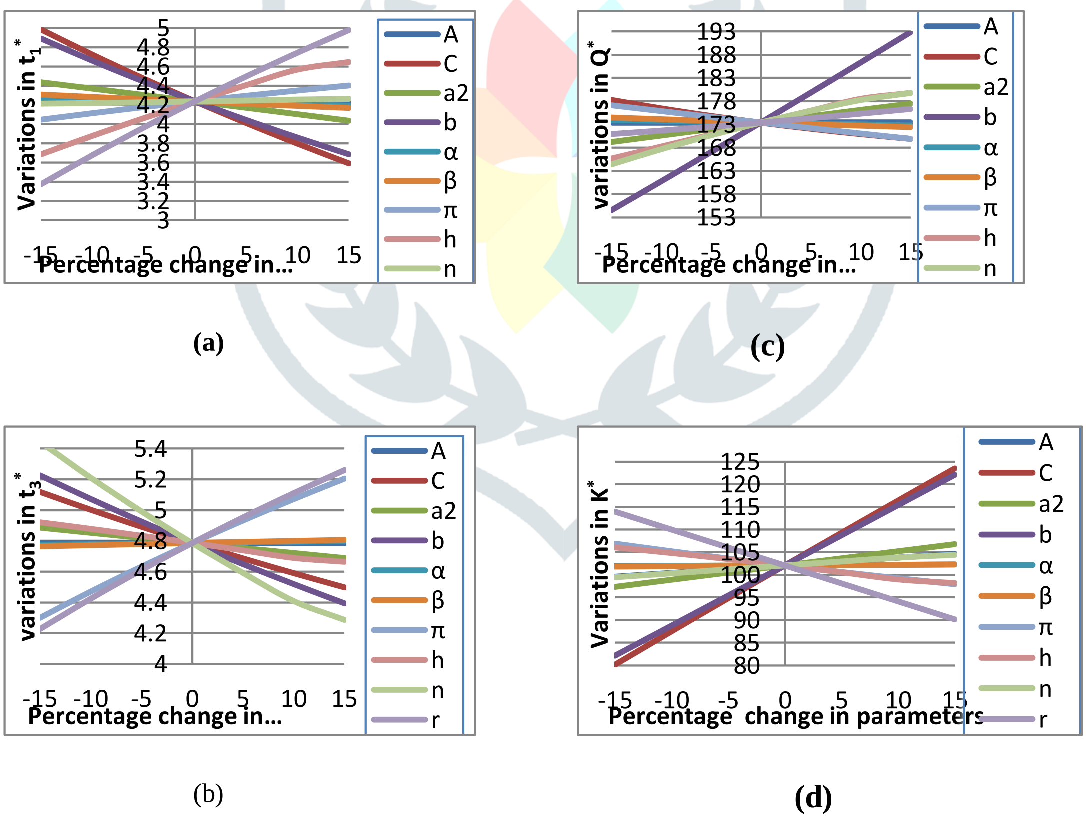


Fig 2: Relationship between parameters and optimal values with shortages

7. INVENTORY MODEL WITHOUT SHORTAGES

In this section, the inventory model for deteriorating items without shortages is developed and analyzed. Here, it is assumed that the shortages are not allowed and the stock level is zero at time t=0. The stock level increases during the period (0, t_1) due to excess replenishment after fulfilling the demand and deterioration. The replenishment stops at time t_1 when the stock level reaches a maximum. The inventory decreases gradually due to demand and deterioration in the interval (t_1, T). At time T the inventory reaches zero. The schematic diagram representing the instantaneous state of inventory is given in figure 3.

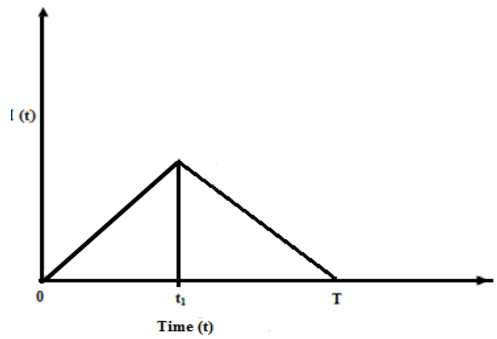


Figure 3.3 Instantaneous state of inventory for model

Let I (t) be the inventory level of the system at time t $(0 \le t \le T)$.

Then the differential equations governing the instantaneous state of I (t) over the cycle of length T are:

$$\frac{d}{dt} I(t) + (\alpha + \beta t) I(t) = (a + b t) - \lambda(t); \qquad 0 \le t \le t_1 \qquad (24) \qquad \frac{d}{dt} I(t) + (\alpha + \beta t) I(t) = -\lambda(t); \qquad t_1 \le t \le T \qquad (7.1)$$

With the boundary conditions I (T) = 0, I (0) = 0

The instantaneous state of inventory at any given time t during the interval (0, T) is

$$I(t) = e^{-(\alpha t + \beta t^2/2)} \int_0^t \left((a + bt) - \frac{r t^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} \right) e^{(\alpha t + \beta t^2/2)} dt; \quad 0 \le t \le t_1 \qquad (7.2)$$

$$I(t) = e^{-(\alpha t + \beta t^2/2)} \int_t^T \frac{r t^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} e^{(\alpha t + \beta t^2/2)} dt \quad ; \quad t_1 \le t \le T \qquad (73)$$

The production quantity during the cycle time (0, T) is given by

$$Q = a t_1 + \frac{b t_1^2}{2} \qquad (7.4)$$

Using equations (7.2) and (7.3), we obtain the stock loss due to deterioration in this interval (0, T) as the

difference between the total quantity produced and the demand met during (0, T) and is given by

$$L(T) = a t_1 + b \frac{t_1^2}{2} - \lambda(t) T \qquad (7.5)$$

This amount of quantity is lost due to the deterioration of the commodity and is a waste. To obtain the optimal operating policies one must reduce the stock loss due to deterioration.

Let K (t_1) is the total cost per unit time. The total cost is the sum of the setup cost, cost of units, and the inventory holding cost. Therefore the total cost is

$$K(t_1) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T} \left[\int_0^{t_1} I(t) dt + \int_{t_1}^{T} I(t) dt \right] \qquad (7.6)$$

Substituting the value of I (t) and Q given in equations (7.2), (7.3) and (7.5) in the equation (7.7) we obtain K (t_1) as

$$K(t_1) = \frac{A}{T} + \frac{C}{T}\left(a t_1 + b \frac{t_1^2}{2} \right) + \frac{h}{T} \left\{ \int_0^{t_1} e^{-(\alpha t + \beta \frac{t^2}{2})} \left(\int_0^t \left((a + bu) - \left(\frac{r u^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} \right) \right) e^{(\alpha u + \beta \frac{u^2}{2})} du \right) dt \right.$$
$$\left. + \int_{t_1}^{T} e^{-(\alpha t + \beta \frac{t^2}{2})} \left(\int_t^T \left(\frac{r u^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} \right) e^{(\alpha u + \beta \frac{u^2}{2})} du \right) dt \right\} \qquad (7.7)$$

On integrating and simplifying equation (4.8) we get

$$K(t_1) = \frac{A}{T}+\frac{C}{T}\left(at_1+b\frac{t_1^2}{2}\right)+\frac{h}{T}\left\{a\int_0^{t_1}\left[e^{-(\alpha t+\beta\frac{t^2}{2})}\int_0^t e^{(\alpha u+\beta\frac{u^2}{2})}du\right]dt\right.$$
$$+b\int_0^{t_1}\left[e^{-(\alpha t+\beta\frac{t^2}{2})}\int_0^t u\, e^{(\alpha u+\beta\frac{u^2}{2})}du\right]dt$$
$$-\frac{r}{nT^{\frac{1}{n}}}\int_0^{t_1}\left[e^{-(\alpha t+\beta\frac{t^2}{2})}\int_0^t u^{\frac{1}{n}-1}\ e^{(\alpha u+\beta\frac{u^2}{2})}du\right]dt \qquad (7.8)$$
$$-\frac{r}{nT^{\frac{1}{n}}}\left[\int_{t_1}^{T}\left(e^{-(\alpha t+\beta\frac{t^2}{2})}\int_{t_1}^{t} u^{\frac{1}{n}-1}\ e^{(\alpha t+\beta\frac{t^2}{2})}du\right)dt\right.$$
$$\left.\left.-\int_{t_1}^{T}\left(e^{-(\alpha t+\beta\frac{t^2}{2})}\int_{t_1}^{T} u^{\frac{1}{n}-1}\ e^{(\alpha u+\beta\frac{u^2}{2})}du\right)dt\right]\right\}$$

8. OPTIMAL PRICING AND ORDERING POLICIES OF THE MODEL

In this section, we obtain the optimal policies of the inventory system under study. To find the optimal values of production time t_1, we equate the first-order partial derivative of K (t_1) to t_1 and equate it to zero. The condition for a minimum of K (t_1) is

$$\frac{\partial^2 K(t_1)}{\partial t_1^2} > 0$$

Differentiate K (t_1) to t_1 and equate it to zero, we get

$$\frac{C}{T}\left[a+2bt_1\right]+\frac{h}{T}\left\{a\, e^{-(\alpha t_1+\beta\frac{t_1^2}{2})}\int_0^{t_1} e^{(\alpha u+\beta\frac{u^2}{2})}du+b\, e^{-(\alpha t_1+\beta\frac{t_1^2}{2})}\int_0^{t_1} u\, e^{(\alpha u+\beta\frac{u^2}{2})}du\right.$$
$$\left.-\frac{r}{nT^{\frac{1}{n}}}\left[e^{-(\alpha t_1+\beta\frac{t_1^2}{2})}\int_0^{t_1} u\, e^{(\alpha u+\beta\frac{u^2}{2})}du+e^{-(\alpha t_1+\beta\frac{t_1^2}{2})}\int_{t_1}^{T} e^{(\alpha u+\beta\frac{u^2}{2})}du\right]\right\}=0 \qquad (8.1)$$

Solving the equation (7.8) we obtain the optimal time at which the replenishment is to be stopped t_1^* of t_1. The optimum ordering quantity Q^* of Q in the cycle of length T is obtained by substituting the optimal values of t_1 in (28) as

$$Q^* = a{t_1}^* + \frac{bt_1^{2*}}{2} \qquad (8.2)$$

9. NUMERICAL ILLUSTRATION

In this section, we discuss a numerical illustration of the model. For demonstrating the solution procedure of the model, the deteriorating Parameter 'α' is considered to vary as 0.1, 0.2, 0.3, the values of other parameters and costs associated with the model are:

A = 190, 200, 210; a = 2, 3, 4; b = 1, 2, 3; β =0 .01, 0.02, 0 .03, C = 9, 10, 11; h = 0.01, 0.02, 0.03; n = 1, 2, 3; r = 1150, 1200, 1250; T = 12 months

Substituting these values of optimal ordering quantity Q*, an optimal value of time and the total cost is computed and presented in Table 3.

As the ordering cost 'A' is increasing from 190 to 210, the optimal value of t_1^*, the optimal ordering quantity Q* are constant. The total cost is increasing from 116.814 to 118.481. When the cost per unit 'C' is increasing from 9 to 11, the optimal value of t_1^* decreases from 10.852 to 10.792, the optimal ordering quantity Q* decreases from 150.338 to 148.874, the total cost is increasing from 105.152 to 130.082.

When the Production parameters 'a' increase from 2 to 4, the optimal value of t_1^* decreases from 10.833 to 10.809, the optimal ordering quantity Q* increases from 139.040 to 160.086, the total cost is increasing from 108.428 to 126.857. As 'b' is increasing from 1 to 3. The optimal value of t_1^* decreases from 10.993 to 10.712, the optimal ordering quantity Q* increases from 93.408 to 204.268, the total cost is increasing from 67.438 to166.620.

As the deteriorating parameter 'α' increases from 0.01 to 0.03, the optimal value of t_1^* increases from 10.818 to 10.824, the optimal ordering quantity Q* increases from 149.504 to 149.634, the total cost increases from 117.285 to 118.004. If the deteriorating parameters 'β' increases from 0.2 to 0.4, the optimal value of t_1^* increases from 10.405 to 11.056, the optimal ordering quantity Q* increases from 139.482 to 155.412, the total cost increases from 102.973 to 126.242.

When the inventory holding cost per unit 'h ' is increasing from 0.2 to 0.4, the optimal value of t
increases from 10.700 to 10.906, the optimal ordering quantity Q* increases from 146.605 to $_1{}^*$ 151.665, the total cost is decreasing from 125.095 to 109.484.

As the indexing parameter ‘n’ is increasing from 1 to 3, the optimal value of t_1^* decreases from 11.023 to 10.700, the optimal ordering quantity Q* decreases from 154.578 to 146.614, the total cost is decreasing from 132.941 to 108.013. When the demand parameter ‘r’ is increasing from 1100 to 1300, the optimal value of t_1^* increases from 10.795 to 10.845, the optimal ordering quantity Q* increases from 148.929 to 150.159, the total cost is decreasing from 119.763 to 115.489.

TABLE 3

OPTIMAL VALUES OF t_1^*, Q^* **and K^* FOR DIFFERENT VALUES OF PARAMETERS WITHOUT SHORTAGES**

A	**C**	**a**	**b**	**α**	**β**	**h**	**T**	**n**	**r**	**t_1^***	**K***	**Q***
200	10	3	2	0.02	0.3	.3	12	2	1200			
190										10.821	116.814	149.569
200										10.821	117.648	149.569
210										10.821	118.481	149.569
	9									10.852	105.152	150.338
	10									10.821	117.648	149.569
	11									10.793	130.082	148.874
		2								10.833	108.428	139.040
		3								10.821	117.648	149.569
		4								10.809	126.857	160.086
			1							10.993	67.438	93.408
			2							10.821	117.648	149.569
			3							10.712	166.620	204.268
				0.01						10.818	117.285	149.504
				0.02						10.821	117.648	149.569
				0.03						10.824	118.004	149.634
					0.2					10.405	102.973	139.482
					0.3					10.821	117.648	149.569
					0.4					11.056	126.242	155.412
						0.2				10.700	125.095	146.605
						0.3				10.821	117.648	149.569
						0.4				10.906	109.484	151.665
								1		11.023	132.941	154.578
								2		10.821	117.648	149.569
								3		10.700	108.013	146.614
									1100	10.795	119.763	148.929
									1200	10.821	117.648	149.569
									1300	10.845	115.489	150.159

10. SENSITIVITY ANALYSIS OF THE MODEL

Sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal policies, by varying each parameter (-15%,-10%,-5%, 0%, 5%, 10%, 15%) at a time for the model under study. The results are presented in Table 4.

As the ordering cost ‘A’ decreases, the optimal value of t_1^*, the optimal ordering quantity Q* are the constants and the total cost is decreasing and if ‘A’ increases, the optimal value of t_1^*, the optimal ordering quantity Q* are the constant and total cost is increasing.

When the cost per unit ‘C’ is decreasing, the optimal value of t_1^*, the optimal ordering quantity Q* are increasing and the total cost is decreasing. If ‘C’ is increasing, the optimal value of t_1^*, the optimal ordering quantity Q* are decreasing, the total cost is increasing. If the Production parameters ‘a’ decreases, the optimal value of $t_1^*$is increasing, the optimal ordering quantity Q* and the total cost are decreasing. If ‘a’ increases, the optimal value of $t_1^*$is decreases, the total cost and the optimal ordering quantity Q* are increasing. If ‘b’ decreases, the optimal value of t_1^* is increasing, the total cost and the optimal ordering quantity Q* are decreasing. If ‘b’ increases, the optimal value of t_1^* is decreasing and the total cost and the optimal ordering quantity Q* are increasing.

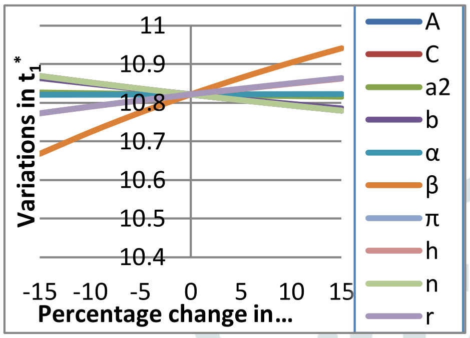
As the deteriorating parameters ‘α’ decreases, the optimal value of t_1^*, the optimal ordering quantity Q* and the total cost are decreasing. If ‘α’ increases, the optimal value of t_1^*, the total cost and the optimal ordering quantity Q* are increasing. If ‘β’ decreases, the optimal value of t_1^*, the optimal ordering quantity Q* and the total cost are decreasing. If ‘β’ increases, the optimal value of t_1^*, the total cost and the optimal ordering quantity Q* are increasing.

As the holding cost per unit ‘h’ decreases, the optimal value of t_1^*, the optimal ordering quantity Q* are decreasing and the total cost is increasing. If ‘h’ increases, the optimal value of t_1^* and the optimal ordering quantity Q* are increasing, and the total cost is decreasing.

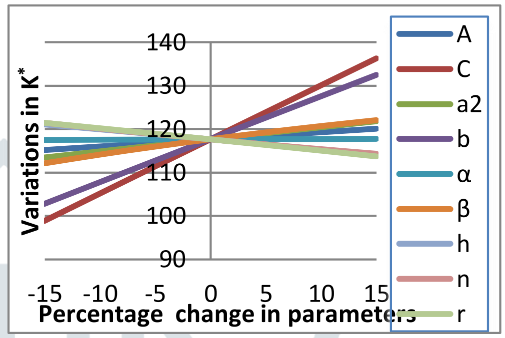
As the indexing parameter ‘n’ decreases, the optimal value of t_1^*, the optimal ordering quantity Q* and total cost are increasing. If ‘n’ increases, the

optimal value of t_1^*, the optimal ordering quantity Q* and total cost are decreasing. As the demand parameter ‘r’ decreases, the optimal value of t_1^* and

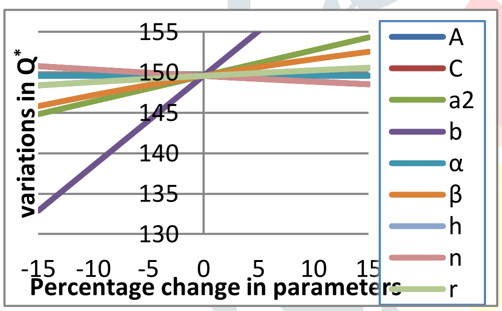
the optimal ordering quantity Q* are decreasing and the total cost is increasing. If ‘r’ increases, the optimal value of t_1^* and the optimal ordering quantity Q* are increasing and the total cost is decreasing.



(a)



(b)



(c)

Fig 4: Relationship between parameters and optimal values without shortages

CONCLUSION:

In this paper, an economic production quantity model for deteriorating items is developed and analyzed with the assumption that the demand is a function of time. It is known that time influences the demand for the commodity. Here, it is assumed that the demand follows power patterns with an indexing parameter, the demand rate may be increasing/decreasing/constant rates. The lifetime of the commodity for some items is finite and dependent on time. The time dependent deterioration is represented by a linear function is of time. The power pattern demand includes a wide variety of demand functions for different types of commodities with suitable cost considerations, the total cost function is obtained. This model is extended to the case of without shortages. From the sensitivity analysis, it is observed that production parameters and demand parameters have a significant influence on the optimal operating policies of the model. This model is useful for analyzing the situations in food processing, agricultural production units where the production, demand and deterioration are all time dependent demand

REFERENCES

[1]. Nahmias 1982, Perishable inventory theory : review, Opsearch, Vol. 30, No. 4, pp 680-708.

[2]. Aggarwal and Goyal 1984, Order level inventory systems with demand pattern for deteriorating items, Operations Research in Managerial Systems, Vol. 14, pp: 176 – 187.

[3]. Raafat 1991, Survey of Literature on Continuously Deteriorating Inventory Models,Journal of the Operational Research Society, Vol. 42, No. 1, pp: 27-37.

[4]. Goyal and Giri 2001, Recent trends in the modelling of deteriorating inventory, European Journal of Operational Research, Vol.134, No.1, pp:1-16.

[5]. Srinivasa Rao, Vivekananda Murty and Eswara Rao 2005. Optimal ordering and pricing policies of inventory models for deteriorating items with generalized Pareto lifetime, Journal of Stochastic Process and its Applications, Vol.8, No.1, pp: 59-72.

[6]. Srinivasa Rao, Prasad Reddy, Gopinath 2006, Inventory model with hypo exponential lifetime having demand as a function of selling price and time. Ultra Science, Vol.18, No.1, pp: 57- 64.

[7]. Uma Maheswara Rao, Venkata Subbaiah, Srinivasa Rao 2010. Production Inventory Models for Deteriorating Items with Stock Dependent demand and Weibull Decay, IST Transaction of Mechanical Systems- Theory and Applications, Vol.1, No.2, pp: 13-23.

[8]. Sunil Kawale, Pravin Bansode 2012, An EPQ Model using Weibull Deterioration for Deterioration Item with Time-Varying Holding Cost, International Journal of Science, Engineering and Technology Research(IJSETR), ISSN: 2278 – 7798, Volume 1, Issue 4, October 2012.

[9]. Vinod Kumar Mishra1, Lal Sahab Singh and Rakesh Kumar2013, An inventory model for deteriorating items with time-dependent demand and time-varying holding cost under partial backlogging. Journal of Industrial Engineering International.doi:10.1186/51- 712X-9-4

[10] . Juanjuan Qina 2015.An EPQ Model with Increasing Demand and Demand Dependent Production Rate Under Trade CreditFinancing.IJSOM,ISSN-Print;2383-1359,ISSN- Online-2383-2525, Vol:1, IssueNo:1, pp.532- 547, www.ijsom.com

[11]. Srinivasa.Rao, Srinivasa Rao,.Kesava Rao 2016.Optimal Operating Policies of an EPQ Model With Dependent Production Rate and Time Dependent Demand Having Pareto Decay. (ICRISEM-16),ISBN:978- 81-932074-1-3

[12] . Sharmila,Uthayakumar2016. An Inventory Model with Three Rates of Production Rate under Stock and Time Dependent Demand for Time Varying Deterioration Rate with hortages International Journal Advanced Engineering,ManagementandScience (IJAEMS), Vol-2,Issue-9,Sept-2016,ISSN:2454-1311,Infogain Publication Infogainpublication.com

[13]. Ardak, Borade 2017.An Economic Production Quantity model. with inventory dependent demandand deterioration. ISSN (Print) 2319-8613, ISSN (Online):0975-4024,Vol 9,IssueNo:2,Apr-May 2017, DOI:10.21817/ijet/2017/v9i2/170902197.

[14]. Aruna Kumari 2017.Optimal Operating Policies of EPQ Model for Deteriorating Items with Time DependentProduction having Production Quantity Dependent Demand.International Journal for Innovative Research in Science & Technology (IJIRST), Volume 3, Issue 12, May 2017, ISSN (online): 2349-6010.

[15]. Ardak, Borade, Renata Stasiak Betlejewska 2017. An EPQ Model for Deteriorating Items with Mix Demand Pattern. International Journal of Mechanical Engineering and Technology, Vol:8,issue:(6),2017,pp59-69, http://www.iaeme.com

[16]. Deepa Khurana, Shilpy Tayal and Singh 2018. An EPQ model for deteriorating items with Variable demand rate and allowable shortages. International journal of Mathematics in Operational Research, Vol. 12, No. 1, 2018, Copyright © 2018 Inderscience Enterprises Ltd.

[17]. Ali Akbar Shaikh , Amalesh Kumar Manna 2020 , An EPQ model for a deteriorating item with Partial Trade Credit Policy for price dependent demand under inflation and reliability. DOI:.2298/YJOR200515036S Received: May 2020 / Accepted: June 2020

[18]Chayanika Rout, Debjani Chakraborty and Adrijit Goswam i2021, Received August 23,2018. Accepted August 14, 2019. A Production inventory model for deteriorating items with backlog dependent demand. RAIRO-Operations Research 55 (2021) S549–S570, RAIRO Operations Rhttps://doi.org/10.1051/ro/2019076, www.rairo-ro.org

[19] Malumfashi · Mohd Tahir Ismail ·Majid Khan Majahar Ali 2022, An EPQ Model for Delayed Deteriorating Items with Two-Phase Production Period, Exponential Demand Rate and Linear Holding Cost Bull. Malays. Math. Sci. Soc. (2022) 45 (Suppl1):S395–S424, https://doi.org/10.1007/s40840-022-01316-x, Received: 6 July 2021 / Revised: 4 May 2022 / Accepted: 8 May 2022 / Published online: 9, June 2022 ©

20] KATARIYA, Dharmesh; SHUKLA, Kunal Tarunkumar. 2023: An EPQ Model for Delay Deteriorating Products With Price, Freshness and Greening Efforts Dependent Demand Under Markdown Strategy. **Yugoslav Journal of Operations Research**, [S.l.],sep.2023.ISSN:2334-6043.Available hhttp:// dx.doi.org/ 10.2298 /YJOR230515023K