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Advance Robust Nonlinear Control Algorithm for Quadrotor Applications

[1]Praveen Kumar Goskula, [2] Lini Mathew, [3] Shimi S.L, [4] Sandeep Gupta

[1]ME Scholar, [2]Professor and Head, [3]Associate Professor, [4] Senior Research Fellow
[1, 2]Department of Electrical Engineering
[1, 2] National Institute of Technical Teachers Training and Research (NITTTR), Chandigarh,
[3] Punjab Engineering College Chandigarh, [4] IIT Kanpur, India

Abstract: One of the main challenges for quadrotor is desired trajectory tracking in presence of external disturbances. This paper addresses the quadrotor tracking control for external disturbances. Quadrotor dynamic modeling is taken by considering drag forces, aerodynamic friction and gyroscopic effects. So the new state space model is represented for the mathematical modeling of quadrotor. Two non linear controllers, sliding mode controller and backstepping controller are combined. The new sliding surface is designed using Lyapunov based adaptive law to compensate for the effect of external disturbances. The new law is asymptotically stable as tracking errors are reduced to zero. The controller algorithm is implemented using MATLAB simulations and compares with the nominal SMC with backstepping controller. And results shows new law is successfully compensate the external disturbances.

Index Terms– **Matlab, sliding mode control (SMC), Backstepping control (BSC)**

I. INTRODUCTION

The quadrotor is also known as quadcopter and it is characterized by a unique four rotor design. It has the ability to take off and land vertically even in a small place. Firstly these are developed mainly for military application purposes but because of its various advantages these are inevitable in many applications like agriculture, aerial photography etc. the critical aspect of the quadrotor is trajectory tracking which is an intense research area in recent years. Because of quadrotor nonlinear dynamics, controlling is a challenging task. In the presence of external disturbances it is difficult to achieve desired trajectory and stability of the system.

Mathematical modeling of the quadrotor is a difficult task, several quadrotor models like non linear model, quaternion model and near hover position model was discussed [1]. These mathematical models are represented in state space form. To implement the control techniques on dynamics of quadrotor state space modeling is easy [2]. Inertia, drag forces, aerodynamic friction and gyroscopic effects are considered for more effective modeling of the quadrotor [3].

Many linear and nonlinear control strategies were applied to the quadrotor Unmanned Aerial Vehicle (UAV), such as PD control, PID control, Trajectory Linearization Control (TLC), Sliding Mode Controller (SMC), Backstepping Controller (BSC) etc [24][25]. PID controller is a most popular controller but main disadvantage is tuning the controller gain which is a tedious process. Most of the PID controllers are focusing on tuning these controller gains. Two degrees of freedom PID controllers with tuning parameters individually without affecting the other parameters proposed [4]. Adaptive PID controller with particle swarm optimization used for automatic tuning based on strictly negative imaginary theory [5]. A nonlinear PID controller with Hurwitz stability theorem is used genetic algorithm is used for tuning the parameters [6]. Intelligent active force control integrated with PID controller is used to stabilize quadrotor and to compensate the external disturbances [7]. Backstepping control is well suited for stabilize nonlinear dynamic systems. It starts by defining a set of virtual control inputs then by using virtual control laws state variables are stabilized using recursive scheme. This paper proposed backstepping controller integrated with an auxiliary input saturation compensator, disturbance observer is employed and finite time stability is derived [8]. An adaptive neural tracking control law based back stepping control is designed which avoids singularity problem of virtual controls [9]. For quadrotor slung load, a nonlinear backstepping controller is used by introducing virtual thrust force for position control [10]. An adaptive backstepping controller is designed to overcome the problem of unknown input gains and improves tracking performance [11]. Robust backstepping control proposed the wind estimator is designed using neural networks levenberg marquart algorithm with back propagation [12]. Sliding mode controller, sliding surface is designed and controller is designed to derive the state trajectory onto the sliding surface and then maintain it there. It is a robust and nonlinear control technique and its main drawback is chattering problem. Both PID and sliding mode controller are designed by using iteration method coefficients are tuned [13]. Non singular fast terminal sliding mode control is proposed and to estimate the unknown external disturbances a nonlinear disturbance observer is designed for robust performance [14]. An adaptive fractional order nonsingular fast terminal sliding mode controller is designed for fast time convergence and reject uncertainties [15]. An adaptive sliding mode controller is proposed and these adaptive laws are used to detect actuator faults and improve the stability [16]. Sliding mode controller is integrated with neural network algorithm, which results in time varying sliding surfaces [17]. To estimate upper bounds of disturbance and physical parameters adaptive second order system based sliding mode controller is designed [18]. Sliding mode controller integrated with fuzzy network is proposed to compensate the external disturbances [19]. Appointed fixed time sliding mode controller is designed and its stability is verified using Lyapunov theorem [20]. Robust sliding mode controller with PID controller is designed observer estimate the disturbances and ensures exponential convergence [21]. Sliding mode controller is integrated with iterative learning control, this algorithm force the state trajectory on to the sliding surface without need of accurate dynamics of quadrotor [22]. A gain scheduling based SMC law is synthesized to compensate the presence of uncertainties in the system and it is integrated with nonlinear disturbance observer to reduce disturbances [23]. This paper presents sliding mode backstepping controller with new Lyapunov based adaptive law to compensate the external disturbances. This control strategy is effective for trajectory tracking of quadrotor as it combines the advantages of both sliding mode controller and back stepping controller. This approach offers robust and accurate trajectory tracking capabilities in the presence of external disturbances and complex dynamics. The main contribution of the paper is summarized as follows. i) The quadrotor model is developed by considering aerodynamic friction torques gyroscopic effects and drag forces. ii) The state space model is designed by considering all system nonlinearities. iii) Backstepping sliding mode controller is designed. iv) Lyapunov based new adaptive law is synthesized to compensate the disturbances. The rest of the paper is arranged as follows. Section II describes the modeling of the quadrotor. Section III describes the controller design and laws. Section IV presents simulation results of the proposed controller. Section V carries the conclusion.

II. QUADROTOR DYNAMIC MODELLING:

In this section we will discuss the mathematical model of a symmetrical rigid quadrotor based on Newton-Euler formulation. Quadrotor is equipped with four rotors that are directed upwards. It is an underactautated system with four inputs to control six degrees of freedom. Input thrust is generated by four propellers, which can be controlled. Six degrees of freedom in space includes translation motion x, y and z in three directions and rotational motion roll, pitch and yaw around three axes. For the quadrotor kinematic and dynamic equations are derived in both the inertial frame and the body fixed frame, assuming that the center of gravity of the quadrotor coincides with the origin of the body fixed frame. The transformation from the inertial reference frame to the body fixed reference frame of the quadrotor is given by a rotational matrix.

$$R_i^b(\phi,\theta,\psi) = \begin{pmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta c\psi - c\phi s\psi & s\phi s\theta s\psi + c\phi c\psi & s\phi c\theta \\ c\phi s\theta c\varphi + s\phi s\psi & c\phi s\theta s\psi - s\phi c\psi & c\phi c\theta \end{pmatrix} \quad (1)$$

where $c \triangleq cos$ and $s \triangleq sin$.

The position derivative vector P is in the inertial frame and the velocity vector V is in the body frame. They can be related to each other through a rotational matrix such as $P = R_i^b(\phi,\theta,\psi)\, V$ where $P = (\dot{x},\ \dot{y},\ \dot{z})^T$ and $V = (u, v, w)^T$, and the angular derivative vector R= $(p,\ q, r)^T$ and angular velocity vector A =$(\dot{\phi}\ \ \dot{\theta}\ \ \dot{\psi})^T$ are related by the equation

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 1 & \mathrm{s}(\phi)\,\mathrm{t}(\theta) & \mathrm{c}(\phi)\,\mathrm{t}(\theta) \\ 0 & \mathrm{c}(\phi) & -\mathrm{s}(\phi) \\ 0 & \mathrm{s}(\phi)\mathrm{sec}(\theta) & \mathrm{c}(\phi)\mathrm{sec}(\theta) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} \quad (2)$$

where $s(.)$= sin(.) , $t(.)$= tan(.) and $c(.)$= cos(.)

Translational dynamic equations of the quadrotor can be written by using Newton's laws as below

$$\dot{V} = \begin{pmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{pmatrix} V \quad (3)$$

Above matrix is skew symmetric matrix.

$$m\dot{P} = F_r + F_g + F_d \quad (4)$$

Where total mass of the quadrotor is represented by m, F_r is the resultant forces which are generated by the four rotors, F_g is the force of the gravity and F_d is the resultant drag forces along translation axis which are given as

$$F_r = \begin{pmatrix} c\phi s\theta c\psi + s\phi s\psi \\ c\phi s\theta s\psi - s\phi c\psi \\ c\phi c\theta \end{pmatrix} U_1 \quad (5)$$

where $c(.)$= cos(.) and $s(.)$= sin(.) respectively

$$F_g = [0 \quad 0 \quad -mg]^T \quad (6)$$

$$F_d = \begin{pmatrix} -K_{dx} & 0 & 0 \\ 0 & -K_{dy} & 0 \\ 0 & 0 & -K_{dz} \end{pmatrix} P \quad (7)$$

Such as K_{dx}, K_{dy} and K_{dz} are the translation drag coefficients.

Rotational dynamic equations of the quadrotor can be derived by using Newton's laws as follows

$$\dot{A} = -A \times J_b A + \tau_r - \tau_a - \tau_g \quad (8)$$

where J_b is the matrix that represents quadrotor constant inertia in a symmetric manner

$$J_b = \begin{pmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{pmatrix} \quad (9)$$

Rotor torques developed by the quadrotor is denoted by τ_r and it is expressed as follows

$$\tau_r = \begin{pmatrix} lU_2 \\ lU_3 \\ U_4 \end{pmatrix} \quad (10)$$

τ_a is the aerodynamic frictions torques expressed as follows

$$\tau_a = \begin{pmatrix} K_{ax} & 0 & 0 \\ 0 & K_{ay} & 0 \\ 0 & 0 & K_{az} \end{pmatrix} R^2 \quad (11)$$

K_{ax}, K_{ay} and K_{az} are the aerodynamic friction coefficients
$\tau_g$is the resultant torque caused by the gyroscopic
effects which is expressed as

$$\tau_g = \sum_{i=1}^{4} R \times J_r (-1)^{i+1} \Omega \quad (12)$$

J_r is the rotor inertia and Ω is the rotor speed of the quadrotor expressed as

$$\Omega = \Omega_1 - \Omega_2 + \Omega_3 - \Omega_4 \quad (13)$$

The control inputs are derived using angular velocities as

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{pmatrix} K_P & K_P & K_P & K_P \\ -K_P & 0 & K_P & 0 \\ 0 & -K_P & 0 & K_P \\ K_d & -K_d & K_d & -K_d \end{pmatrix} \begin{bmatrix} {\Omega_1}^2 \\ {\Omega_2}^2 \\ {\Omega_3}^2 \\ {\Omega_4}^2 \end{bmatrix} \quad (14)$$

where K_P and K_d are the thrust and drag coefficients respectively.
Quadrotor uses four rotors powered by DC motors and the rotor model is expressed as

$$\sum_{i=1}^{4} \dot{\Omega}_i = b \sum_{i=1}^{4} V_i - \beta_0 - \beta_1 \sum_{i=1}^{4} \Omega_i - \beta_2 \sum_{i=1}^{4} \Omega_i^2 \quad (15)$$

III. CONTROLLER DESIGN

The above quadrotor mathematical model can be represented by the state space model as

$$\dot{X} = f(X, U)$$

The symbols X and U represent the state vector and control inputs, respectively

$$X = [\phi \quad \dot{\phi} \quad \theta \quad \dot{\theta} \quad \psi \quad \dot{\psi} \quad x \quad \dot{x} \quad y \quad \dot{y} \quad z \quad \dot{z}]^T \quad (16)$$

$$U = [U_1 \quad U_2 \quad U_3 \quad U_4]^T \quad (17)$$

$$\ddot{\phi} = \frac{(J_y - J_z)}{J_x} qr + \frac{l}{J_x} U_2 - \frac{1}{J_x}\left(K_{ax}\dot{\phi}^2 - J_r \Omega \dot{\theta}\right) \quad (18)$$

$$\ddot{\theta} = \frac{J_z - J_x}{J_y} pr + \frac{l}{J_y} U_3 - \frac{1}{J_y}\left(K_{ay}\dot{\theta}^2 + J_r \Omega \dot{\phi}\right) \quad (19)$$

$$\ddot{\psi} = \frac{J_x - J_y}{J_z} pq + \frac{1}{J_z} U_4 - \frac{1}{J_z} K_{az}\dot{\psi}^2 \quad (20)$$

$$\ddot{x} = (c\phi s\theta c\psi + s\phi s\psi)\frac{U_1}{m} - \frac{K_{dx}}{m}\dot{x} \quad (21)$$

$$\ddot{y} = (c\phi s\theta s\psi - s\phi c\psi)\frac{U_1}{m} - \frac{K_{dy}}{m}\dot{y} \quad (22)$$

$$\ddot{z} = (c\phi c\theta)\frac{U_1}{m} - g - \frac{K_{dz}}{m}\dot{z} \quad (23)$$

This non linear dynamic model can be represented in state space model with external disturbance as follows.

$$\dot{\phi} = \dot{x}_1 = x_2 \quad (24)$$

$$\ddot{\phi} = \dot{x}_2 = a_1 x_4 x_6 + a_2 x_2^2 + a_3 \Omega x_4 + b_1 U_2 + d_\emptyset \quad (25)$$

$$\dot{\theta} = \dot{x}_3 = x_4 \quad (26)$$

$$\ddot{\theta} = \dot{x}_4 = a_4 x_2 x_6 + a_5 x_4^2 + a_6 \Omega x_2 + b_2 U_3 + d_\theta \quad (27)$$

$$\dot{\psi} = \dot{x}_5 = x_6 \quad (28)$$

$$\ddot{\psi} = \dot{x}_6 = a_7 x_2 x_4 + a_8 x_6^2 + b_3 U_4 + d_\psi \quad (29)$$

$$\dot{x} = \dot{x}_7 = x_8 \quad (30)$$

$$\ddot{x} = \dot{x}_8 = a_9 x_8 + \left(U_x * \frac{U_1}{m}\right) + d_x \quad (31)$$

$$\dot{y} = \dot{x}_9 = x_{10} \quad (32)$$

$$\ddot{y} = \dot{x}_{10} = a_{10} x_{10} + \left(U_y * \frac{U_1}{m}\right) + d_y \quad (33)$$

$$\dot{z} = \dot{x}_{11} = x_{12} \quad (34)$$

$$\ddot{z} = \dot{x}_{12} = a_{11} x_{12} + \left(U_1 * \frac{(Cx_1 * Cx_3)}{m}\right) - g + d_z \quad (35)$$

where

$$a_1 = \frac{J_y - J_z}{J_x}, a_4 = \frac{J_z - J_x}{J_y}, a_7 = \frac{J_x - J_y}{J_z} \quad (36)$$

$$a_2 = \frac{-K_{ax}}{J_x}, a_5 = \frac{-K_{ay}}{J_y}, a_8 = \frac{-K_{az}}{J_z} \quad (37)$$

$$a_3 = \frac{-J_r}{J_x}, a_6 = \frac{-J_r}{J_y} \quad (38)$$

$$a_9 = \frac{-K_{dx}}{m}, a_{10} = \frac{-K_{dy}}{m}, a_{11} = \frac{-K_{dz}}{m} \quad (39)$$

$b_1 = \frac{l}{Jx}, b_2 = \frac{l}{Jy}, b_3 = \frac{1}{Jz}$ (40)

$U_x = Cx_1 * Sx_3 * Cx_5 + Sx_1 * Sx_5$ (41)

$U_y = Cx_1 * Sx_3 * Cx_5 - Sx_1 * Cx_5$ (42)

Backstepping Sliding mode control:

This section introduces the SMC technique combined with Back-Stepping control technique and introduces an adaptive reaching law to create a robust controller for position trajectory tracking. As it combines advantages of both SMC and Back-stepping controllers, the sliding mode control is an effective approach for addressing nonlinear tracking problems that involve model uncertainties and external disturbances and back-stepping controller offers precise tracking and stable performance for quadrotor. An adaptive reaching law is proposed which dynamically adjusts the control inputs while handling external disturbances to ensure robust performance and to guide a system to a desired trajectory.

1. Roll Control:

Roll Error is defined as

$e_1 = x_{1d} - x_1$ (43)

where e_1 is error, x_{1d} is desired roll and x_1 is actual roll

Differentiating the roll error

$\dot{e}_1 = \dot{x}_{1d} - \dot{x}_1$ (44)

Substituting from equation no.24

$\dot{e}_1 = \dot{x}_{1d} - x_2$ (45)

The sliding surface $S_\emptyset$ is defined and v_1 is virtual control

$e_2 = S_\emptyset = x_2 - v_1$ (46)

$x_2 = S_\emptyset - v_1$ (47)

$\dot{e}_1 = \dot{x}_{1d} - S_\emptyset + v_1$ (48)

The Lyapunov candidate chosen for this is

$V_1 = \frac{1}{2}e_1^2$ (49)

Derivative of V_1 is

$\dot{V}_1 = e_1\dot{e}_1$ (50)

$\dot{V}_1 = e_1(\dot{x}_{1d} + v_1 - S_\emptyset)$ (51)

The virtual control v_1 is designed to stabilize Lyapunov function as

$v_1 = -\dot{x}_{1d} - c_1e_1$ (52)

$\dot{V}_1 = e_1(-c_1e_1 - S_\emptyset)$ (53)

$\dot{V}_1 = - c_1e_1^2 - e_1 S_\emptyset < 0$ (54)

where c_1 is positive constant, and the subsystem is asymptotically stable.

The sliding surface time derivative is

$\dot{e}_2 = \dot{S}_\emptyset = \dot{v}_1 + \dot{x}_2$ (55)

Lyapunov candidate chosen for this system is

$V_2(e_1, S_\emptyset) = \frac{1}{2}(e_1^2 + S_\emptyset^2)$ (56)

Time derivative of the Lyapunov function is

$\dot{V}_2(e_1, S_\emptyset) = e_1\dot{e}_1 + S_\emptyset\dot{S}_\emptyset$ (57)

Necessary sliding condition required to stabilize is

$\dot{S}_\emptyset = -q_1 sign(S_\emptyset) - k_1S_\emptyset$ (58)

$\dot{V}_2(e_1, S_\emptyset) = - c_1e_1^2 - e_1S_\emptyset + S_\emptyset(-q_1 sign(S_\emptyset) - k_1S_\emptyset)$ (59)

$\dot{V}_2(e_1, S_\emptyset) = - c_1e_1^2 - e_1S_\emptyset - q_1 sign(S_\emptyset)\, S_\emptyset - k_1S_\emptyset^2$ (60)

$\dot{V}_2(e_1, S_\emptyset) = - c_1e_1^2 - e_1S_\emptyset - q_1 sign(S_\emptyset)\, S_\emptyset - k_1S_\emptyset^2 \quad < 0$ (61)

System is asymptotically stable and control input can be obtained by

$\dot{S}_\emptyset = -q_1 sign(S_\emptyset) - k_1S_\emptyset$ (62)

$\dot{S}_\emptyset = -\ddot{x}_{1d} - c_1\dot{e}_1 + \dot{x}_2$ (63)

$-q_1 sign(S_\emptyset) - k_1S_\emptyset = - \ddot{x}_{1d} - c_1\dot{e}_1 + a_1x_4x_6 + a_2x_2^2 + a_3\Omega x_4 + b_1U_2$ (64)

$U_2 = \frac{1}{b_1}(-q_1 sign(S_\emptyset) - k_1S_\emptyset + \ddot{x}_{1d} + c_1\dot{e}_1 - a_1x_4x_6 - a_2x_2^2 - a_3\Omega x_4)$ (65)

In this control input an adaptive control law which is proposed to compensate the external disturbances.

$q_1 = k_{bar1} * |S_\emptyset|$ (66)

where k_{bar1} is a controller gain and q_1 is a state variable whose value changes.

2. *Pitch Control:*

Pitch error is defined as

$e_3 = x_{3d} - x_3$ (67)

Where e_3 is error, x_{3d} is desired pitch and x_3 is actual pitch

Differentiating the pitch error

$\dot{e}_3 = \dot{x}_{3d} - \dot{x}_3$ (68)

Substituting from equation no.26

$\dot{e}_3 = \dot{x}_{3d} - x_4$ (69)

The sliding surface S_θ is defined and v_3 is virtual control

$e_4 = S_\theta = x_4 - v_3$ (70)

$x_4 = S_\theta - v_3$ (71)

$\dot{e}_3 = \dot{x}_{3d} - S_\theta + v_3$ (72)

The Lyapunov candidate chosen for this is

$V_3 = \frac{1}{2}e_3^2$ (73)

Derivative of V_3 is

$\dot{V}_3 = e_3\dot{e}_3$ (74)

$\dot{V}_3 = e_3\,(\dot{x}_{3d} + v_3 - S_\theta)$ (75)

Virtual control v_3 is designed to stabilize Lyapunov function as

$v_3 = -\dot{x}_{3d} - c_3 e_3$ (76)

$\dot{V}_3 = e_3\,(-c_3 e_3 - S_\theta)$ (77)

$\dot{V}_3 =- c_3 e_3^2 - e_3\,S_\theta < 0$ (78)

c_3 is positive constant and the subsystem is asymptotically stable.

The sliding surface time derivative is

$\dot{e}_4 = \dot{S}_\theta = \dot{v}_3 + \dot{x}_4$ (79)

Lyapunov candidate chosen for this system is

$V_4(e_3,S_\theta) = \frac{1}{2}(e_3^2 + S_\theta^2)$ (80)

Time derivative of the Lyapunov function is

$\dot{V}_4(e_3,S_\theta) = e_3\dot{e}_3 + S_\theta\dot{S}_\theta$ (81)

Necessary sliding condition required to stabilize is

$\dot{S}_\theta = -q_2 sign(S_\theta) - k_2 S_\theta$ (82)

$\dot{V}_4(e_3,S_\theta) =- c_3 e_3^2 - e_3 S_\theta + S_\theta(-q_2 sign(S_\theta)$
$- k_2 S_\theta)$ (83)

$\dot{V}_4(e_3,S_\theta) =- c_3 e_3^2 - e_3 S_\theta - q_2 sign(S_\theta)\,S_\theta - k_2 S_\theta^2$ (84)

$\dot{V}_4(e_3,S_\theta) =- c_3 e_3^2 - e_3 S_\theta - q_2 sign(S_\theta)\,S_\theta - k_2 S_\theta^2 < 0$ (85)

System is asymptotically stable and control input can be obtained by

$\dot{S}_\theta = -q_2 sign(S_\theta) - k_2 S_\theta$ (86)

$\dot{S}_\theta = -\ddot{x}_{3d} - c_3\dot{e}_3 + \dot{x}_4$ (87)

$-q_2 sign(S_\theta) - k_2 S_\theta = -\ddot{x}_{3d} - c_3\dot{e}_3 + a_4 x_2 x_6 + a_5 x_4^2 + a_6\Omega x_2 + b_2 U_3$ (88)

$U_3 = \frac{1}{b_2}(-q_2 sign(S_\theta) - k_2 S_\theta + \ddot{x}_{3d} + c_3\dot{e}_3 - a_4 x_2 x_6 - a_5 x_4^2 - a_6\Omega x_2)$ (89)

In this control input an adaptive control law which is proposed to compensate the external disturbances.

$q_2 = k_{bar2} * |S_\theta|$ (90)

where k_{bar2} is a controller gain and q_2 is a state variable whose value changes.

3. *Yaw Control:*

Yaw error is defined as

$e_5 = x_{5d} - x_5$ (91)

Where e_5 is error, x_{5d} is desired yaw and x_5 is actual yaw

Differentiating the yaw error

$\dot{e}_5 = \dot{x}_{5d} - \dot{x}_5$ (92)

Substituting from equation no.28

$\dot{e}_5 = \dot{x}_{5d} - x_6$ (93)

The sliding surface S_ψ is defined and v_5 is virtual control

$e_6 = S_\psi = x_6 - v_5$ (94)

$x_6 = S_\psi - v_5$ (95)

$\dot{e}_5 = \dot{x}_{5d} - S_\psi + v_5$ (96)

The Lyapunov candidate chosen for this is

$V_5 = \frac{1}{2}e_5^2$ (97)

Derivative of V_5 is

$\dot{V}_5 = e_5\dot{e}_5$ (98)

$\dot{V}_5 = e_5\,(\dot{x}_{5d} + v_5 - S_\psi)$ (99)

Virtual control v_5 is designed to stabilize Lyapunov function as

$v_5 = -\dot{x}_{5d} - c_5 e_5$ (100)

$\dot{V}_5 = e_5(-c_5 e_5 - S_\psi)$ (101)

$\dot{V}_5 =- c_5 e_5^2 - e_5\,S_\psi < 0$ (102)

c_5 is positive constant and the subsystem is asymptotically stable.

The sliding surface time derivative is

$\dot{e}_6 = \dot{S}_\psi = \dot{v}_5 + \dot{x}_6$ (103)

Lyapunov candidate function chosen for this system is

$V_6(e_5,S_\psi) = \frac{1}{2}(e_5^2 + S_\psi^2)$ (104)

Time derivative of the Lyapunov function is

$\dot{V}_6(e_5,S_\psi) = e_5\dot{e}_5 + S_\psi\dot{S}_\psi$ (105)

Necessary sliding condition required to stabilize is

$\dot{S}_\psi = -q_3 sign(S_\psi) - k_3 S_\psi$ (106)

$\dot{V}_6(e_5, S_\psi) = -c_5 e_5^2 - e_5 S_\psi + S_\psi(-q_3 sign(S_\psi) - k_3 S_\psi)$ (107)

$\dot{V}_6(e_5, S_\psi) = -c_5 e_5^2 - e_5 S_\psi - q_3 sign(S_\psi) S_\psi - k_3 S_\psi^2$ (108)

$\dot{V}_6(e_5, S_\psi) = -c_5 e_5^2 - e_5 S_\psi - q_3 sign(S_\psi) S_\psi - k_3 S_\psi^2 < 0$ (109)

System is asymptotically stable and control input can be obtained by

$\dot{S}_\psi = -q_3 sign(S_\psi) - k_3 S_\psi$ (110)

$\dot{S}_\psi = -\ddot{x}_{5d} - c_5 \dot{e}_5 + \dot{x}_6$ (111)

$-q_3 sign(S_\psi) - k_3 S_\psi = -\ddot{x}_{5d} - c_5 \dot{e}_5 + a_7 x_2 x_4 + a_8 x_6^2 + b_3 U_4$ (112)

$U_4 = \frac{1}{b_3}(-q_3 sign(S_\psi) - k_3 S_\psi + \ddot{x}_{5d} + c_5 \dot{e}_5 - a_7 x_2 x_4 - a_8 x_6^2)$ (113)

In this control input an adaptive control law which is proposed to compensate the external disturbances.

$q_3 = k_{bar3} * |S_\psi|$ (114)

where k_{bar3} is a controller gain and q_3 is a state variable whose value changes.

4. *Horizontal X-Position Control:*

X-position error is defined as

$e_7 = x_{7d} - x_7$ (115)

Where e_7 is error, x_{7d} is desired x-axis position and x_7 is actual x-axis position

Differentiating the x-axis position error is

$\dot{e}_7 = \dot{x}_{7d} - \dot{x}_7$ (116)

Substituting from equation no.30

$\dot{e}_7 = \dot{x}_{7d} - x_8$ (117)

The sliding surface S_x is defined and v_7 is virtual control

$e_8 = S_x = x_8 - v_7$ (118)

$x_8 = S_x - v_7$ (119)

$\dot{e}_7 = \dot{x}_{7d} - S_x + v_7$ (120)

The Lyapunov candidate chosen for this is

$V_7 = \frac{1}{2} e_7^2$ (121)

Derivative of V_7 is

$\dot{V}_7 = e_7 \dot{e}_7$ (122)

$\dot{V}_7 = e_7 (\dot{x}_{7d} + v_7 - S_x)$ (123)

Virtual control v_7 is designed to stabilize Lyapunov function as

$v_7 = -\dot{x}_{7d} - c_7 e_7$ (124)

$\dot{V}_7 = e_7(-c_7 e_7 - S_x)$ (125)

$\dot{V}_7 = -c_7 e_7^2 - e_7 S_x < 0$ (126)

c_7 is positive constant and the subsystem is asymptotically stable.

The sliding surface time derivative is

$\dot{e}_8 = \dot{S}_x = \dot{v}_7 + \dot{x}_8$ (127)

Lyapunov candidate chosen for this system is

$V_8(e_7, S_x) = \frac{1}{2}(e_7^2 + S_x^2)$ (128)

Time derivative of the Lyapunov function is

$\dot{V}_8(e_7, S_x) = e_7 \dot{e}_7 + S_x \dot{S}_x$ (129)

Necessary sliding condition to be stabilize is

$\dot{S}_x = -q_4 sign(S_x) - k_4 S_x$ (130)

$\dot{V}_8(e_7, S_x) = -c_7 e_7^2 - e_7 S_x + S_x(-q_4 sign(S_x) - k_4 S_x)$ (131)

$\dot{V}_8(e_7, S_x) = -c_7 e_7^2 - e_7 S_x - q_4 sign(S_x) S_x - k_4 S_x^2$ (132)

$\dot{V}_8(e_7, S_x) = -c_7 e_7^2 - e_7 S_x - q_4 sign(S_x) S_x - k_4 S_x^2 < 0$ (133)

System is asymptotically stable and control input can be obtained by

$\dot{S}_x = -q_4 sign(S_x) - k_4 S_x$ (134)

$\dot{S}_x = -\ddot{x}_{7d} - c_7 \dot{e}_7 + \dot{x}_8$ (135)

$-q_4 sign(S_x) - k_4 S_x = -\ddot{x}_{7d} - c_7 \dot{e}_7 + a_9 x_8 + \left(U_x * \frac{U_1}{m}\right)$ (136)

$U_x = \frac{m}{U_1}(-q_4 sign(S_x) - k_4 S_x + \ddot{x}_{7d} + c_7 \dot{e}_7 - a_9 x_8)$ (137)

In this control input an adaptive control law which is proposed to compensate the external disturbances.

$q_4 = k_{bar4} * |S_x|$ (138)

where k_{bar4} is a controller gain and q_4 is a state variable whose value changes.

5. *Horizontal Y-Position Control:*

Y-position error is defined as

$e_9 = x_{9d} - x_9$ (139)

Where e_9 is error, x_{9d} is desired y-axis position and x_9 is actual y-axis position

Differentiating the y-axis position error is

$\dot{e}_9 = \dot{x}_{9d} - \dot{x}_9$ (140)

Substituting from equation no.32

$\dot{e}_9 = \dot{x}_{9d} - x_{10}$ (141)

The sliding surface S_y is defined and v_9 is virtual control

$e_{10} = S_y = x_{10} - v_9$ (142)

$x_{10} = S_y - v_9$ (143)

$\dot{e}_9 = \dot{x}_{9d} - S_y + v_9$ (144)

The Lyapunov candidate chosen for this is

$V_9 = \frac{1}{2} e_9^2$ (145)

Derivative of V_9 is

$\dot{V}_9 = e_9 \dot{e}_9$ (146)

$\dot{V}_9 = e_9 (\dot{x}_{9d} + v_9 - S_y)$ (147)

Virtual control v_9 is designed to stabilize Lyapunov function

$v_9 = -\dot{x}_{9d} - c_9 e_9$ (148)

$\dot{V}_9 = e_9 (- c_9 e_9 - S_y)$ (149)

$\dot{V}_9 =- c_9 e_9^2 - e_9 S_y < 0$ (150)

where c_9 is positive constant the subsystem is asymptotically stable.

The sliding surface time derivative is

$\dot{e}_{10} = \dot{S}_y = \dot{v}_9 + \dot{x}_{10}$ (151)

Lyapunov candidate chosen for this system is

$V_{10}(e_9, S_y) = \frac{1}{2}(e_9^2 + S_y^2)$ (152)

Time derivative of the Lyapunov function is

$\dot{V}_{10}(e_9, S_y) = e_9 \dot{e}_9 + S_y \dot{S}_y$ (153)

Necessary sliding condition required to stabilize is

$\dot{S}_y = -q_5 sign(S_y) - k_5 S_y$ (154)

$\dot{V}_{10}(e_9, S_y) =- c_9 e_9^2 - e_9 S_y + S_y(-q_5 sign(S_y) - k_5 S_y)$ (155)

$\dot{V}_{10}(e_9, S_y) =- c_9 e_9^2 - e_9 S_y - q_5 sign(S_y) S_y - k_5 S_y^2$ (156)

$\dot{V}_{10}(e_9, S_y) =- c_9 e_9^2 - e_9 S_y - q_5 sign(S_y) S_y - k_5 S_y^2 \quad < 0$ (157)

System is asymptotically stable and control input can be obtained by

$\dot{S}_y = -q_5 sign(S_y) - k_5 S_y$ (158)

$\dot{S}_y = -\ddot{x}_{9d} - c_9 \dot{e}_9 + \dot{x}_{10}$ (159)

$-q_5 sign(S_y) - k_5 S_y = -\ddot{x}_{9d} - c_9 \dot{e}_9 + a_{10} x_{10} + \left(U_y * \frac{U_1}{m}\right)$ (160)

$U_y = \frac{m}{U_1}(-q_5 sign(S_y) - k_5 S_y + \ddot{x}_{9d} + c_9 \dot{e}_9 - a_{10} x_{10})$ (161)

In this control input an adaptive control law which is proposed to compensate the external disturbances.

$q_5 = k_{bar5} * |S_y|$ (162)

where k_{bar5} is a controller gain and q_5 is a state variable whose value changes

6. *Height Control:*

Z-position error is defined as

$e_{11} = x_{11d} - x_{11}$ (163)

where e_{11} is error, x_{11d} is desired z-axis position and x_{11} is actual z-axis position

Differentiating the z-axis position error

$\dot{e}_{11} = \dot{x}_{11d} - \dot{x}_{11}$ (164)

Substituting from equation no.34

$\dot{e}_{11} = \dot{x}_{11d} - x_{12}$ (165)

The sliding surface S_z is defined and v_{11} is virtual control

$e_{12} = S_z = x_{12} - v_{11}$ (166)

$x_{12} = S_z - v_{11}$ (167)

$\dot{e}_{11} = \dot{x}_{11d} - S_z + v_{11}$ (168)

The Lyapunov candidate chosen for this is

$V_{11} = \frac{1}{2} e_{11}^2$ (169)

Derivative of V_{11} is

$\dot{V}_{11} = e_{11} \dot{e}_{11}$ (170)

$\dot{V}_{11} = e_{11} (\dot{x}_{11d} + v_{11} - S_z)$ (171)

Virtual control v_{11} is designed to stabilize Lyapunov function as

$v_{11} = -\dot{x}_{11d} - c_{11} e_{11}$ (172)

$\dot{V}_{11} = e_{11} (- c_{11} e_{11} - S_z)$ (173)

$\dot{V}_{11} =- c_{11} e_{11}^2 - e_{11} S_z < 0$ (174)

where c_{11} is positive constant the subsystem is asymptotically stable.

The sliding surface time derivative is

$\dot{e}_{12} = \dot{S}_z = \dot{v}_{11} + \dot{x}_{12}$ (175)

Lyapunov candidate chosen for this system is

$V_{12}(e_{11}, S_z) = \frac{1}{2}(e_{11}^2 + S_z^2)$ (176)

Time derivative of the Lyapunov function is

$\dot{V}_{12}(e_{11}, S_z) = e_{11}\dot{e}_{11} + S_z\dot{S}_z$ (177)

Necessary sliding condition to be stabilize is

$\dot{S}_z = -q_6 sign(S_z) - k_6 S_z$ (178)

$\dot{V}_{12}(e_{11}, S_z) = -c_{11}e_{11}^2 - e_{11}S_z + S_z(-q_6 sign(S_z) - k_6 S_z)$ (179)

$\dot{V}_{12}(e_{11}, S_z) = -c_{11}e_{11}^2 - e_1 S_z - q_6 sign(S_z)\, S_z - k_6 S_z^2$ (180)

$\dot{V}_{12}(e_{11}, S_z) = -c_{11}e_{11}^2 - e_{11}S_{11} - q_6 sign(S_z)\, S_z - k_6 S_z^2 < 0$ (181)

System is asymptotically stable and control input can be obtained by

$\dot{S}_z = -q_6 sign(S_z) - k_6 S_z$ (182)

$\dot{S}_z = -\ddot{x}_{11d} - c_{11}\dot{e}_{11} + \dot{x}_{12}$ (183)

$-q_6 sign(S_z) - k_6 S_z = -\ddot{x}_{11d} - c_{11}\dot{e}_{11} + a_{11}x_{12} + \left(U_1 * \frac{(Cx_1 * Cx_3)}{m}\right) - g$ (184)

$U_1 = \frac{m}{(Cx_1 * Cx_3)}(-q_6 sign(S_z) - k_6 S_z + \ddot{x}_{11d} + c_{11}\dot{e}_{11} - a_{11}x_{12} + g)$ (185)

In this control input an adaptive control law which is proposed to compensate the external disturbances.

$q_6 = k_{bar6} * |S_z|$ (186)

where k_{bar6} is a controller gain and q_6 is a state variable whose value changes.

IV. SIMULATION RESULTS

In this section, the performance of the proposed algorithm is compared with nominal SMC with backstepping controller in MATLAB simulation. The initial conditions $\phi(t_0), \theta(t_0), \psi(t_0), x(t_0), y(t_0)$ and $z(t_0)$ are set as (0, 0, 0, 0, 0, 0). The desired trajectory for the attitude and altitude ($\phi_d, \theta_d, \psi_d, x_d, y_d$ and z_d) of the quadrotor are chosen as $(0, 0, 0, sin(t), cos(t)$ and $0.1 * t)$. External disturbances $d_\emptyset = 2 * \sin(t), d_\theta = 2 * \sin(t), d_\psi = 2 * \sin(t), d_x = 2 * \sin(t), d_y = 2 * \sin(t)$ and $d_z = 2 * \sin(t)$ are added. The quadrotor physical parameters are given in [26] and control gain parameters are given in the table 4.1.

Table 4.1: Parameters of the controller

The parameters of the controller	Numerical value
k_{bar1}, k_{bar2} and k_{bar3}	2
k_{bar4}, k_{bar5} and k_{bar6}	1.5
q_1, q_2, q_3, q_4, q_5 and q_6	2

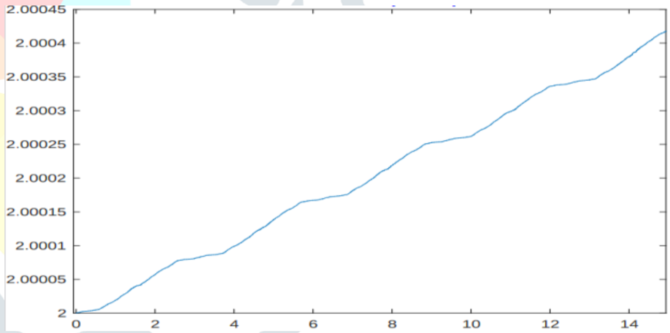

Fig.1 Adaptive q_1 value

To show the effectiveness of proposed adaptive control algorithm for trajectory tracking problem of quadrotor, simulation is conducted with a helical reference trajectory with red color lines and Actual trajectory with blue lines as shown in Fig.2 and Fig.3. The Fig.1 shows that the value of adaptive reaching law gain q_1 is not constant and it is varying as per the proposed adaptive law in order to adjust the control input so that they can compensate the external disturbances. The Fig.4 to Fig.5 shows the output along the X-axis with Sliding Mode Back-Stepping Controller without adaptive control law and with adaptive control law.

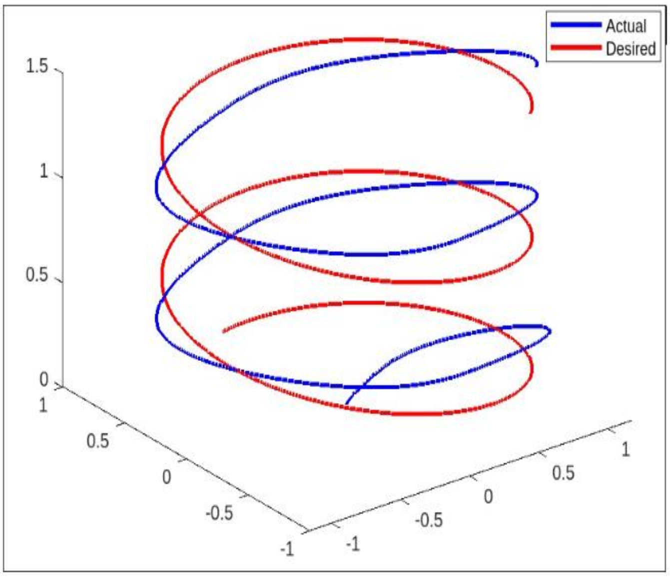

Fig.2 Trajectory tracking in three dimensions for Nominal Sliding Mode Back-Stepping Controller

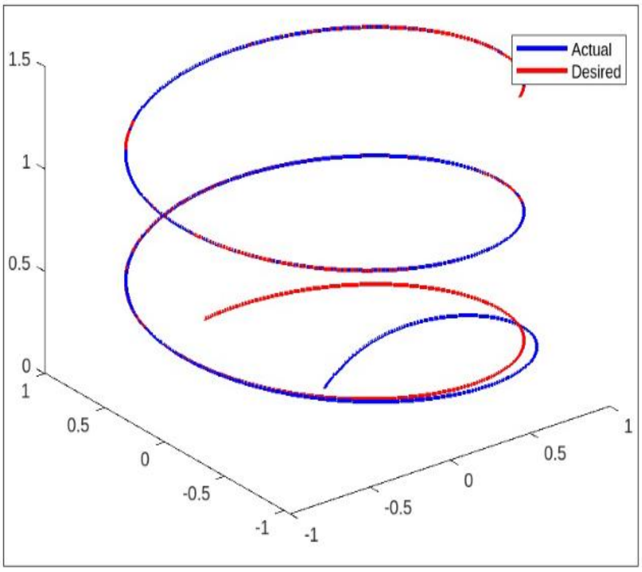

Fig.3 Trajectory tracking in three dimensions for Adaptive control Law

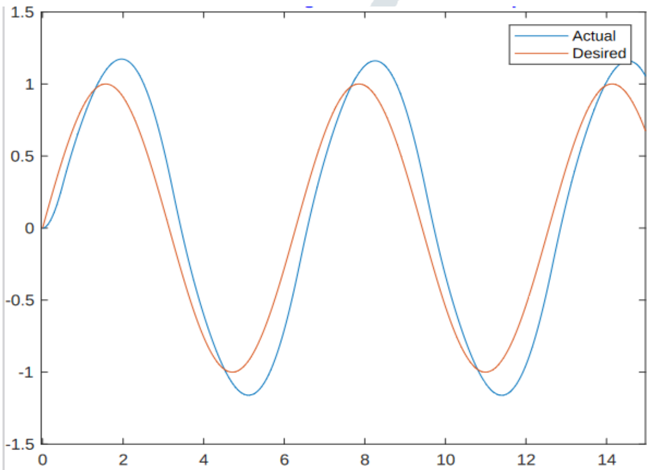

Fig.4. Output of X-axis without Adaptive Control Law

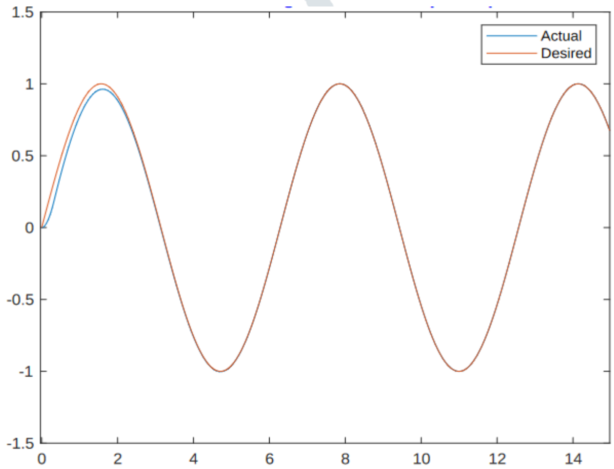

Fig.5. Output of X-axis with Adaptive Control Law

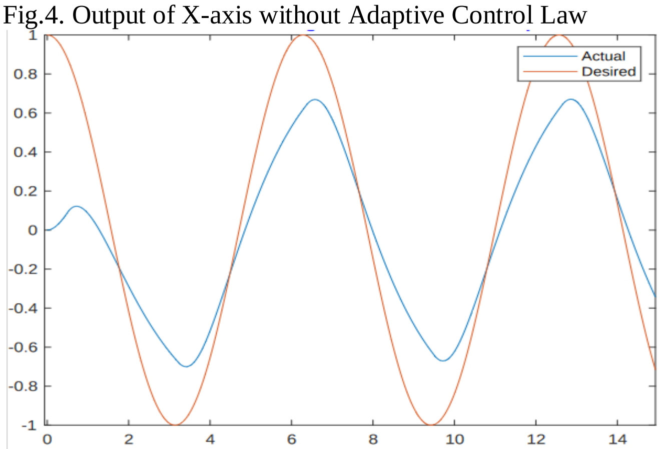

Fig.6. Output of Y-axis without Adaptive Control Law

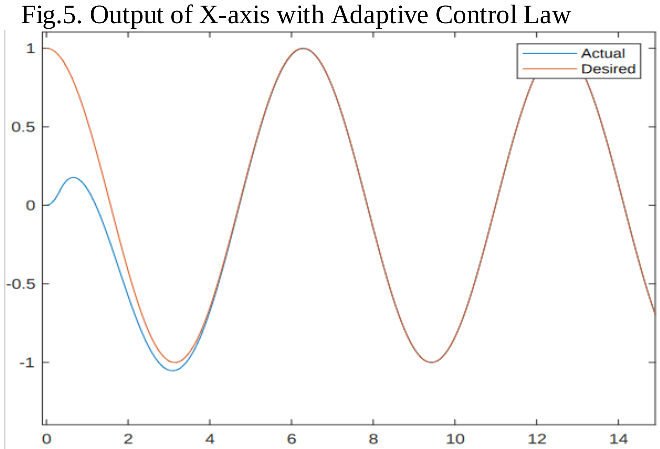

Fig.7. Output of Y-axis with Adaptive Control Law

The Fig.6 to Fig.7 shows the output along the Y-axis with Sliding Mode Back-Stepping Controller without adaptive control law and with adaptive control law.

From the results one conclude that the system behavior of the proposed controller is robust against external disturbances. Also results obtained from the experiments suggest that performance of the proposed controller is better than nominal SMC with backstepping controller.

V. CONCLUSIONS

This article investigates the trajectory tracking problem of the quadrotor UAV in the presence of external disturbances. First nominal SMC with backstepping controller with reaching law is designed in MALAB. However, the system is unable to deal with external disturbances to deal with this problem, an adaptive reaching law is designed that successfully estimates the external disturbances and improves trajectory tracking performance.

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