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AN INVESTIGATION ON A FEW PROPERTIES OF THE SUBGROUP LATTICE OF 2 × 2 MATRICES OVER Z_{13}

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Abstract: In this article, the properties of the subgroup lattice of the group of 2 × 2 matrices over Z_{13} like modularity, semi modularity, super modularity distributivity, consistency, the GD condition, pseudo complemented and super solvability have been validated.

Keywords: Lattice, Subgroup lattice, Lattice properties.

I. Introduction

Allow L(H) as the Subgroup Lattice of H, where H is $SL_2(Z_k)$.

If $\mathcal{G}=GL_2(Z_k)=\{\begin{pmatrix} x & y \\ z & w \end{pmatrix}: x, y, z, w \in Z_k , xw-yz \neq 0\}$ and

$G=SL_2(Z_k) = \{\begin{pmatrix} x & y \\ z & w \end{pmatrix} \in \mathcal{G} : xw-yz = 1\}$, then H is a subgroup $\mathcal{G}$.

Regarding order of groups, we will show that, $o(\mathcal{G}) = k(k^2-1)(k-1)$ [1] and $o(G) = k(k^2-1)$.[1]

For complete reference we provide the breakupof L(H)while k=13 [2].Thus, we will investigate regarding to the entire said properties in L(H) of this article.

II. Basics

Lattice: Definition 1

A Poset L is said to be a lattice if inf {u, v} and sup {u, v} exists for all u, v∈ L.

Modular Lattice: Definition 2

For a lattice L, **L is modular** if $r \leq u$ implies that $u \wedge (v \vee r) = (u \wedge v) \vee r$ for all u, v, r ∈L.

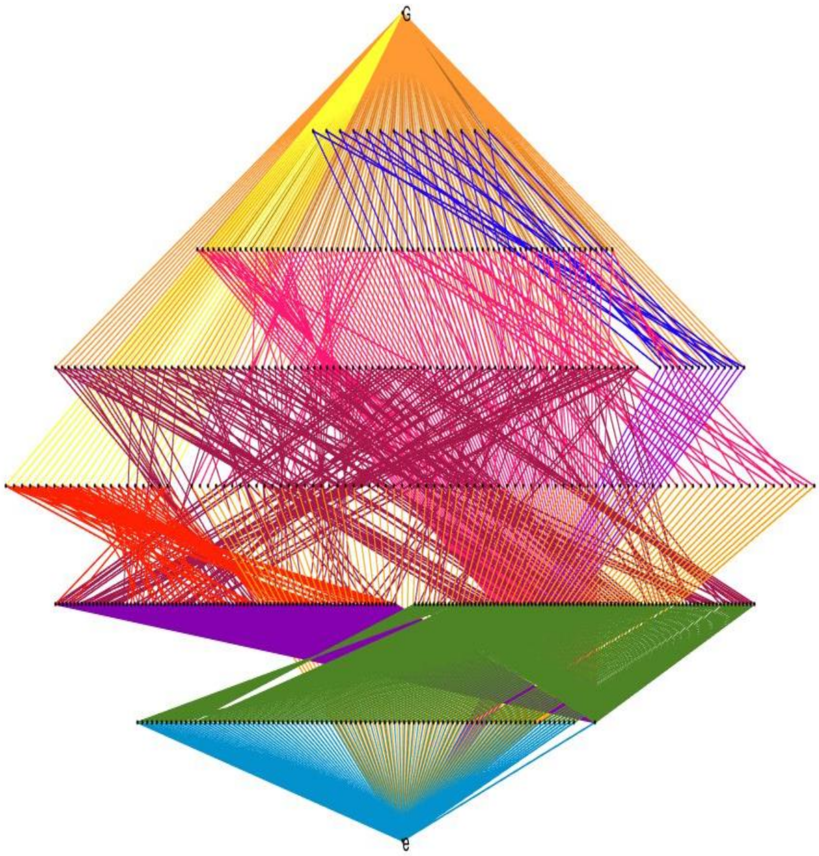
Upper-semi modular: Definition 3

For a lattice L, L is an**upper-semi modular** if $u \vee v$ covers u and v, $u \neq v$ and u and v cover $u \wedge v$.

Distributive lattice: Definition 2.5

For a lattice L, L is **distributive** if $u \vee (v \wedge r) = [(u \vee v) \wedge (u \vee r)]$ for all u, v, r∈ L.

Now, we present the drawing of L(H) when k=13 [2] as shown in pic.1.



Pic.1: L(H) when k = 13

Row I : (Left to Right**)** H_1 to H_{14}

Row II : (Left to Right**)** E_1 to E_{78}

Row III : (Left to Right**)** C_1 to C_{91} and D_1 to D_{14}

Row IV : (Left to Right**)** B_1 to B_{21} and A_1 to A_{78}

Row V : (Left to Right**)** Y_1 to Y_{91} and Z_1 to Z_{91}

Row VI : (Left to Right**)** X_1 to X_{91} and $\mathcal{H}_1$

III. Main Properties

Property 3.1

If k =13, then L(G) is not modular,

Proof:

From the diagram shown in Pic.1, we take three subgroups E_1, A_1, $H_1 \epsilon L(H)$.

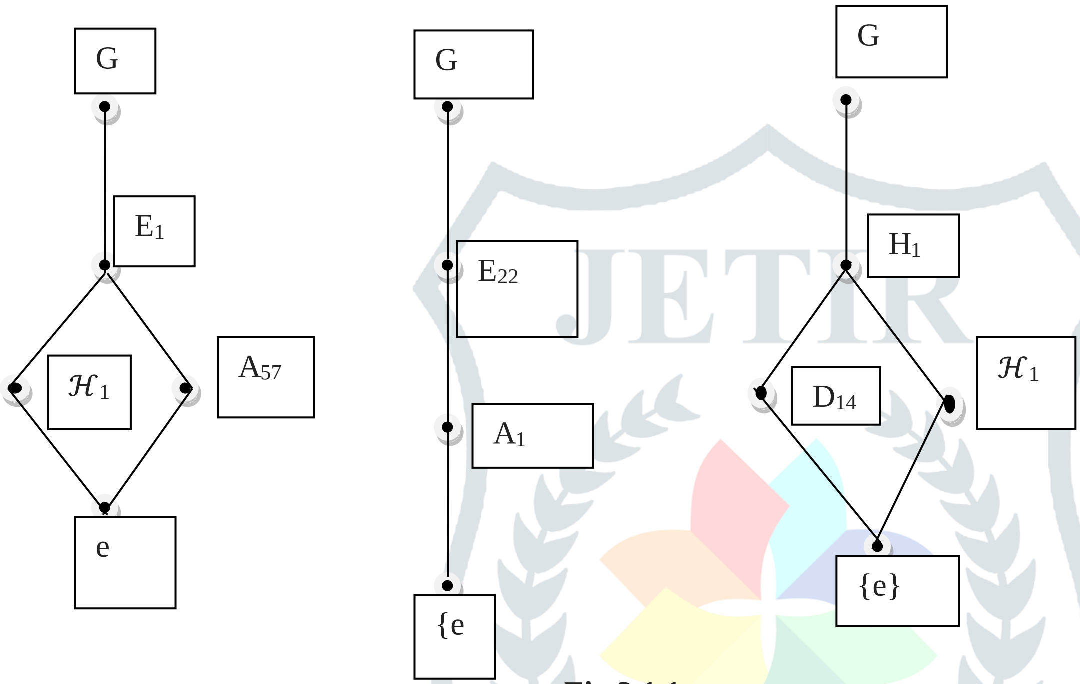


Fig 3.1.1

$E_1 \vee (A_1 \wedge H_1) = E_1 \vee \{e\} = E_1$

But, $(E_1 \vee A_1) \wedge H_1 = G \wedge H_1 = H_1$

Hence $E_1 \vee (A_1 \wedge H_1) \neq (E_1 \vee A_1) \wedge H_1$

Otherwise, $(E_1 \wedge H_1) \vee (A_1 \wedge H_1) = \{e\} \vee \{e\} = e$.

But, $[(E_1 \wedge H_1) \vee A_1] \wedge H_1 = (\mathcal{H}_1 \vee A_1) \wedge H_1 = G \wedge H_1 = H_1$.

Therefore, $(E_1 \wedge H_1) \vee (A_1 \wedge H_1) \neq [(E_1 \wedge H_1) \vee A_1] \wedge H_1$

Consequently, L(G) is not modular when p = 13.

Property 3.2

L(G) is not upper semi modular if k=13.

Proof:

As of the diagram shown in Pic.1, we take two subgroups B_1, $Y_1 \in L(G)$.

$B_1 \wedge Y_1 = \mathcal{H}_1$.which is covered by B_1 while $B_1 \vee Y_1 = G$. which does not cover $Y_{1.}$

Therefore L(G) is not upper semi modular when p = 13.

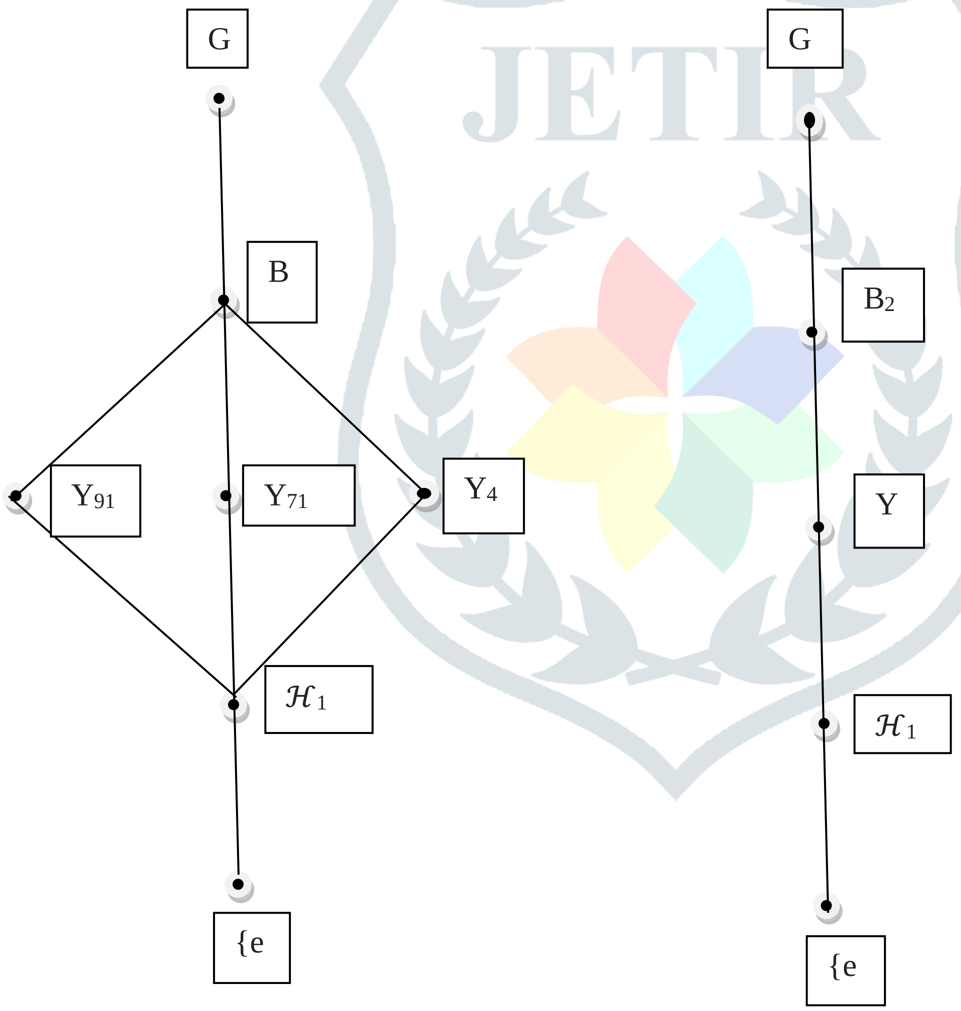


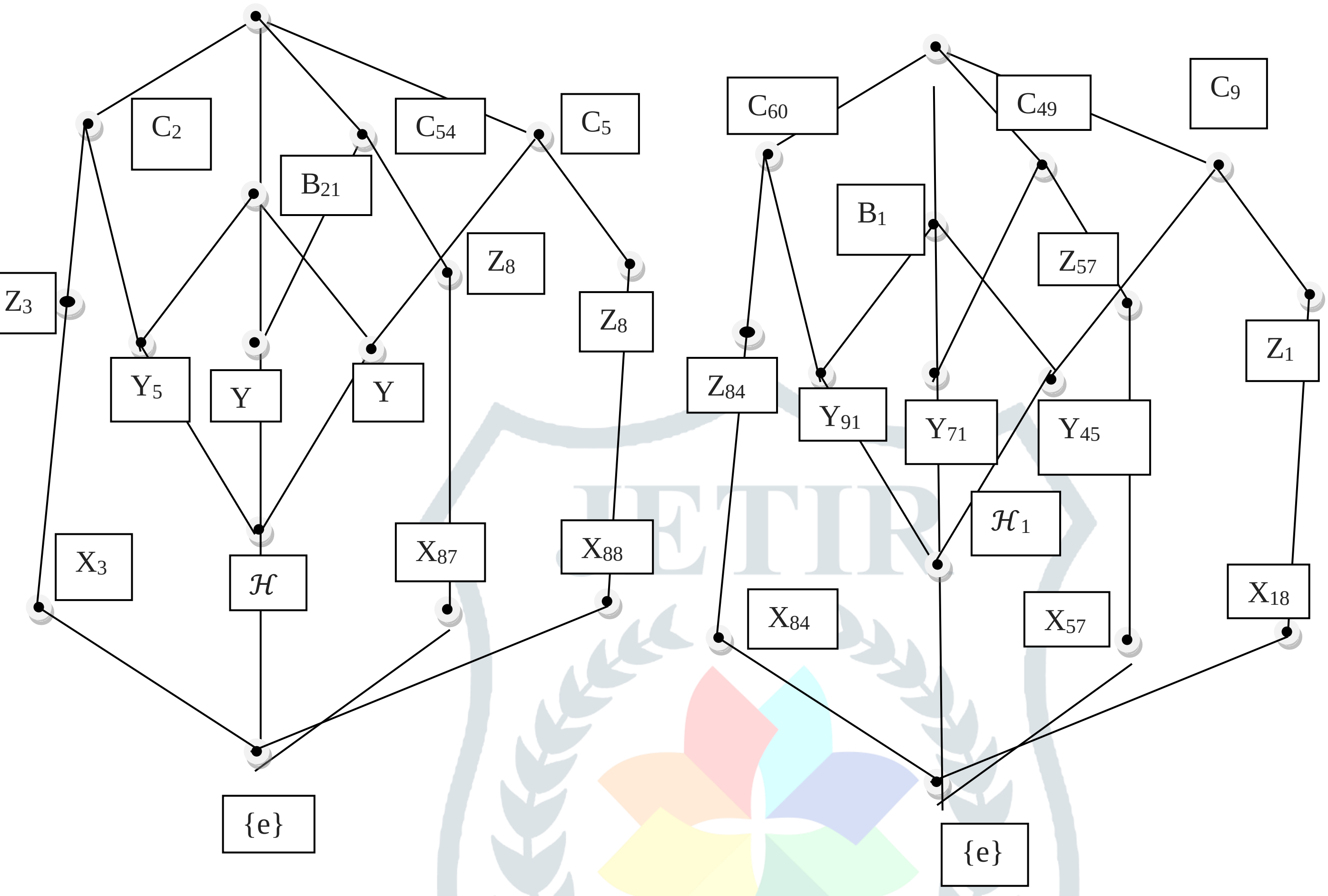
Fig 3.2.1

Property 3.3

If k =13, then L(G) is not super modular.

Proof:

00From Pic.1, Let Y_1, Z_1, B_1, C_1, ∈ L(G).Then



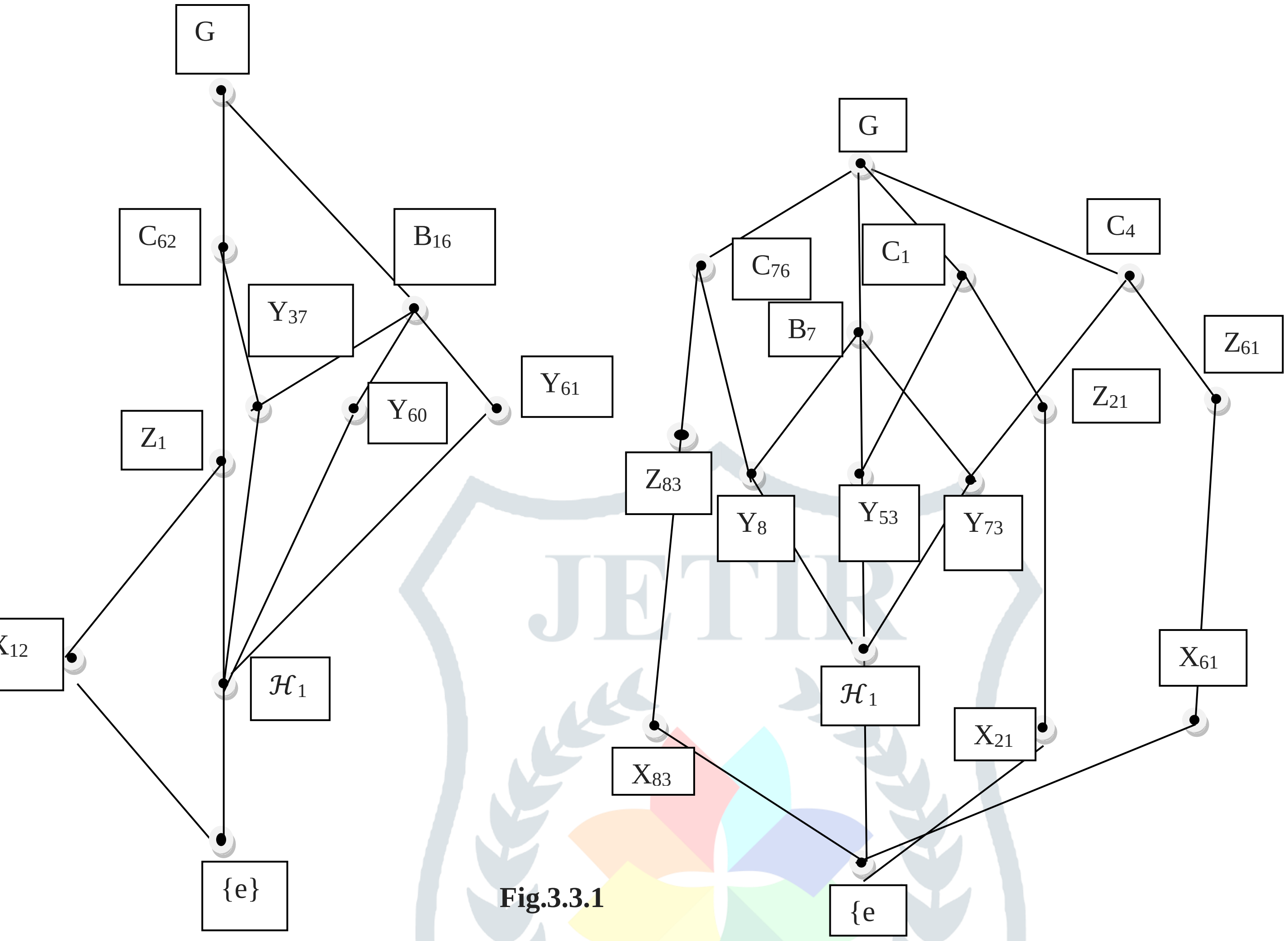


Fig.3.3.1

$(Y_1 \vee Z_1) \wedge (Y_1 \vee B_1) \wedge (Y_1 \vee C_1) = G \wedge G \wedge G = G$

But, $Y_1 \vee [Z_1 \wedge B_1 \wedge (Y_1 \vee C_1)] \vee [B_1 \wedge C_1 \wedge (Y_1 \vee Z_1)] \vee [Z_1 \wedge C_1 \wedge (Y_1 \vee B_1)]$

$= Y_1 \vee [Z_1 \wedge B_1 \wedge G] \vee [B_1 \wedge C_1 \wedge G] \vee [Z_1 \wedge C_1 \wedge G]$

$= Y_1 \vee \mathcal{H}_1 \vee \mathcal{H}_1 \vee \mathcal{H}_1$

$= Y_1$.

Therefore, $(Y_1 \vee Z_1) \wedge (Y_1 \vee B_1) \wedge (Y_1 \vee C_1) \neq Y_1 \vee [Z_1 \wedge B_1 \wedge (Y_1 \vee C_1)] \vee [B_1 \wedge C_1 \wedge (Y_1 \vee Z_1)] \vee [Z_1 \wedge C_1 \wedge (Y_1 \vee B_1)]$

Consequently, L(G) is not super modular when p = 13.

Property 3.4

If k =13, then L(G) is not distributive.

Proof:

From Pic.1, We take three subgroups $Z_1, B_1, C_1 \in L(G)$.

We observe from Pic 1,

$Z_1 \vee (B_1 \wedge C_1) = Z_1 \vee \mathcal{H}_1 = Z_1$.

But, $(Z_1 \vee B_1) \wedge (Z_1 \vee C_1) = G \wedge G = G$.

Therefore, $Z_1 \vee (B_1 \wedge C_1) \neq (Z_1 \vee B_1) \wedge (Z_1 \vee C_1)$.

Therefore, verified the property.

Property 3.5

If k =13, then L(G) is not consistent.

Proof:

We opt the join – irreducible element $D_1 \in L(G)$ for the case k=13. We find that when k = 13, $\mathcal{H}_1 \vee D_1 = G = C_1 \vee C_2$ in the upper interval $[\mathcal{H}_1, G]$,

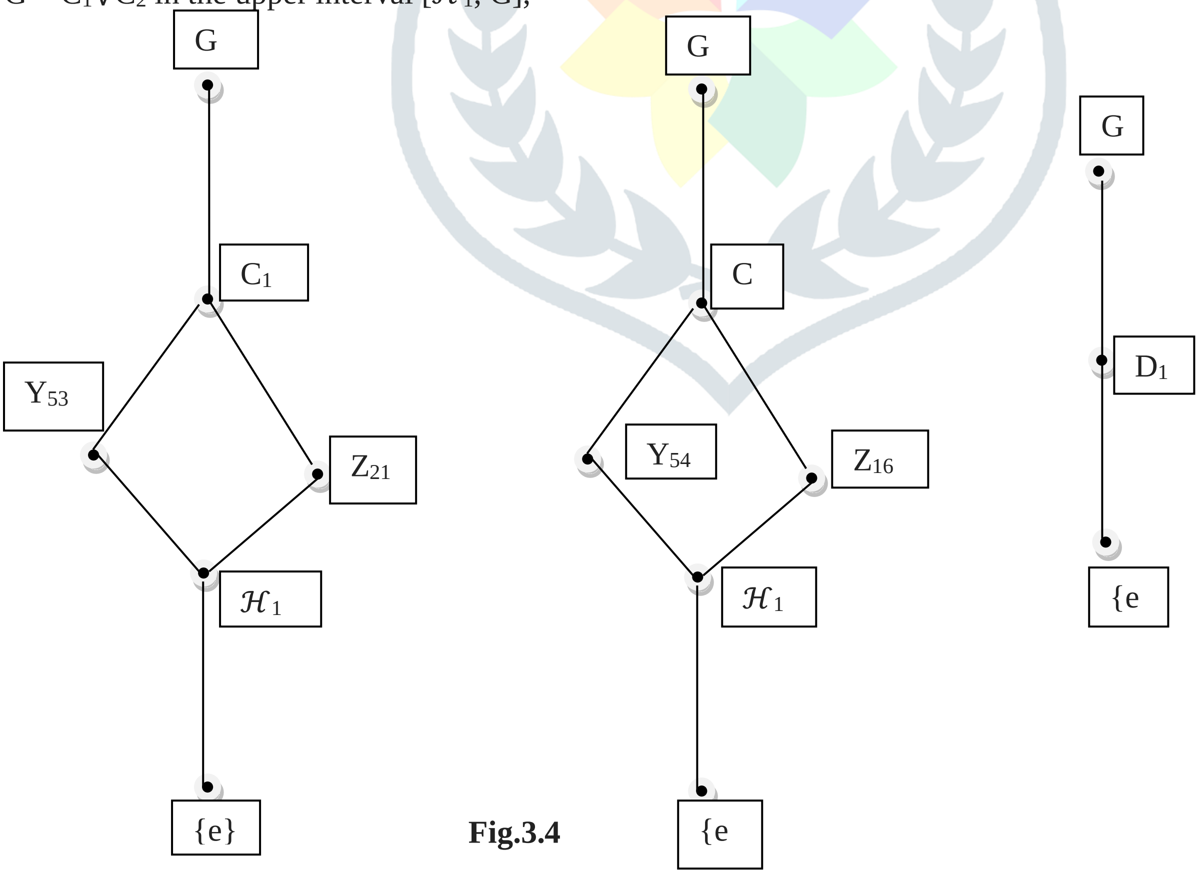


Fig.3.4

So, L(G) is not consistent.

Property 3.6.

If k =13, then the GD condition is not true in L(G).

Proof:

From Fig.1, we consider subgroups $\mathcal{H}_{1}, X_{1}, X_{2} \in L(G)$.

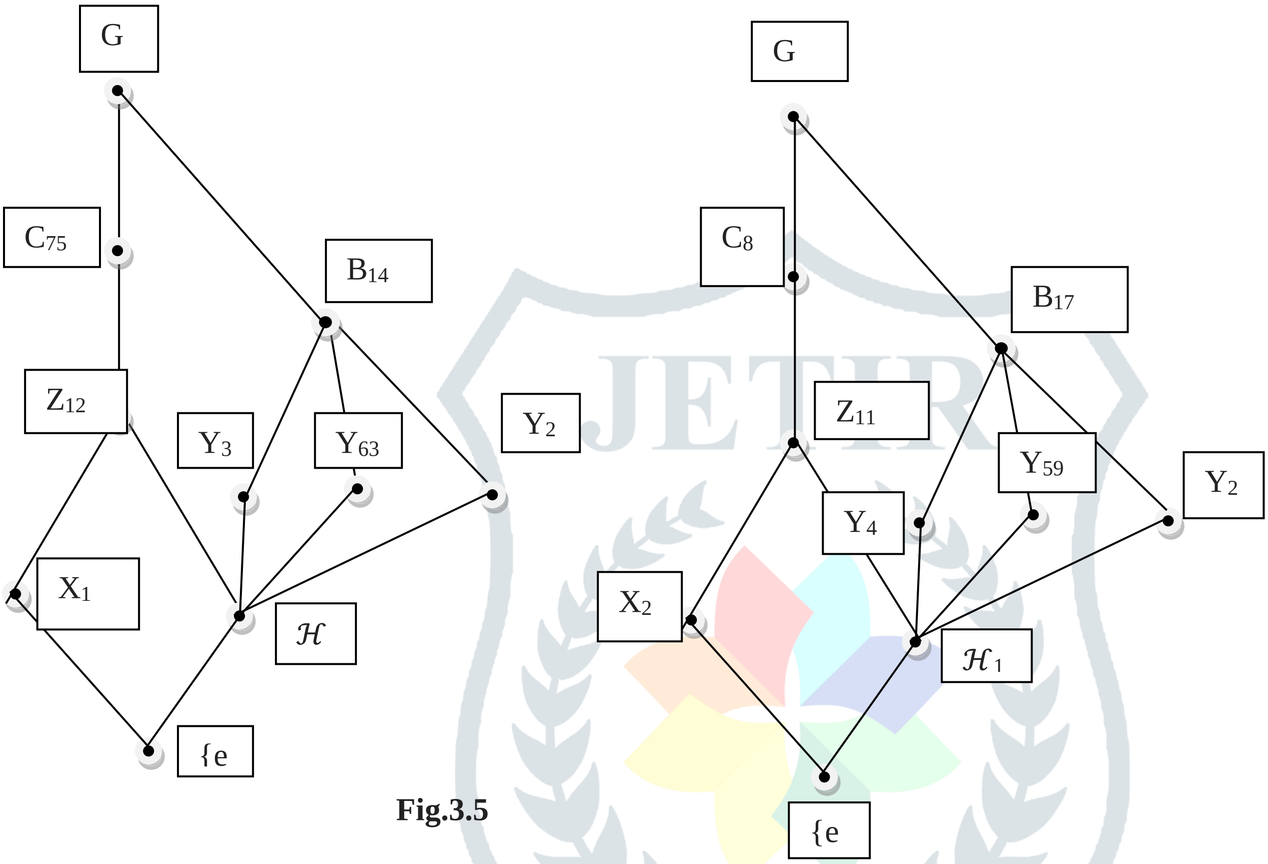


Fig.3.5

Now let $\mathcal{H}_1 \wedge X_1 = 0$ and $(\mathcal{H}_1 \vee X_1) \wedge X_2 = 0$

Then, $\mathcal{H}_1 \wedge (X_1 \vee X_2) = \mathcal{H}_1 \wedge G = \mathcal{H}_1 \neq 0$

$\mathcal{H}_1 \wedge (X_1 \vee X_2) \neq 0$.

Hence the General disjointness condition is not true in L(G) when p = 13.

Property 3.7

If k =13, then L(G) is not pseudo complemented.

Proof:

From Pic.1, we consider one subgroup $Y_{17} \in L(G)$.

Then, $Y_{17} \wedge D_1 = 0$ and if for any $\mathcal{H}_1 \in L(G)$ such that $\mathcal{H}_1 \subset D_1$.

But, $\mathcal{H}_1 \wedge Y_{17} = \mathcal{H}_1 \neq 0$.

Therefore, $\mathcal{H}_1 \wedge Y_{17} \neq 0$.

Consequently an element $D_1 \in L(G)$ is not pseudo complement of $Y_{17} \in L(G)$.

Hence L(G) is not pseudo complemented when p = 13.

.Thus, verified the property.

Property 3.8

Every atom is non – modular if k=13.

Proof:

Let an atoms A_2, A_3, A_4 say A_2.

We have $D_{14} \subset H_1$

Now, $D_{14} \vee (A_2 \wedge H_1) = D_{14} \vee \{e\} = D_{14}$

But, $(D_{14} \vee A_2) \wedge H_1 = G \wedge H_1 = H_1$

Therefore, $D_{14} \vee (A_2 \wedge H_1) \neq (D_{14} \vee A_2) \wedge H_1$

Therefore A_2 is not modular in L(H) when p = 13.

Similarly we can prove that A_3 and A_4 are not modular.

By Similar argument, we can prove that all the other atoms in L(G) when p = 13 are not modular.

Hence there is no atom in L(G) when p= 13, which is modular.

Property 3.9

If k =13, then L(G) is not super solvable.

Proof:

By Property 3.8, we have, no atom in L(G) is modular, so there is no maximal chain in L(G) with modular elements.

Therefore, L(G) is not super solvable.

4. Conclusion

In this article, the properties of L(G) over Z_{13} like modularity, semi modularity, super modularity distributivity, consistency, the GD condition, pseudo complemented and super solvability have been proved and validated.

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