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Techniques to solve General Equation of Second Degree in 2D

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Abstract : The general equation of the second degree in two dimensions (2D) is given by: $\boldsymbol{ax^2+2hxy+by^2+2gx+2fy+c=0}$. This represents a conic section, which can be an ellipse, hyperbola, parabola, or a pair of intersecting lines, depending on the coefficients. Solving this equation involves determining the type of conic section and finding its properties such as center, axes, and orientation. In this paper a developed technic has been introduced for solving problems on general equation of the second degree in two dimensions.

Index Terms **– Coordinate Geometry, General Equation of Second Degree, Canonical form.**

1. Introduction

The roots of geometry can be traced back to ancient civilizations such as the Egyptians and Babylonians, who used geometric principles for practical purposes like land measurement and construction. Greek mathematicians, especially Euclid (circa 300 BC), formalized the study of geometry in works like "The Elements," which systematized knowledge about shapes, sizes, and the properties of space. The formal foundation of coordinate geometry was laid by the French mathematician and philosopher René Descartes (1596-1650). Descartes' seminal work "La Géométrie," published in 1637 as an appendix to his "Discourse on the Method," introduced the idea of using a coordinate system to describe geometric shapes algebraically. Descartes proposed the use of a grid system, which later became known as the Cartesian coordinate system, to locate points on a plane using pairs of numerical coordinates. Around the same time, Pierre de Fermat (1601-1665), a French lawyer and mathematician, independently developed similar ideas. Fermat's work on analytic geometry was instrumental in advancing the study of curves and the properties of algebraic equations. The integration of algebra with geometry allowed mathematicians to solve geometric problems using algebraic equations and vice versa. This was a revolutionary shift from the purely synthetic approach of the Greeks. The ability to describe curves like parabolas, ellipses, and hyperbolas algebraically paved the way for further advancements in mathematics, including calculus and differential geometry. The development of calculus by Isaac Newton (1643-1727) and Gottfried Wilhelm Leibniz (1646-1716) was heavily influenced by the principles of coordinate geometry. Calculus provided powerful tools for analyzing curves and understanding their properties. Leonhard Euler (1707-1783), a prolific Swiss mathematician, made significant contributions to analytic geometry, including the development of formulas and methods to analyze conic sections and other geometric shapes. Carl Friedrich Gauss (1777-1855) extended the ideas of analytic geometry to higher dimensions, laying the groundwork for modern geometry and the study of surfaces and manifolds. In the 19th century, mathematicians like Nikolai Lobachevsky (1792-1856) and János Bolyai (1802-1860) explored non-Euclidean geometries, which further expanded the scope of coordinate geometry beyond the Euclidean framework.

2. Classification of General Equation of Second Degree in 2D

General equation of second degree $ax^2+2hxy+by^2+2gx+2fy+c=0$.

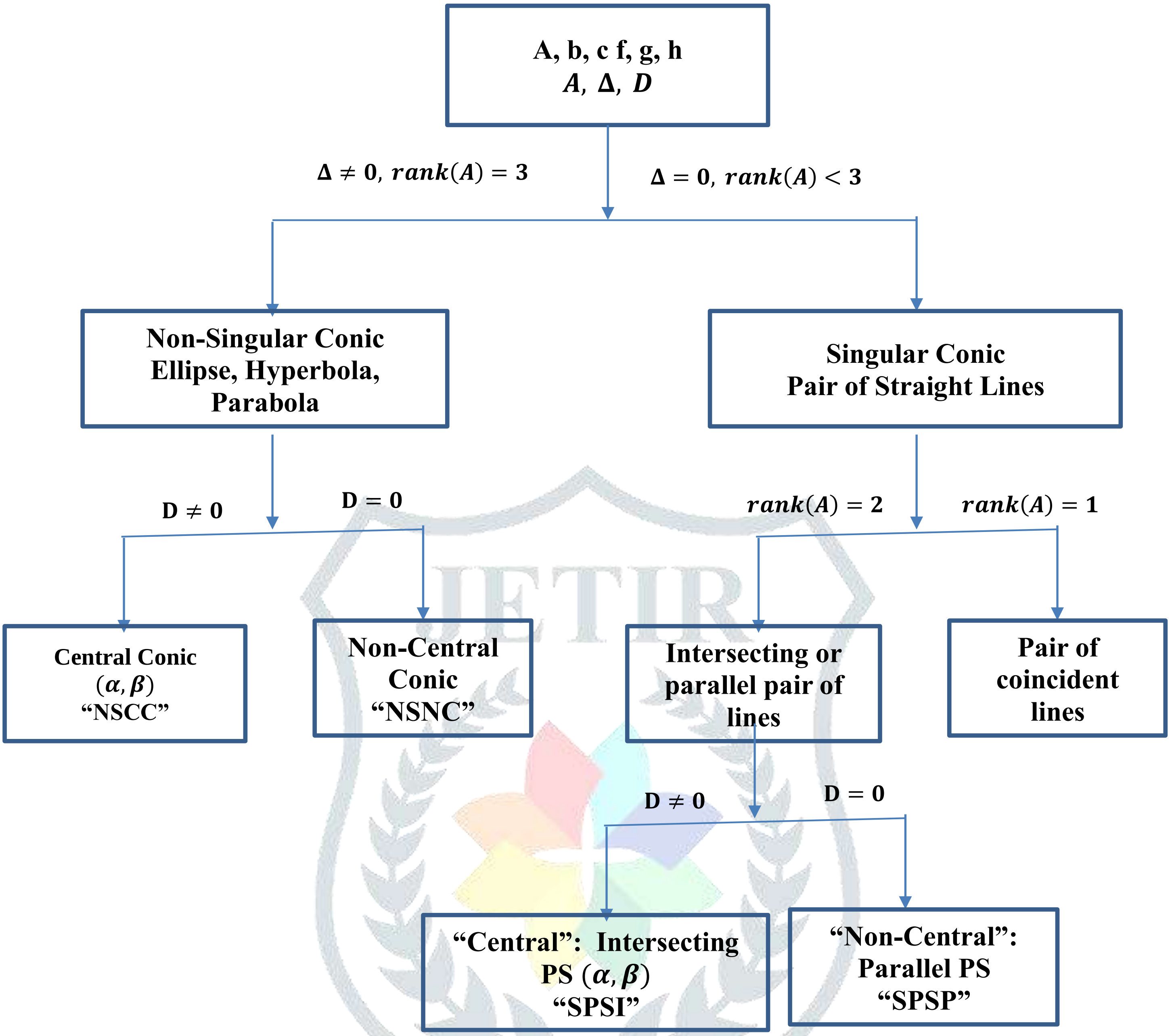
$$A=\begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix},\qquad \Delta=\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=abc+2fgh-af^2-bg^2-ch^2$$

$$D=\begin{vmatrix} a & h \\ h & b \end{vmatrix}=ab-h^2$$

Table 2.1 Classification of 2nd degree equation

Sl. No.	Value of		Rank of A	Nature	Canonical form	Trick
	Δ	D				
1.	$\neq 0$	$-$	**3**	**Non-singular curve**		
1.1	$\neq 0$	$= 0$	3	Parabola	$y^2 = 4\alpha x$ or $x^2 = 4\beta y$	**NSNC**
1.2	< 0	< 0	3	Hyperbola	$\frac{x^2}{A^2} - \frac{y^2}{B^2} = -1$	**NSCC**
1.3	< 0	> 0	3	Ellipse	$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$	**NSCC**
1.4	> 0	< 0	3	Hyperbola	$\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$	**NSCC**
1.5	> 0	> 0	3	Imaginary Ellipse	$\frac{x^2}{A^2} + \frac{y^2}{B^2} = -1$	**NSCC**
2.	$= 0$	$-$	**2**	**Singular curve**		
2.1	$= 0$	> 0	2	Point ellipse or Imaginary st. line	$Ax^2 + By^2 = 0$	**SPSI**
2.2	$= 0$	< 0	2	Pair of intersecting st. lines	$y^2 - k^2x^2 = 0$	**SPSI**
2.3	$= 0$	$= 0$	2	Pair of parallel st. lines	$y^2 = m^2\ (m \neq 0)$ $y^2 = n^2\ (n \neq 0)$	**SPSP**
3.	$= 0$	$= 0$	**1**	**Pair of coincident st. lines**	$y^2 = 0$ **or** $x^2 = 0$	**SPSC**

3. Outline to bring into canonical form or normal form of General Equation of Second Degree equation in 2D



3.1. NSCC: Non Singular Central Conic ($\Delta \neq 0, rank(A) = 3, D \neq 0$)

(i) Find centre of the conic (α, β) by solving equations

$$a\alpha + h\beta + g = 0$$
$$h\alpha + b\alpha + f = 0$$

(ii) First transformation: shifting of origin at (α, β) in parallel coordinates

$$(x, y) \to (x', y')$$
$$x = x' + \alpha, y = y' + \beta$$

(iii) Second transformation: Rotation of axes through an angle θ

$$(x', y') \to (X, Y)$$
$$x' = Xcos\theta - Ysin\,\theta\ (Trick{:}\, CM\ S)$$
$$y' = Xsin\,\theta + Ycos\,\theta\ (Trick{:}\, SP\ C)$$

(iv) Removal of product term XY and obtain θ (least possible value).

3.2. NSNC: Non Singular Non Central Conic ($\Delta \neq 0, rank(A) = 3, D = 0$)

(i) Bring second degree terms in perfect square

(ii) Add a constant λ to the squared term and equalize expression

(iii) Find λ such that the straight lines represented by the expressions within square and outside square are perpendicular.

(iv) These two lines serve as new rectangular axes (X, Y). Relate old coordinates (x, y) and new coordinates (X, Y) by considering a point P on the curve.

3.3. SPSI: Singular Pair of Straight lines (intersecting) $(\mathbf{\Delta = 0}, \boldsymbol{rank(A) = 2}, \boldsymbol{D \neq 0})$

(i) Find 'centre' (**point of intersection**) (α, β) by solving equations

$$a\alpha + h\beta + g = 0$$
$$h\alpha + b\alpha + f = 0$$

(ii) First transformation: shifting of origin at (α, β) in parallel coordinates

$$(x, y) \rightarrow (x', y')$$
$$x = x' + \alpha, y = y' + \beta$$

(iii) Second transformation: Rotation of axes through an angle θ

$$(x', y') \rightarrow (X, Y)$$
$$x' = Xcos\theta - Ysin\,\theta\ (Trick: CM\ S)$$
$$y' = Xsin\,\theta + Ycos\,\theta\ (Trick: SP\ C)$$

(iv) Removal of product term XY and obtain θ (least possible value).

3.4. SPSP: Singular Pair of Straight lines (Parallel) $(\mathbf{\Delta = 0}, \boldsymbol{rank(A) = 2}, \boldsymbol{D = 0})$ **(at least one minor of order 2 of** $\boldsymbol{A}$ **not 0)**

(i) No Shift of origin required.

(ii) First transformation: Rotation of axes through an angle θ

$$(x', y') \rightarrow (X, Y)$$
$$x' = Xcos\theta - Ysin\,\theta\ (Trick: CM\ S)$$
$$y' = Xsin\,\theta + Ycos\,\theta\ (Trick: SP\ C)$$

(iii) Removal of product term XY and obtain θ (least possible value).

3.5. SPSC: Singular Pair of Straight lines (Coincident) $(\mathbf{\Delta = 0}, \boldsymbol{rank(A) = 1}, \boldsymbol{D = 0})$ **as well as other minors of order 2 are 0)**

(i) No Shift of origin required.

(ii) First transformation: Rotation of axes through an angle θ

$$(x', y') \rightarrow (X, Y)$$
$$x' = Xcos\theta - Ysin\,\theta\ (Trick: CM\ S)$$
$$y' = Xsin\,\theta + Ycos\,\theta\ (Trick: SP\ C)$$

(iii) Removal of product term XY and obtain θ (least possible value).

4. Impact and Applications

The development of two-dimensional coordinate geometry had profound implications for mathematics, science, and engineering. The unification of algebra and geometry enabled a systematic approach to solving geometric problems, making it easier to model and analyze physical phenomena. The use of coordinate systems became fundamental in physics, particularly in mechanics and electromagnetism, where the position and movement of objects could be described mathematically. Coordinate geometry is essential in computer graphics, robotics, and computer-aided design (CAD), where precise mathematical descriptions of shapes and movements are crucial. The concepts of coordinate geometry laid the foundation for more advanced fields like vector calculus, linear algebra, and differential geometry, which are vital in modern science and engineering.

5. Conclusion

Two-dimensional coordinate geometry, born out of the pioneering work of Descartes and Fermat, represents a monumental shift in mathematical thinking. It bridged the gap between algebra and geometry, transforming how we understand and interact with the geometric world. This field continues to be a cornerstone of mathematics, influencing a wide range of scientific and technological advancements.

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