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An Optimization Technique for Solving Multi-Objective Linear Fractional Programming Problem

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Abstract: - This paper introduces a method for transforming a multi-objective linear fractional programming problem (MOLFPP) into a single-objective linear fractional programming problem (SOLFPP) using median of maximin and minimax techniques. The efficacy of the proposed approach is demonstrated through a numerical illustration.

Keywords: - Multi objectives linear fractional programming problem (MOLFPP), median of maximin and minimax.

1. Introduction: Linear Fractional Programming pertains to a category of mathematical optimization problems characterized by linear relationships among variables. These problems necessitate linear constraints and involve optimizing an objective function expressed as a ratio of two linear functions, such as profit/cost, output/employee etc.. This methodology finds application across various domains including production planning, financial analysis, corporate planning, healthcare management, hospital planning etc. A survey of multi-criteria linear programming issues (MCLP) is presented in [3], proposing an approach to construct multi-criteria functions within constraints ensuring that the optimal value of each problem exceeds zero. Sulaiman and Sadiq investigated the multi-criteria function using mean and median methodologies [5]. Additionally, Sulaiman and Salih examined the multi-criteria fractional programming problem employing mean and median techniques [6]. Nahar Samsun et al. advocated a novel geometric averaging technique for optimizing the objective function, consolidating multiple objective functions into a single one [2]. In 2016, Sulaiman et al. proposed a fresh technique utilizing the Harmonic mean of objective function values to tackle multi-criteria linear programming issues [4].Recently in 2021 Mojtaba Borza & Azmin Sham Rambely [1] Proposed a new method to solve multi- objective Linear fractional problems.

To further expand upon this research, we have introduced the concept of Multi-Objective Linear Fractional Programming Problems (MOLFPP) and proposed an algorithm for resolving linear fractional programming issues pertaining to multi-objective functions. Our method leverages the median of maximin and minimax techniques. We substantiate the effectiveness of our approach through a numerical demonstration.

2. Mathematical form of LFPP:

The mathematical form of LFP problem is given as follows:

$$\text{Max. } Z = \frac{(c^T X+\alpha)}{(d^T X+\beta)}$$

Subject to:

$$AX \leq b$$
$$X \geq 0$$

where

i) X, c and d are n × 1 vector,
ii) b is an m × 1 vector,
iii) c^T, d^T denote transpose of vectors,
iv) A is an m × n matrix and
v) α , β are scalars.

3. Multi-Objective Linear Fractional Programming Problem:

The mathematical form of MOLFPP is given as follows:

$$\left.\begin{array}{l} \text{Max. } z_1 = \frac{c_1{}^T X+\alpha_1}{d_1{}^T X+\beta_1} \\ \text{Max. } z_2 = \frac{c_2{}^T X+\alpha_2}{d_2{}^T X+\beta_2} \\ \quad \vdots \\ \text{Max. } z_r = \frac{c_r{}^T X+\alpha_r}{d_r{}^T X+\beta_r} \\ \text{Min. } z_{r+1} = \frac{c_{r+1}{}^T X+\alpha_{r+1}}{d_{r+1}{}^T X+\beta_{r+1}} \\ \quad \vdots \\ \text{Min. } z_s = \frac{c_s{}^T X+\alpha_s}{d_s{}^T X+\beta_s} \end{array}\right] \quad (3.1)$$

subject to:

$$AX \leq b \quad (3.2)$$
$$X \geq 0 \quad (3.3)$$

where

i) b is an m-dimensional vector of constants,
ii) X is an n-dimensional column vector of decision variables,
iii) r is number of objective functions to be maximized,
iv) s is the number of objective functions to be maximized and minimized
v) (s-r) is the number of objective functions that is minimized.
vi) A is an m× n matrix of constants,

vii) c_i, d_i (where i = 1,2,…,s) are n-dimensional vectors of constants and

viii) α_i, β_i(where i = 1,2,…,s) are scalars.

All vectors are assumed to be column vectors unless transposed(T)

4. Method for Solving MOLFPP:

4.1 Median of maximin and minimax Technique:

Step1: First, we solve each objective function by using Kanti Swarup's Fractional Algorithm (KSFA) [7].

Step2: Next, we assign a name to the optimum value of each objective function Max z_i say φ_i, i = 1,2,…r and Min z_i say φ_i, i = r+1, r+2,…,s .

Step3: Choose $m_1 = min\{\varphi_i\}$, ∀ i = 1, 2,…,r and $m_2 = max\{\varphi_i\}$, ∀ i= r+1,…,s then calculate

$$Md = \text{Median}\left(\left| m_j \right|\right), \qquad j = 1, 2$$

Step4: Optimize the combined objective function by using KSFA[7] under the same constraints (3.2) and (3.3) as follows:

$$\text{Max. Z} = \frac{\left(\sum_{i=1}^{r} Max\ z_i - \sum_{i=r+1}^{s} Min\ z_i\right)}{Md} \qquad (4.1)$$

5. Numerical Example:

5.1. Example.

$$\text{Max. } z_1 = \frac{3x_1 - 2x_2}{x_1 + x_2 + 1}$$

$$\text{Max. } z_2 = \frac{9x_1 + 3x_2}{x_1 + x_2 + 1}$$

$$\text{Max. } z_3 = \frac{3x_1 - 5x_2}{2x_1 + 2x_2 + 2}$$

$$\text{Min. } z_4 = \frac{-6x_1 + 2x_2}{2x_1 + 2x_2 + 2}$$

$$\text{Min. } z_5 = \frac{-3x_1 - x_2}{x_1 + x_2 + 1}$$

Subject to:

$$x_1 + x_2 \leq 2, \quad 9x_1 + x_2 \leq 9, \quad x_1, x_2 \geq 0$$

Solution: After finding the value of each of individual objective functions by using KSFA[7], the results are given below:

Table 1

i	φ_i	x_i	m_1	m_2	Md
1	3/2	(1,0)			
2	9/2	(1,0)	3/4		
3	3/4	(1,0)			9/8
4	$-3/2$	(1,0)		$-3/2$	
5	$-3/2$	(1,0)			

Formulate the combine objective function as follows:

Max. Z = $\frac{(\sum_{i=1}^{r} Max\ z_i - \sum_{i=r+1}^{s} Min\ z_i)}{Md}$ where Md = Median ($|m_j|$), $j = 1, 2$

Max. $Z = \frac{312x_1 - 24x_2}{18x_1 + 18x_2 + 18}$

subject to:

$$x_1 + x_2 \leq 2, \quad 9x_1 + x_2 \leq 9, \quad x_1, x_2 \geq 0$$

Hence the optimal solution is

Max. Z = 8.67, $x_1 = 1$, $x_2 = 0$.

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