



Solution of fractional derivative and Eulerian integral formulae of multivariable H-Function containing general class of polynomial

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Abstract

A large number of fractional derivative and Eulerian integral formulae of multivariable H-Function containing general class of polynomial have been presented. Here, in this paper, our aim is to establishing three fractional integral formulas involving the products of the multivariable H-function and a general class of polynomials by using generalized fractional integration operators given by Saigo and Maeda [M. Saigo, N. Maeda, Varna, Bulgaria, (1996), 386–400]. All the results derived here being of general character, they are seen to yield a number of results (known and new) regarding fractional integrals. In this paper we use fractional differential operators $D_{k,\alpha,x}^n$ and ${}_a D_x^\mu$ to derive a number of key formulas of multivariable H-function. We use the generalized Leibnitz's rule for fractional derivatives in order to obtain one of the aforementioned formulas, which involve a product of two multivariable's H-function. It is further shown that each of these formulas yield interesting new formulas for certain multivariable hypergeometric function such as generalized Lauricella function (Srivastava-Daoust) and Lauriella hypergeometric function some of these application of the key formulas provide potentially useful generalization of known result in the theory of fractional calculus.

Keywords: Fractional differential operator, Generalized fractional integral operators, multivariable H-function, general class of polynomials.

Subject Classification 2020 MSC: 26A33, 33C45, 33C60, 33C70.

1. INTRODUCTION AND DEFINITIONS

Fractional calculus which are derivatives and integrals of arbitrary (real and complex) orders have found many applications in a variety of fields ranging from natural science to social science. In recent years, it has turned out that many phenomena in engineering, physics, chemistry and other sciences can be described very successfully by means of models using mathematical tools deduced from fractional calculus

Fractional derivatives are also used in modeling many chemical processes, mathematical biology and many other problems in physics and engineering (see, e.g., [3]–[1], [4], [5], [6]). Under various fractional calculus operators, the computations of image formulas for special functions of one or more variables are important from the point of view of the usefulness of these results in the evaluation of generalized integrals and the solution of differential and integral equations (see, e.g., [7], [8],[9], [10], [11], [12], [13] and [14] and so on). Motivated essentially by diverse applications of fractional calculus we establish three image formulas for the product of multivariable H-function and general class of polynomials involving left and right sided fractional integral operators of Saigo-Meada [15]. By virtue of the unified nature of our results, a large number of new and known results involving Saigo, Riemann-Liouville and Erdélyi-Kober fractional integral operators and several special functions are shown to follow as special cases of our main results. The generalized fractional integral operators of arbitrary order involving Appell function F_3 in the kernel were defined and investigated by Saigo and Maeda [15, p. 393, Eqs. (4.12) and (4.13)]

The fractional derivative of special function of one and more variables is important such as in the evaluation of series,[10,15] the derivation of generating function [12,chap.5] and the solution of differential equations [4,14;chap-3] motivated by these and many other avenues of applications, the fractional differential operators $D_{k,\alpha,x}^n$ and ${}_x D_x^\mu$ are much used in the theory of special function of one and more variables .

We use the fractional derivative operator defined in the following manner [16]

$$D_{k,\alpha,x}^n(x^\mu) = \prod_{r=0}^{n-1} \left[\frac{\sqrt{\mu + rk + 1}}{\sqrt{\mu + rk - \alpha + 1}} \right] x^{\mu+nk} \quad \dots(1.1)$$

Where $\alpha \neq \mu + 1$ and α and k are not necessarily integers

The present work is an attempt in the direction of obtaining fractional calculus formula by utilizing series expression method, introduced by srivastava [22]. The name general class of polynomials, itself indicates the importance of the results, because we can derive a number of fractional calculus formulae for various classical orthogonal polynomials.

Differential operator ${}_x D_x^\mu$ is defined by [5, p.49; 3; 9; 17, P-356]

$${}_a D_x^\mu f(x) = \begin{cases} \frac{1}{\sqrt{-\mu}} \int_a^x (x-t)^{-\mu-1} f(t) dt, & [\text{Re}(\mu) < 0] \\ \frac{d^m}{dx^m} {}_a D_x^{\mu-m} f(x), & [0 \leq \text{Re}(\mu) < m] \end{cases} \dots(1.2)$$

Where m is a positive integer

For $\alpha = 0$, (1.2) Defines the classical Riemann-Liouville fractional derivative of order μ (or- μ) when $\alpha \rightarrow \infty$ (1.2) may be identified with the definition of the well known Weyl fraction derivative of order μ (or- μ) [1,chap.13);3] the special case of fractional calculus operator ${}_a D_x^\mu$ when $\alpha=0$ is written as D_x^μ thus we have

$$D_x^\mu = {}_0 D_x^\mu \dots (1.3)$$

In this paper we drive several fractional derivative formulas involving multivariable H-function which as defined by srivastav and panda [8, p.271 (4.1) et. Seq.] and studied systematically by then [6,7,8 also 11] for this multivariable H-function we adopt the contracted notations (due essentially to Srivastava and panda [23] thus following the various conventions and notations explained fairly and fully in their earlier works [6,7,8see also 11,13]

$$H[z_1, \dots, z_r] = H_{p,q: p_1, q_1, \dots, p_r, q_r}^{0, n: m_1, n_1, \dots, m_r, n_r} \left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \left| \begin{matrix} (a_j, \alpha_j^{(1)} \dots \alpha_j^{(r)})_{1,p} : (c_j^{(1)}, \gamma_j^{(1)})_{1,p_1} \dots (c_j^{(r)}, \gamma_j^{(r)})_{1,p_r} \\ (b_j, \beta_j^{(1)} \dots \beta_j^{(r)})_{1,q} : (d_j^{(1)}, \delta_j^{(1)})_{1,q_1} \dots (d_j^{(r)}, \delta_j^{(r)})_{1,q_r} \end{matrix} \right. \right] \dots(1.4)$$

Denote the H-function of r-variables z_1, z_2, \dots, z_r here for convenience

$$(a_j, \alpha_j^1, \dots, \alpha_j^{(r)})_{1,p} \text{ Abbreviates the p- member array} \dots(1.5)$$

While $(c_j^{(i)}, \gamma_j^{(i)})_{1,p_i}$ Abbreviates the array of p_i pairs of parameters

$$(\alpha_j^{(i)}, \gamma_j^{(i)}), \dots, (\alpha_{p_i}^{(i)}, \gamma_{p_i}^{(i)}) \quad ; \quad (i=1, \dots, r) \dots (1.6)$$

and so on, suppose, as usual that the parameters

$$a_j, \quad j=1, \dots, p; \quad c_j^{(i)}, \quad j=1, \dots, p_i; \\
 b_j, \quad j=1, \dots, q; \quad d_j^{(i)}, \quad j=1, \dots, q_i; \quad \forall i \in (i=1, \dots, r) \dots(1.7) \text{ complex number and}$$

the associated coefficients

$$\alpha_j, j = 1, \dots, p; \gamma_j^{(i)}, j = 1 \dots p_i; \\ \beta_j, j = 1, \dots, q; \delta_j^{(i)}, j = 1 \dots q_i; \quad \forall i \in (1, \dots, r) \quad \dots (1.8)$$

Are positive real numbers such that

$$\Lambda_i = \sum_{j=1}^r \alpha_j^{(i)} - \sum_{j=1}^q \beta_j^{(i)} + \sum_{j=1}^{p_i} \gamma_j^{(i)} - \sum_{j=1}^{q_i} \delta_j^{(i)} \leq 0 \quad \dots (1.9)$$

and

$$\Omega_i = - \sum_{j=n+1}^p \alpha_j^{(i)} - \sum_{j=1}^q \beta_j^{(i)} + \sum_{j=1}^{n_i} \gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} \gamma_j^{(i)} + \sum_{j=1}^{m_i} \delta_j^{(i)} - \sum_{j=m_i+1}^{q_i} \delta_j^{(i)} > 0; \quad \forall i \in [1, \dots, r] \quad \dots (1.10)$$

Where the integers $n, p, q, m_i, n_i, p_i, q_i$ are constrained by the inequalities $0 \leq n \leq p, q \geq 0, 1 \leq m_i \leq q_i, 0 \leq n_i \leq p_i [i = 1, \dots, r]$ and the equality (1.10) holds true for suitably restricted values of the complex variables z_1, \dots, z_r

Then it is known that the multiple Mellin-Barnes counter integral [11, p.251 (c.1)] representing the multivariable H-function (1.4) converges absolutely under the condition (1.10) when

$$|\arg(z_i)| < \frac{1}{2} \Lambda \Omega_i, \quad \forall i \in [1, \dots, r] \quad \dots (1.11)$$

$$H[z_1, \dots, z_r] = \begin{cases} 0(|z_1|^{\xi_1}, \dots, |z_r|^{\xi_r}), & (\max |z_i| \rightarrow 0) \\ 0(|z_1|^{\eta_1}, \dots, |z_r|^{\eta_r}), & (\eta = 0; \min |z_i| \rightarrow \infty) \end{cases} \quad \dots (1.12)$$

Where with $i=1, \dots, r$

$$\xi_i = \min \left\{ \text{Re} \left(\frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right\}, \quad (j = 1, \dots, m_i) \\ \eta_i = \max \left\{ \text{Re} \left(\frac{c_j^{(i)} - 1}{\gamma_j^{(i)}} \right) \right\}, \quad (j = 1, \dots, n_i) \quad \dots (1.13)$$

Provided that each of the inequalities (1.10), (1.11) and (1.12) holds true.

Throughout the present paper we assume that the convergence and existence condition corresponding appropriately to the ones detained above are satisfied by each of the various H-function involved in our results which are presented in the following sections

(i) The H-Function Defined by Saxena and kumbhat [21] is an extension of Fox's H-Function on specializing the parameters, H-Function can be reduced to almost all the known special function as well as unknown

The Fox's H-Function of one variable is defined and represented in this Paper as follows [see Srivastava et al [22], pp 11-13]

$$H[x] = H_{P,Q}^{M,N} \left[x / \begin{matrix} (a_j, \alpha_j)_{1, P} \\ (b_j, \beta_j)_{1, Q} \end{matrix} \right] = \frac{1}{2\pi\omega} \int_{\theta=N-1} \theta(\xi) x^\xi d\xi \quad \dots (1.14)$$

$$\theta(\xi) = \frac{\prod_{i=1}^n \Gamma b_i - \beta_j \xi \prod_{j=1}^N \Gamma 1 - a_j - \alpha_j \xi}{\prod_{i=M=1}^Q \Gamma 1 - b_j + \beta_j \xi \prod_{j=N+1}^P \Gamma a_j - \alpha_j \xi} \dots(1.15)$$

For condition of the H-Function of one variable (1.13) and on the contour L we

refer to srivastava et al [25]

Let be complex numbers, and let $x \in (0, \infty)$ Following Saigo [24] Fractional integral $\text{Re}(\alpha) > 0$ and derivative $\text{Re}(\alpha) < 0$ of first kind of a function $f(x)$ on are defined respectively in the forms:

$$(f) = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} {}_2F_1(\alpha + \beta; \dots) \dots(1.16) \quad \text{Re}(\alpha) > 0$$

$$(f) , 0 < \text{Re}(\alpha) + n < 1 \quad (n = 1, 2, 3, \dots),$$

Where ${}_2F_1(a, b; c; z)$ is Gauss's hypergeometric function

Let α, β, η and λ be complex numbers. Then there hold the following formulae. . the R.H.S. has a definite meaning

$$= \dots(1.17)$$

provided that $\text{Re}(\alpha) > \max [0, \text{Re}(\beta - \eta)] - 1$

2.MAIN RESULT

In this section we shall prove our main formulas on fractional derivative and Eulerian integral of multivariable H- Function containing general class of polynomial

Results -1

$$D_n^u \{ x^k S_{N_1 N_2}^{M_1 M_2} \{ y_1 x^{u_1} y_2 x^{u_2} \} I_{0,x}^{\alpha, \beta, \eta} \{ t^\lambda H [z_1 x^{\lambda_1} \dots z_1 x^{\lambda_1}] H [w_1 x^{\rho_1} \dots w_1 x^{\rho_1}] \}$$

$$= x^{k+\lambda-u} \sum_{J_1=0}^{N_1/M_1} \sum_{J_2=0}^{N_2/M_2} \sum_{m=0}^{\infty} \frac{(-N_1)_{m_1 k_1} (-N_2)_{m_2 k_2}}{U_1 U_2} \binom{u}{m} A[N_1 J_1, N_2 J_2] y_1^{J_1} y_2^{J_2}$$

$$\frac{[\beta - \lambda][\eta - \lambda]}{[-\lambda][\alpha + \beta + \eta - \lambda]} H_{P+1}^{0, n+1, m_1, n_1, \dots, m_r, n_r}_{q+1, p_1, q_1, \dots, p_r, q_r}$$

$$\left[\begin{array}{l} z_1 x^{\lambda_1} \\ \vdots \\ z_r x^{\lambda_r} \end{array} \left| \begin{array}{l} (\beta - \lambda, \lambda_1, \dots, \lambda_r) (a_j, \alpha_j^i, \dots, \alpha_j^{(r)})_{1,p} (c_j^i, r_j^i)_{p,1} \dots (c_j^r, r_j^r)_{1,p_r} \\ \vdots \\ (\beta + u - m - \lambda, \lambda_1, \dots, \lambda_r) (b_j, \beta_j^i, \dots, \beta_j^{(r)})_{1,q} (d_j^i, \delta_j^i)_{1,q_1} \dots (d_j^r, \delta_j^r)_{1,q_r} \end{array} \right. \right]$$

$$\times H_{P+1}^{0, n+1, m_1, n_1, \dots, m_s, n_s}_{q+1, p_1, q_1, \dots, p_s, q_s}$$

$$\left[\begin{array}{l} w_1 x^{\rho_1} \\ \vdots \\ w_s x^{\rho_s} \end{array} \left| \begin{array}{l} (-k, p_1, \dots, p_s) (-g_j, G_j^i, \dots, G_j^s)_{1,p} (u_j^i, U_j^i)_{1,p} \dots (u_j^s, U_j^s)_{1,p_s} \\ \vdots \\ (m - k, P_1, \dots, P_s) (h_j, H_j^i, \dots, H_j^s)_{1,q} (v_j^i, V_j^i)_{1,q_1} \dots (v_j^s, V_j^s)_{1,q_s} \end{array} \right. \right] \dots (2.1)$$

Provided that (in addition to the appropriate convergence and existence condition) that $\min(k, \lambda, \rho, \sigma) > 0$, $\xi_i < 1$ and $\text{Re}(k + \rho_j - \mu + 1) > 0$

$$\min \{ \nu_1, \nu_2, \rho_i, \sigma_i, \delta_i \} > 0 \quad (i = 1, \dots, r);$$

$$\max \left\{ \left| \arg \left(\frac{x^{\nu_1}}{a} \right) \right|, \left| \arg \left(\frac{x^{\nu_2}}{b} \right) \right| \right\} < \pi \quad \text{Re}(k) + \sum_{i=1}^r \rho_i \xi_i > -1$$

Where $\xi_i = (i = 1, \dots, r)$ are given in (1.18)

Results -2

$$D_x^u \{ x^k (x^\nu + a)^\lambda S_N^M [y x^u] H[z_1 x^{\lambda_1} \dots z_r x^{\lambda_r}]$$

$$H[w_1 x^{\rho_1} (x^\nu + a)^{\sigma_1} \dots w_r x^{\rho_r} (x^\nu + a)^{\sigma_r}]$$

$$= a^\lambda x^{k-u} \sum_{j=0}^{N/M} \sum_{l,m=0}^{\infty} \left(\frac{x^\nu}{a} \right)^l \frac{(-N)_{MJ}}{l!} \frac{A(N,J)y^j}{l!} \binom{\mu}{m}$$

$$H_{p+1,q+1}^{0n+1} \left[\begin{matrix} z_1 x^{\lambda_1} \\ \vdots \\ z_r x^{\lambda_r} \end{matrix} \right]_{p_1,q_1,\dots,p_r,q_r}^{m_1,n_1,\dots,m_r,n_r} \left[\begin{matrix} (-k-uj-vl, \lambda_1, \dots, \lambda_r) (a_j, \alpha_j^1, \dots, \alpha_j^{(r)})_{1,p} : (c_j^1, \gamma_j^1)_{1,p_1}, \dots, (c_j^{(r)}, \gamma_j^{(r)})_{1,p_r} \\ (-k-uj+u-m-vl, \lambda_1, \dots, \lambda_r) (b_j, \beta_j^1, \dots, \beta_j^{(r)})_{1,q} : (d_j^1, \delta_j^1)_{1,q_1}, \dots, (d_j^{(r)}, \delta_j^{(r)})_{1,q_r} \end{matrix} \right]$$

$$\times H_{P+2,Q+2}^{0N+2} \left[\begin{matrix} w_1 x^{\rho_1} a^{\sigma_1} \\ \vdots \\ w_r x^{\rho_r} a^{\sigma_r} \end{matrix} \right]_{(m,P_1,\dots,P_r),(-\lambda,\sigma_1,\dots,\sigma_r)}^{M_1,N_1,\dots,M_r,N_r} \left[\begin{matrix} (g_j, G_j^1, \dots, G_j^{(s)})_{1,P} (u_j^1, U_j^1)_{1,P_1}, \dots, (u_j^{(s)}, U_j^{(s)})_{1,P_s} \\ (h_j, H_j^1, \dots, H_j^{(s)})_{1,Q} (v_j^1, V_j^1)_{1,Q_1}, \dots, (v_j^{(s)}, V_j^{(s)})_{1,Q_s} \end{matrix} \right]$$

....(2.2)

Provided that (in addition to the appropriate convergence and existence condition) that $\min(k, \lambda, \rho, \sigma) > 0$, $\xi_i < 1$ and $\text{Re}(k + \rho_j - \mu + 1) > 0$

$$\min \{ \nu_1, \nu_2, \rho_i, \sigma_i, \delta_i \} > 0 \quad (i = 1, \dots, r);$$

$$\max \left\{ \left| \arg \left(\frac{x^{\nu_1}}{a} \right) \right|, \left| \arg \left(\frac{x^{\nu_2}}{b} \right) \right| \right\} < \pi \quad \text{Re}(k) + \sum_{i=1}^r \rho_i \xi_i > -1$$

Where $\xi_i = (i = 1, \dots, r)$ are given in (2.1)

Results -3

$$D_{k,a,n}^n \{ x^u (x^{\nu_1} + a)^\lambda (b - x^{\nu_2})^{-\delta} S_{N_1 N_2}^{M_1 M_2} [y_1 x^{u_1} \cdot y_2 x^{u_2}]$$

$$H [z_1 x^{\rho_1} (x^{\nu_1} + a)^{\sigma_1} (b - x^{\nu_2})^{-\delta_1} \dots z_r x^{\rho_r} (x^{\nu_1} + a)^{\sigma_r} (b - x^{\nu_2})^{-\delta_r}]$$

$$\begin{aligned}
 &= a^\lambda b^{-\delta} \sum_{J_1=0}^{N_1/M_1} \sum_{J_2=0}^{N_2/M_2} \sum_{l,m=0}^{\infty} \frac{(-N_1)_{m_1 j_1}}{l J_1} \frac{(-N_2)_{m_2 j_2}}{l J_2} A[N_1 J_1, N_2 J_2] \\
 &\frac{\left(\frac{x^{v_1}}{a}\right)^l \left(\frac{x^{v_2}}{b}\right)^m}{l! m!} x^{u+u_1 j_1+u_2 j_2+n k} H_{p^1+n+2, q^1+n+2, p_1^1, q_1^1, p_2^1, q_2^1}^{0, n^1+n+2, m_1^1, n_1^1, m_2^1, n_2^1} \\
 &\left[\begin{array}{l} (-\lambda, \sigma_1 \sigma_2)(1-\delta-m, \delta_1 \delta_2)(-u-u_1 j_1-u_2 j_2-v_1 l-v_2 m-g k, \rho_1 \rho_2)_{g=0, n-1} \\ \vdots \\ (-\lambda+l, \sigma_1 \sigma_2)(1-\delta_1, \delta_1 \delta_2)(-u-u_1 j_1-u_2 j_2-v_1 l-v_2 m-g k+\alpha, \rho_1 \rho_2)_{g=0, n-1} \end{array} \right] \\
 &\quad (a_j; c) \dots\dots(2.3)
 \end{aligned}$$

$$(b^j \cdot \beta_j^1, \dots)$$

Provided that (in addition to the appropriate convergence and existence condition) that $\min(k, \lambda, \rho, \sigma) > 0 < 1$ and $\text{Re}(k + \rho j - \mu + 1) > 0$

$$\min\{v_1, v_2, \rho_i, \sigma_i, \delta_i\} > 0 \quad (i = 1, \dots, r);$$

$$\max\left\{\left|\arg\left(\frac{x^{v_1}}{a}\right)\right|, \left|\arg\left(\frac{x^{v_2}}{b}\right)\right|\right\} < \pi \quad \text{Re}(k) + \sum_{i=1}^r \rho_i \xi_i > -1$$

Where $\xi_i = (i = 1, \dots, r)$ are given in (3.1)

Proof:-Result -1 First using The Fox's H-Function of multivariable is defined in (1.14) then apply Fractional integral formula (1.17) we shall utilize following definition introduced by srivastava [26] or general class of polynomials for two variables

$$= \left\{ \sum_{J_1=0}^{N_1/M_1} \sum_{J_2=0}^{N_2/M_2} \frac{(-N_1)_{m_1 k_1}}{l J_1} \frac{(-N_2)_{m_2 k_2}}{l J_2} A[N_1 J_1, N_2 J_2] y_1^j \right\}$$

Where m is an arbitrary positive integer and the coefficient (are arbitrary constant real or complex

Now using by mellin barnes type contour integral for H- function for multivariable

We use the fractional derivative operator () and after simplification we get required result .

Proof:-Result -2 For the proof of this result we shall utilize following definition introduced by Fox's H-Function (1.14)

we shall using the following definition introduced by srivastava [9] or general class of polynomials

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Where m is an arbitrary positive integer and the coefficient (are arbitrary constant real or complex

Now using by mellin barnes type contour integral for H- function for multivariable

We use the fractional derivative operator () and after simplification we get required result .

Proof:-Result -3 We use the fractional derivative operator () and after simplification. then we shall utilize following definition introduced by srivastava [26] or general class of polynomials for two variables

Now Taking by mellin barnes type contour integral for H- function for multivariables and then using beta function we get required result.

3.Conclusion

In this paper we get fractional differential operator formulae involving special function and general class of polynomials. Here we presented two very generalized and unified theorems associated with the generalized fractional integral operators given by Saigo-Maeda. The main fractional integrals whose integrands being the products of multivariable H-functions and a general class of polynomials. The main results may find potentially useful applications in a variety of areas.

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5.References

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