# A COMPARATIVE STUDY ON THE SIGNIFICANCE OF COMPUTER ORIENTED NUMERICAL ANALYSIS IN SOLVING PROBLEMS USING DIFFERENTIATION AND INTEGRATION

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***Abstract*—** The basic goal of this proposal is to show a segment of the central speculative results identifying with the critical locales of numerical examination. This paper is an undertaking of examination on the genuine progressing essentialness of Numerical Methods in the Equation clarification, differential equations and integration.

***Keywords*—Numerical methods, various categories applications, solutions, equations, eigen value, interpolation, approximation**

## I. INTRODUCTION

A numerical method in numerical analysis is a arithmetical contrivance projected to look after numerical problems. The implementation of a numerical procedure with a appropriate amalgamation ensure in a encoding language is called as a numerical algorithm. Numerical approaches for regular differential circumstances are practiced to realize numerical approximations to the provision of ordinary differential equations. Their operation is also called "numerical integration". In this section we discussed various methods and applications on Numerical differentiation and intergration initial value problems(IVP) for ordinary differential equations Boundary Value Analysis(BVA)

The categories of numerical methods are:

1. Differentiation and integration based on numerical methods

- Computation of divided difference and finding derivatives
- Applying Trapezoidal Rule to compute integration
- Numerical integration by simpson's $\frac{1}{3}$ rule
- Romberg's method
- Double integrals using trapezoidal rule

In next section various methods are discussed . In this paper it is attempted to consolidate the various applications of these methods in engineering and Science-technology.

## II. BACKGROUND

In this section we discuss mainly about the differentiation and integration.

2.1 Computation of divided difference and finding derivatives:

The expression derived by divided difference can be differentiated to obtain any order of derivative of the divided difference method.

2.2 Applying Trapezoidal Rule to compute integration:

There are two methods that might be used to find the estimation of a positive integral, one is called as the Trapezoidal rule and the other one is known as the Simpson's rule. The Trapezoidal rule is a method for approximating a definite integral $\int_a^b f(x)dx$ where $a$ and $b$ are the two values of $x$, using linear approximations of function. The distance between two $x_i$ is taken and the half of it is multiplied to the sum of individual values of $y$ (may be multiplied to a constant) at a particular $x_i$. This gives the closest approximation to the estimation of that integral. The Trapezoidal rule is only applicable for two instances;

a) $n = 1$
b) $n > 1$

Here,

$n = number\ of\ points - 1$

Formula:

$$\int_a^b f(x)dx = \frac{h}{2}[f(a) + f(b)]$$

$\int_a^b f(x)dx = \frac{h}{2}[y_0 + y_1]$

In general,

$\int_{x_0}^{x_n} f(x)dx = \frac{h}{2}[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1}]$

Where,
$h = x_i - x_{i-1}$
$y_0 = value\ of\ the\ function\ at\ the\ first\ point\ x_0$
$y_n = value\ of\ the\ function\ at\ the\ first\ point\ x_n$
Numerical integration by simpson's $\frac{1}{3}$ rule:

This rule is used to find an approximate value for the integral of a function $f(x)$ using quadratic polynomials. It can be derived by Lagrange interpolation polynomial. It can only be used for the instances where $n > 1$

Here,
$n = numberofpoints - 1$
The Simpson $\frac{1}{3}$ rule is

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{3}[y_0 + y_n + 2\big(y_2 + y_4 + \cdots + y_{last\ even\ number\ before\ n}\big) + 4(y_1 + y_2 + \cdots + y_{last\ odd\ number\ before\ n})]$$

Here,
$y_0 = f(x_0)$
$y_n = f(x_n)$
$h = x_i - x_{i-1}$

Romberg's method:

This is an extrapolation formula of the Trapezoidal rule which is used in integration. The method gives a more accurate result by reducing the true error.

In this method we find the third integral using two estimates of an integral such that it is more accurate than the previous results.

$$I_{t_1} = \frac{h_1}{2}[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})]$$

Now,

We double the number of $x$ by choosing a middle value of $x$ between the existing $x$ and finding the corresponding values of $y$ for the newly obtained $x$.

$$I_{t_2} = \frac{h_2}{2}[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})]$$

The Romberg's method:

$$I_{r_1} = I_{t_2} + \frac{1}{3}(I_{t_2} - I_{t_1})$$

For better accuracy this method can be repeated by adding more $x$.
In general:

$$I_{r_n} = I_{t_{n+1}} + \frac{1}{3}(I_{t_{n+1}} - I_{t_{n+2}})$$

Double integrals using trapezoidal rule:
The Simpson's rule and trapezoidal rule for ordinary integrals can be extended to multiple integrals.
For an expression:

$$\int_c^d \int_a^b f(x,y)dxdy = I$$

$$x = a\ to\ x = b\ and\ y = c\ to\ y = d$$

We know, by trapezoidal rule;

$$I = \frac{b-a}{2}\int_c^d [f(a,y) + f(b,y)]dy$$

$$I = \frac{hk}{4}[f(a,c) + f(a,d) + f(b,c) + f(b,d)]$$

Here,

$$h = x_i - x_{i-1}$$
$$k = y_i - y_{i-1}$$

## III. SIGNIFICANCE OF NUMERICAL METHOD IN DIFFERENTIATION AND INTEGRATION

3.1 Computation of divided difference and finding derivatives:
1. On sampled-data models used for nonlinear systems.
2. Derivative study of Hyperspectral Data.
3. Comparison of Photovoltaic Array Maximum Power Point Tracking methods.
4. New design with stability analysis of fuzzy proportional-derivative control systems.
5. Multipole-accelerated capacitance extraction algorithms meant for 3D structures by means of multiple dielectrics.
6. Comparison of time domain algorithms used for estimating aortic characteristic impedance in humans.
7. AdaBoost used for Text Detection in Natural Scene.
8. Combining source transformation with operator overloading techniques to compute derivatives for MATLAB programs.
9. The analysis of hyperspectral data by means of Savitzky-Golay filtering-theoretical basis.
10. Efficient Techniques used for Finite Element Analysis of Electric Machines.

3.2 Applying Trapezoidal Rule to compute integration:
1. A projection technique used for generalized eigenvalue problems by means of numerical integration.
2. Discretization schemes used for fractional-order differentiators with integrators.
3. Electrooptical effects in silicon.
4. Testing Of Trapezoidal Integration by means of Damping used for The Solution Of Power Transient Problems.
5. Parallel Solution of Transient Problems by means of Trapezoidal Integration.
6. Restraint of numerical oscillations in the EMTP power systems.
7. A connection of numerical techniques designed for modeling electromagnetic dispersive medium.
8. Novel IIR differentiator as of the Simpson integration rule.
9. Simultaneous Numerical Solution of Differential-Algebraic Equations.
10. Cooperative control of portable sensor networks:Adaptive angle moving in a disseminated situation.

3.3 Numerical integration by simpson's $\frac{1}{3}$ rule:
1. Trigonometric Approximations used for Bessel Functions.
2. A New Approach utilized for the Design of Wideband Digital Integrator with Differentiator.
3. A piecewise harmonic balance method used for determination of periodic response of nonlinear systems.
4. On the digital approximation of moment invariants.
5. Computing integrals concerning the matrix exponentia.
6. Novel steady relentless higher request s-to-z changes.
7. Closed-Form Design of Digital IIR Integrators by methods for Numerical Integration Rules with Fractional Sample Delays
8. An optimal lognormal estimation to lognormal sum distributions.
9. Upper and lower bounds on the result of the algebraic Riccati equation.
10. Removal of numerical oscillations in power system dynamic simulation.

3.4 Romberg's method:

1. Beyond Nyquist: Well-organized Sampling of Sparse Bandlimited Signals.
2. Stable signal recovery from partial and inaccurate measurements.
3. Imaging by means of Compressive Sampling.
4. Modeling as well as simulation of interconnection delays with crosstalks in high-speed integrated circuits.
5. Number of Faults for each Line of Code.
6. Robust uncertainty principles: exact signal reconstruction from highly partial frequency information.
7. Robust Signal Recovery from partial Observations.
8. Rate-distortion optimized image compression by means of wedgelets.
9. Multiscale classification by means of complex wavelets as well as hidden Markov tree models.
10. Fast and Accurate Algorithms used for Re-Weighted $\ell 1$-Norm Minimization.

3.5 Double integrals using trapezoidal rule:

1. On limits of wireless communications while using multiple dual-polarized antennas.
2. Digital Computer Solution of Electromagnetic Transients in Single as well as Multiphase Networks.
3. Mathematical models of hysteresis.
4. Error Probability used for Direct-Sequence Spread-Spectrum Multiple-Access Communications.
5. Efficient Coverage Control used for Mobile Sensor Networks by means of Guaranteed Collision Avoidance.
6. Estimation of walking features from foot inertial sensing.
7. Error Probability used for Direct-Sequence Spread-Spectrum Multiple-Access Communications.
8. Calculation of radiation patterns involving numerical double integration.
9. Evaluation of Weakly Singular Integrals by means of Generalized Cartesian Product system Based on the Double Exponential Formula.
10. Computation of energy transaction allocation factors.

## IV. Merits and Demerits

| Methods | Advantages | Disadvantages |
|---|---|---|
| Comutation of divided difference and finding derivatives | The method provides the derivative without an additional differentiation, only requires an additional value ($u = \frac{x-x_0}{h}$).<br><br>The order of derivative is further simplified for special cases such as $x = x_0$. | Accuracy decreases when the differences of dependent variables are not small.<br><br>Complexity increases when the number of dependent variables. |
| Applying Trapezoidal Rule to compute integration | This technique works well for periodic functions namely trigonometric functions.<br><br>Improved accuracy than simple trapezoidal rule and it is easy to utilize. | For this method the convergence rate is low and need to have additional sub intervals. |
| Numerical integration by simpson's $\frac{1}{3}$ rule | Precise for cubic polynomials.<br><br>This method ismore accurate than composite trapezoidal rule. | This rule is useful for only even number of finite intervals between the limits.<br><br>In this method, it is mandatory to use a huge number of ordinates to get a fine estimation to the real integral. |
| Romberg's method | Better precision than any other techniques.<br><br>In this method, user without any trouble can choose an appropriate step size and order. | Romberg integration method is used succesfully only when integrand satisfies the hypotheses of the Euler–Maclaurin.<br><br>This method cannot deal with uneven intervals. |
| Double integrals using trapezoidal rule | Simplifies the solution to the evaluation of double integration which are very difficult or impossible analytically.<br><br>The method provides with a formula that only requires the dependent variables, actual integration is not required. | Provides an approximate result and not the exact result.<br><br>Accuracy is directly dependent on the number subintervals thus increases time and complexity for a more accurate result. |

## V. Results of Applying Numerical Method in Differentiation and Integration Based Problems

The table shown below shows the error while applying computer oriented numerical analysis with the increase in the value of n. This is generated with Simpson's method and function is $\int \log_e dx$

| Value of n | Error |
|---|---|
| 1 | 0.462119 |
| 2 | 0.114895 |
| 3 | 5.25E-06 |
| 4 | 0.121511 |
| 5 | 0.004807 |
| 6 | 3.28E-07 |
| 7 | 0.069955 |
| 8 | 0.030369 |
| 9 | 5.8E-08 |
| 10 | 0.049115 |

TABLE 2: Different intervals and computed error values .

The Graph generated is shown below .

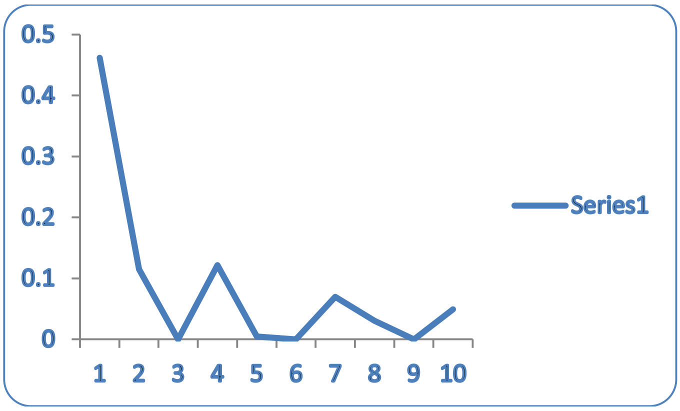


*Figure 1 Plot of Errors with increased value of number of intervals.*

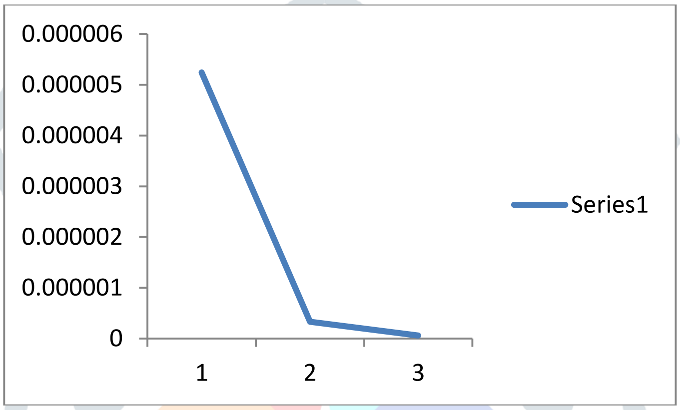


*Figure 2: Change in the value of error with decrease in the number of intervals*

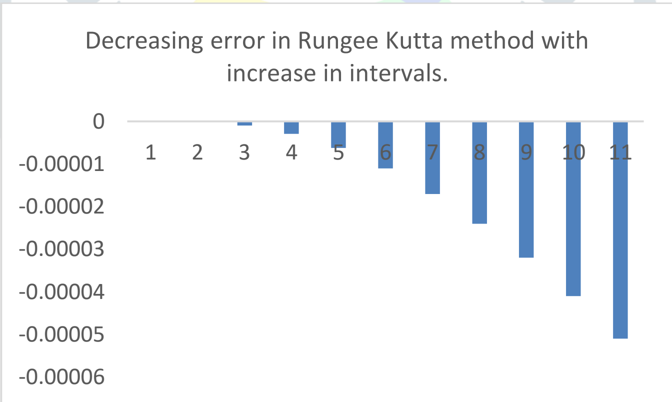


*Figure 3 Change in error in case of Rungee Kutta method*

| x | y | Mean y value | Error |
|---|---|---|---|
| **0** | 1 | 1 | 0 |
| **1** | 1.5625 | 1.28125 | -1.5E-07 |
| **2** | 3.999999 | 2.781245 | -9.2E-07 |
| **3** | 10.5625 | 7.281245 | -2.9E-06 |
| **4** | 24.99999 | 17.781245 | -6.2E-06 |
| **5** | 52.56249 | 38.781195 | -1.1E-05 |
| **6** | 99.99998 | 76.281195 | -1.7E-05 |
| **7** | 175.5625 | 137.7812 | -2.4E-05 |
| **8** | 289 | 232.28125 | -3.2E-05 |
| **9** | 451.5625 | 370.28125 | -4.1E-05 |
| **10** | 675.9999 | 563.7812 | -5.1E-05 |

Table 2 : Table shows the comparison on change in y value and error computation with increased value of intervals.

## VI. CONCLUSION

Data Analysis is one of the most salient sectors in the development of a new aspect or in finding a solution for an existing problem. Identification of significant data is the paramount of Data Analysis. Through this paper, we have tried to obtain such significant data by applying methods. These values of data can be easily identified using the aforementioned methods, each of which presents a different characteristic. In this paper, we have begun with the major idea of differential condition, some genuine applications where the concern is emerging and explanation of several current numerical techniques for their answer.

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