Bi-ideals of Γ- near subtraction semigroups

P. Meenakshi[1], N. Meenakumari[2]
[(1,2)] PG & Research Department of Mathematics
A. P. C. Mahalaxmi College for Women, Thoothukudi,Tamil Nadu, India.
[1]meena.bushy@gmail.com
[2]meenakumarin123@gmail.com

ABSTRACT

Schein, B. M [9] introduced the set theoretic subtraction '-' which is analogous to subtraction algebra and it is developed by Abbott, J. C.[1], Zelinka,B.[12] discussed the problem of Schein [9] relating the structure of multiplication in a subtraction semigroup. Kim, K. H [5]et. al. studied an ideal of a subtraction semigroup. Dheena, P.[3] and his colleagues defined the concept of near-subtraction semigroups and discussed their properties. Alandkar, S.J[2] gave the definition of Γ-near subtraction semigroup and some of its properties.

In this paper, using the concept of bi-ideals in near subtraction semigroups, we introduce the notion of bi-ideals in Γ-near subtraction semigroups. We show that the set of all bi-ideals of a Γ-near subtraction semigroup form a moore system. Also we proved that the intersection of a bi-ideals of Γ-near subtraction semigroup and sub-Γ-near subtraction semigroup is again a bi-ideal of X. Throughout this paper, by a Γ- near subtraction semigroup X, we mean a zero symmetric Γ-near subtraction semigroup.

Keywords:

Quasi ideals, bi-ideal, Γ-near subtraction semigroup.

1.Introduction

Throughout this paper X stands for a Zero- Symmetric Γ-near subtraction semigroup. Γ-near subtraction semigroup was introduced by Dr. S. J. Alandkar[2]. For basic terminology in near subtraction semigroup, we refer to Dheena [3] and for Γ-near subtraction semigroup, we refer to Dr. S. J. Alandkar[2]. Tamizh Chelvam and Ganesan [11]introduced the notion of bi-ideals in near-rings. In this paper we introduce the notion of bi-ideals in Γ-near subtraction semigroups.

2.Preliminaries

A nonempty set X together with a single binary operation "-" is said to be a subtraction algebra if it satisfies the following identities: for any x, y, z ∈ X, i) x – (y – x) = x; ii) x – (x – y) = y – (y – x); iii) (x – y) – z = (x – z) – y. A nonempty set X together with two binary operations "-" and "**.**" is said to be a subtraction semigroup if it satisfies the following axioms: for any x, y, z∈ X, i)(X, **.**) is a semigroup ii) (X, -) is a subtraction algebra; iii)x(y – z) = xy – xz and (x – y)z = xz – yz. A nonempty set X together with two binary operations "-" and "**.**" is said to be a near-subtraction semigroup(right) if it satisfies the following axioms: for any x, y, z∈ X, i)(X, **.**) is a semigroup ii) (X, -) is a subtraction algebra; iii) (x – y)z = xz – yz. A Γ-near subtraction semigroup is a triple (X, -, γ), $\gamma \epsilon$ Γ, where Γ is a non-empty set of binary operators on X, such that (X, -, γ) is a near-subtraction semigroup $\forall \gamma \epsilon \Gamma$. In practice, we called simply Γ-near subtraction semigroup instead of right Γ-near subtraction semigroup. Similarly we can define a Γ-near subtraction semigroup(left). It is clear that $0\gamma a = 0$ for all a∈ X and $\forall \gamma \epsilon \Gamma$. A nonempty subset S of a subtraction algebra X is said to be subalgebra of X, if x - y∈ S whenever x, y ∈ S. A subalgebra M of (X, -) with M ΓM⊆M is called a sub Γ-near subtraction semigroup of X. A nonempty subset A of X is called i) a left Γ-subalgebra of X if A is a subalgebra of (X, -) and X ΓA⊆A. ie.) $X\gamma A \subseteq A$ for all $\gamma \epsilon \Gamma$. ii) a right Γ-subalgebra of X if A is a subalgebra of (X, -) and AΓX⊆A. ie., $A\gamma X \subseteq A$ for all $\gamma \epsilon \Gamma$. $X_0 = \{x \in X / x\gamma 0 = 0$ for all $\gamma \epsilon \Gamma\}$ is called the zero-symmetric part of X. X is called Zero-Symmetric, if $X = X_0$. An element a∈ X is called idempotent if $a\gamma a = a$ for all $\gamma \epsilon \Gamma$. A family ζ of subsets of a set A is called a Moore system, if i)A∈$\boldsymbol{\zeta}$ and ii) $\boldsymbol{\zeta}$ is closed under arbitrary intersection. $X_d = \{n \in X / n\gamma(x + y) = n\gamma x + n\gamma y$ for all x, y∈X and for all $\gamma \epsilon \Gamma\}$ is the set of all distributive elements of X. X is called distributive if $X = X_d$.

3.Bi- ideals in Γ-near subtraction semigroup

In this section, we introduce the notion of bi-ideals in Γ-near subtraction semigroup and we obtain some preliminary results .

Definition 3.1

A subalgebra Q of (X, -) is said to be a quasi-ideal of X if (QΓX)∩(XΓQ)⊆Q.

Definition 3.2

A subalgebra B of (X, -) is said to be a bi- ideal of X if BΓXΓB⊆B.

Proposition 3.3

The set of all bi- ideals of a Γ-near subtraction semigroup X form a Moore system on X.

Proof:

Let $\{B_i\}_{i\in I}$ be a set of bi- ideals in X. Let $B = \cap_{i\in I} B_i$. Then BΓXΓB $\subseteq B_i\Gamma X\Gamma B_i \subseteq B_i \subseteq B, \forall\ i \in I$. Therefore B is a bi- ideal of X.

Remark 3.4

Every quasi-ideal is a bi- ideal.

Proof:

For, if Q is a quasi-ideal, then (QΓX)∩(XΓQ)⊆Q. Now, QΓXΓQ = QΓ(X∩X)ΓQ = (QΓX) ∩ (XΓQ) ⊆ Q. Therefore Q is a bi- ideal.

Proposition 3.5

Let X be a zero-symmetric Γ-near subtraction semigroup in which every quasi-ideal is idempotent. Then for left Γ-subalgebra L and right Γ-subalgebra R of X, RΓL=R∩L⊆ $L\Gamma R$ is true.

Proof:

Let A and B be two quasi-ideals in X. Then by proposition 3.3, A∩B is also a quasi-ideal. By the assumption on quasi-ideals we have A∩ B = (A ∩ B)Γ(A ∩ B) ⊆ (AΓB) ∩ (BΓA). On the other hand, we have (AΓB) ∩ (BΓA) ⊆ (AΓX) ∩ (XΓA) ⊆ A and analogously, (AΓB) ∩ (BΓA) ⊆ B . Hence (AΓB) ∩ (BΓA) ⊆ A ∩ B. Hence A ∩ B = (AΓB) ∩ (BΓA). Since one sided Γ-subalgebra are always quasi-ideals, by the above argument, we have R∩ L = (RΓL) ∩ (LΓR) ⊆ RΓL for a left Γ-subalgebra L and right Γ-subalgebra of X. Trivially RΓL⊆ R∩ L and so RΓL=R∩ L ⊆ LΓR.

Proposition 3.6

Let R and L be respectively right and left Γ-subalgebras of X. Then any subalgebra B of X such that RΓL⊆ B ⊆ R∩L is a bi- ideal of X.

Proof:

For a subalgebra B of (X, -) with RΓL ⊆ B ⊆ R∩L, we have BΓXΓB ⊆ (R∩L)ΓXΓ(R∩L) ⊆ RΓXΓL ⊆ RΓL⊆ B and so B is a bi- ideal of X.

Proposition 3.7

If B is a bi- ideal of X and S is a sub Γ-near subtraction semigroup of X, then B∩S is a bi- ideal of S.

Proof:

Since B is a bi- ideal of X, BΓXΓB ⊆ B. Let C= B ∩ S. Now, CΓSΓC = (B∩S)ΓSΓ(B∩S)⊆ (BΓSΓB)∩S ⊆ B∩S = C. Hence C is a bi-ideal of S.

Proposition 3.8

Let X be a zero-symmetric Γ- near subtraction semigroup. If B is a bi- ideal of X, then Bγx and x'γB for all γ ∈Γ are bi- ideal of X where x, x' ∈ X and x' is distributive element in X .

Proof:

Clearly Bγn is a subalgebra of (X, -) ∀ γ ∈ Γ. Also (B γ x)ΓXΓ(B γ x)⊆ BΓXΓ(Bγx) ⊆ Bγx and so we get that Bγx is a bi- ideal of X. Since x' is distributive in X, x'γB is a subalgebra of (X, -) for all γ ∈Γ (x'γB)ΓXΓ(x'γB)⊆ x'γB and hence x'γB is a bi- ideal of X.

Corollary 3.9

If B is a bi-ideal of zero-symmetric Γ- near subtraction semigroup X and b is a distributive element in X, then bγBγc is a bi- ideal of X for c ∈ X and for all γ ∈Γ.

Proposition 3.10

If B is a bi- ideal and sub Γ- near subtraction semigroup of a Zero symmetric Γ- near subtraction semigroup X and C is a bi- ideal of the Γ- near subtraction semigroup B such that $C^2 = C$, then C is a bi- ideal of the Γ- near subtraction semigroup X.

Proof:

Since C is a bi- ideal of the Γ- near subtraction semigroup B we have, CΓBΓC⊆C Now, CΓXΓC = C^2 ΓXΓ C^2 =CΓ(CΓXΓC)ΓC ⊆ CΓ(BΓXΓB)ΓC ⊆ CΓBΓC ⊆ C. Hence C is a bi- idealof the Γ- near subtraction semigroup X.

Theorem 3.11

Let X be a Γ- near subtraction semigroup. Let B be a bi- ideal of the Γ- near subtraction semigroup X and A be a non-empty subset of X, then following are true.

i) BΓA is a bi- ideal of the Γ- near subtraction semigroup X.
ii) AΓB is a bi- ideal of the Γ- near subtraction semigroup X.

Proof:

i) We see that (BΓA)Γ(BΓA) = (BΓAΓB)ΓA and (BΓA)ΓXΓ(BΓA)=(BΓAΓXΓB)ΓA. Since B is a bi- idealof the Γ- near subtraction semigroup X, (BΓA)Γ(BΓA) = (BΓAΓB)ΓA ⊆ BΓA and (BΓA)ΓXΓ(BΓA) = (BΓAΓXΓB)ΓA ⊆ (BΓXΓB)ΓA ⊆ BΓA. Therefore BΓA is a bi- ideal of the Γ- near subtraction semigroup X.
ii) Proof is similar to i) of theorem 3.11.

References

[1] **Abbott J. C.**, *Sets, Lattices and Boolean Algebras*, Allyn and Bacon, Boston 1969.

[2] **Alandkar S. J.,** *A Note on Γ- near subtraction semigroups*, Indian Streams Research Journal, Vol.6, Issue-6(2016)

[3] **Dheena P. and Satheesh Kumar G.**, *On strong regular near-subtraction semigroups* Commun. Korean Math.Soc.22(2007),pp.323-330.

[4] **Kaushik J. P. and Moin Khan**, *On bi-Γ-ideal in Γ-semirings*, Int. J. Contemp. Math. Sciences, Vol. 3, no.26(2008),pp.1255-1260

[5] **Kim K. H. Roh E. H. and Yon Y. H.**, *A Note on Subtraction Semigroups*, Scientiae Mathematicae Japonicae Online, Vol.10,(2004),pp.393-401.

[6] **Mahalakshmi V., Maharasi S., Jayalakshmi S.,** *On bi-ideals of Near-Subtraction Semigroup*, Advances in Algebra ISSN 0973-6964, Vol. 6(1)(2013), pp. 35-43.

[7] **Meenakshi, P. and Meenakumari N.**, *On semigamma bi-ideals in Γ-seminear-rings*, International Journal of Current Research Vol.9, Issue 10(2017), pp.59172-59175.

[8] **Pilz Gunter** , *Near-Rings*, North Holland, Amsterdam, 1983.

[9] **Schein B. M.,** *Difference Semigroups*, Comm. In Algebra 20(1992),pp. 2153-2169.

[10] **Tamizh Chelvam T. And Meenakumari N.,** *Bi-ideals of Gamma Near-Rings*, Southeast Asian Bulletin of Mathematics, 27(2004), pp. 983-988.

[11] **Tamil Chelvam and N. Ganesan** *On Bi-ideals of Near-Rings*, Indian J. Pure appl. Math. 18(11)(1987),pp.1002-1005 .

[12] **Zelinka B.,** *Subtraction Semigroups*, Math. Bohemica, 120(1995), pp. 445-447.