FUZZY PRE CONVERGENCE OF FUZZY NETS

N. Suvathi[1], Dr. M. Muthukumari[2],
[1]II – M.Sc. Mathematics ,
A.P.C. Mahalaxmi College for Women ,
Thoothukudi – 2.
Tamil Nadu , India.
(suvathichandran@gmail.com)
[2]Assistant Professor of Mathematics,
PG& Research Department of Mathematics,
A.P.C. Mahalaxmi College for Women,
Thoothukudi – 2.
Tamil Nadu , India.
(mmkumari27@gmail.com)

ABSTRACT

Nets and Filters are important tools in a topological space. Mathematicians introduced pre open sets in topological space. Recently Fuzzy Nets are introduced in topological space. The convergence of Fuzzy Nets and many properties of Fuzzy Nets are studied. In this paper we introduce Fuzzy pre convergence of Fuzzy Nets in topological spaces.

KEYWORDS: Fuzzy Nets, Fuzzy pre convergence.

.1. Introduction

Many concepts in metric spaces are studied using convergent sequences. Sequences are inadequate to study some properties of topological spaces. In the year 1922 E.H. Moore and H.L. Smith generalized the concept of sequences in the name Moor-Smith sequences. Later on Kelley coined the term Net for such sequences. In the year 1963 N. Levine introduced Semi open sets. In the year 1965 A. Zadeh introduced the concept of Fuzzy sets. In the year 1982 A.S. Mash hour and others introduced pre open sets. In the year 2014 M. Muthu kumari and others introduced the concepts of Fuzzy Nets and studied convergence properties.

In this paper we study pre convergence of Nets and Fuzzy Nets.

1.2 Preliminaries

1.2.1 Let D be a non empty set. ≤ be a relation in D. D is called a directed set if

1. ≤ is reflexive
2. ≤ is transitive
3. For $a, b \epsilon D$, $\exists\ C \epsilon D$ such that $a \leq c$ and $b \leq c$

(i.e.) Any two elements of D have upper bound in D.

1.2.2 Let X be a non empty set. Let D be a directed set. A function $f : D \rightarrow X$ is called a net. f(λ) is denoted as x_λ.

1.2.3 Let $f : D \rightarrow X$ be a net where X is a topological space. Let $a \epsilon X$. f is said to converge to a if for every open set U containing a, $\exists\,\lambda_0 \in$ D, such that $x_\lambda \in U$ for all $\lambda \geq \lambda_0$.

1.2.4 Let X be a non empty set. A function A : X → [0,1] is called a fuzzy set on X.

1.2.5 Let X be a topological space. Let A⊂X. A is called a pre open set if A ⊂ int clA. Clearly every open set is pre open but not conversely.

1.2.6 Let X be a non empty set. Let D be a directed set. A function A : D × X → [0,1] is called a fuzzy Net.

1.2.7 Let X be a topological space. Let A : D × X → [0,1] be a fuzzy Net. Let a∈X. A is said to fuzzy converge to a if

1. For each λ∈D, ∃ x∈X such that A(λ, x) = 1.
2. For each open set U containing a, there exists $\lambda_0 \in D$ such that

$$\lambda \geq \lambda_0 \text{ and } A(\lambda, x) = 1 \;\Rightarrow\; x \in U.$$

2. Pre convergence of nets

In this section we define pre convergence of nets in topological spaces.

2.1 Definition

Let D be a directed set and X be a topological space. Let f : D → X be a net. Let f(λ) = x_λ. Let a∈X, f is said to pre converge to a if for every pre open set U containing a, $\exists\lambda_0 \in D$ such that $x_\lambda \in$U for all $\lambda \geq \lambda_0$. (i.e.) (x_λ) is eventually in every pre open set U containing a.

2.2 Example

X = {a, b, c} T = {φ, {a}, {a, b}, {a, c}, X}

Let D = {{a}, {a, b}, {a, c}, X}. In D define ≤ as A ≤ B if A ⊃ B (i.e.) B ≤ A if B ⊃ A

Define a net f : D → X as {a} → a,{a, b} → b, {a, c} → c,X → b.

Claim: f pre converges to a.

Pre open sets are {a}, {a, b}, {a, c}, X.

Take λ_0 = {a}.

$$\lambda \geq \lambda_0 \Rightarrow A \geq \{a\} \Rightarrow A \subset \{a\}$$

$$\Rightarrow A = \{a\} \Rightarrow \lambda = \{a\}$$

Now

$$x_\lambda = f(\lambda) = f(\{a\}) = a$$

For any pre open set U contains a, a∈U

Hence $x_\lambda \in$ U $\forall\, \lambda \geq \lambda_0$

∴ f – pre converges to a.

2.3 Theorem:

If a net pre converges to a then it converges to a.

Proof:

Let f : D → X be a net and let a∈X.

Let f pre converge to a. Then f is eventually in every pre open set contains a.

Let U be an open set contains a. Since every open set is pre open, U is a pre open set contains a.

Hence f is eventually in U.

Hence f converges to a.

2.4 Result:

Converse is not true.

f converges to a does not imply that f pre converges to a.

2.5 Example

$X = \{a, b, c\}$ $T = \{\varphi, \{a, b\}, X\}$

Let $D = \{\{a, b\}, X\}$. Define $\leq$ in D as $A \leq B$ if $A \supset B$. Clearly D is a directed set. Define $f : D \to X$ as $\{a, b\} \to b$ and $X \to b$.

Claim: f converges to a.

Consider open sets contain a. They are $\{a, b\}, X$.

1. Take $U = \{a, b\}$ Take $\lambda_0 = \{a, b\}$

$\lambda \geq \lambda_0 \Rightarrow A \geq \{a, b\} \Rightarrow A \subset \{a, b\}$
$\Rightarrow A = \{a, b\} \Rightarrow \lambda = \{a, b\}$

Now,

$x_\lambda = f(\lambda) = f(\{a, b\}) = b \in U$

2. Take $U = X$ Take $\lambda_0 = \{a, b\}$ $x_\lambda = b \in U$

Hence f converges to a.

Claim: f does not pre converge to a.

Consider set $\{a, c\}$ which contains a

$U = \{a, c\}$, $cl\, U = X$, $int\, cl\, U = X$.

$U \subset int\, cl\, U$.

Hence U is pre open.

$\lambda = \{a, b\} \Rightarrow x_\lambda = f(\lambda) = f(\{a, b\}) = b \notin U$

$\lambda = X \Rightarrow x_\lambda = f(\lambda) = f(\lambda) = b \notin U$

$x_\lambda \notin U \quad \forall \lambda$

Hence f does not pre converge to a.

3. Fuzzy pre convergence of Fuzzy nets.

In this section we define Fuzzy pre convergence of fuzzy net in topological space.

3.1 Definition

Let (X, T) be a topological space. Let $A : D \times X \to [0,1]$ be a fuzzy net on X. Let $a \in X$. A is said to fuzzy pre converge to a if

1. For each $\lambda \in D$, $\exists$ at least one x in X such that $A(\lambda, x) = 1$.

2. For each pre open set containing a, $\exists\, \lambda_0 \in D$ such that $\lambda \geq \lambda_0$ and $A(\lambda, x) = 1 \Rightarrow x \in U$.

3.2 Example

$X = \{a, b, c\}$ $T = \{\varphi, \{a\}, \{a, b\}, \{a, c\}, X\}$

$D = \{\{a\}, \{a, b\}, \{a, c\}, X\}$. In D define $\leq$ as $A \leq B$ if $A \supset B$. $(D, \leq)$ is a directed set. Define a fuzzy net $A : D \times X \to [0,1]$ as

$(\{a\}, a) \to 1$

$(\{a\}, b) \to 0.5$

$(\{a\}, c) \to 0.5$

$(\{a, b\}, a) \to 0.5$

$(\{a, b\}, b) \to 1$

$(\{a, b\}, c) \to 0.6$

$(\{a, c\}, a) \to 0.6$

$(\{a, c\}, b) \to 0.6$

$(\{a, c\}, c) \to 1$

$(X, a) \to 0.7$

$(X, b) \to 0.1$

$(X, c) \to 0.7$

Claim: A fuzzy pre converges to a.

Pre open sets containing a are $\{a\}, \{a, b\}, \{a, c\}, X$.

Take $\lambda_0 = \{a\}$

$\lambda \geq \lambda_0 \Rightarrow \lambda \geq \{a\} \Rightarrow \lambda \subset \{a\} \Rightarrow \lambda = \{a\}$

$\lambda \geq \lambda_0$ and $A(\lambda, x) = 1 \Rightarrow \lambda = \{a\}$ and $A(\lambda, x) = 1$

$\Rightarrow A(\{a\}, x) = 1 \Rightarrow x = a \Rightarrow x \in U$

For all pre open set contains a.

Hence A fuzzy pre converges to a.

3.3 Theorem

Let A be a fuzzy net on a topological space X. If A fuzzy pre converges to a then A fuzzy converges to a.

Proof:

Let A fuzzy pre converge to a.

Let U be an open set contains a. Since every open set is pre open, U is a pre open set contain a, $\exists\, \lambda_0 \in D$ such that

$\lambda \geq \lambda_0$ and $A(\lambda, x) = 1 \Rightarrow x \in U$.

Hence A fuzzy converges to a.

3.4 Result

Converse is not true

A fuzzy net fuzzy converges to a does not imply that A fuzzy pre converges to a.

3.5 Example

$$X = \{a, b, c\} \quad T = \{\varphi, \{a, b\}, X\}$$

$$D = (\{a, b\}, X)$$

Define $\leq$ in D as $A \leq B$ if $A \supset B$. Clearly D is a directed set. Define a fuzzy net $A : D \times X \to [0,1]$ as

$(\{a, b\}, a) \to 0.5$

$(\{a, b\}, b) \to 1$

$(\{a, b\}, c) \to 0.5$

$(X, a) \to 0.6$

$(X, b) \to 1$

$(X, c) \to 0.6$

Claim: A converges to a.

Pre open sets containing a are $\{a, b\}, X$.

$U = \{a, b\}$, $\lambda_0 = X$, $A(\lambda_0, x) = 1$

$\Rightarrow A(X, x) = 1 \Rightarrow x = b \Rightarrow b \epsilon U$

$\lambda \geq \lambda_0$ and $A(\lambda, x) = 1$

$\Rightarrow A(\{a, b\}, x) = 1 \Rightarrow x = b$ and $b \epsilon U$

$\therefore \exists \lambda_0$ such that $\lambda \geq \lambda_0$ and $A(\lambda, x) = 1$

$$\Rightarrow x \epsilon U$$

$\therefore$ A fuzzy converges to a.

Claim: A does not pre converge to a.

Take $U = \{a\}$; $int\ cl\ U = int\ X = X$

$$U \subset int\ cl\ U$$

$\therefore$ U is pre open.

U is a pre open set contains a.

Take $\lambda_0 = X \quad A(\lambda_0, x) = 1$

$\Rightarrow A(X, x) = 1 \Rightarrow x = b$ but $b \notin U$

Take $\lambda_0 = \{a, b\} \quad A(\lambda_0, x) = 1$

$\Rightarrow A(\{a, b\}, x) = 1 \Rightarrow x = b$ but $b \notin U$

A does not pre converge to a.

Reference:

1. Chang, C.L., " Fuzzy topological spaces", Journal of Mathematical Analysis Application, 24(1968), 182-190.

2. Zadeh, L.A., "Fuzzy sets", Information and control, 8(1965), 338-353.

3. Muthukumari, M., Nagarajan,A., Murugalingam, M., "Fuzzy nets", International Journal of Mathematical Analysis, Vol.8(2014), no.35, 1715-1721.