INTERVAL VALUED C-PRIME FUZZY BI-IDEALS OFΓ-NEAR-RINGS

[1]N.Priyanka, [2]N.Meenakumari
[1]M.Phil Scholar, A.P.C. Mahalaxmi College for Women, Thoothukudi,Tamil Nadu, India.
e-mail Id: banupriyanka110@gmail.com
[2]PG and Research Department of Mathematics, A.P.C. Mahalaxmi College for Women, Thoothukudi, Tamil Nadu, India.
e-mail Id: meenakumari.n123@gmail.com

ABSTRACT:

A fuzzy set in a set M is a function μ: M→[0,1]. A fuzzy set in M is called a fuzzy bi-ideal of M if (i) $\mu(x-y) \geq min\{\mu(x),\mu(y)\}$ for all x, y ∈M. (ii) $\mu\ (x\alpha y\beta z) \geq min\ \{\mu(x),\mu(z)\}$ for all $x, y, z \in$ M and $\alpha, \beta \in \Gamma$. A fuzzy bi-ideal μ of M is called c-prime if for all $x, y \in M, \gamma \in \Gamma,\ \mu(x\gamma y) \leq max\{\mu(x),\mu(y)\}$.An interval valued fuzzy subset $\tilde{\mu}$ of M is called an interval valued fuzzy bi-ideal of M if (i) $\tilde{\mu}(x-y) \geq min^i\{\tilde{\mu}(x),\tilde{\mu}(y)\}$ for all $x, y \in$ M. (ii) $\tilde{\mu}(x\alpha y\beta z) \geq min^i\{\tilde{\mu}(x),\tilde{\mu}(z)\}$ for all $x, y, z \in$ M and $\alpha, \beta \in \Gamma$. In this paper, we introduce interval valued c-prime fuzzy bi-ideals in Γ-near-rings and obtain some of their properties.

KEYWORDS:

Γ-near-rings, bi-ideal, c-prime fuzzy bi-ideal, interval valued c-prime fuzzy bi-ideal.

1. Introduction

Zadeh introduced the concept of fuzzy sets in 1965 [15] and also generalized it to interval valued fuzzy subsets [16]. Gamma-near-ring was introduced by Satyanarayana [9] in 1984. The concept of bi-ideals of gamma-near-rings was applied to gamma-near-rings by Tamizhchelvam et al. [10]. Meenakumari [6] introduced C-Prime fuzzy bi-ideals in gamma-near-rings and discussed some of its properties. V. Chinnadurai [4] introduced interval valued fuzzy ideals of gamma-near-rings and also developed interval valued fuzzy weak bi-ideals of gamma-near-rings [3].

In this paper, we define a new notion called interval valued c-prime fuzzy bi-ideals in gamma-near-rings. We also investigate some of its properties and illustrate with examples.

2.Preliminaries

Definition2.1 [8]

A *near-ring* is an algebraic system $(R, +,\ .)$ consisting of a non empty set R together with two binary operations called + and . such that $(R, +)$ is a group not necessarily abelian and $(R\ ,\ .)$ is a semigroup connected by the following distributive law :$(x+z)\,.\,y = x\,.\,y + z\,.\,y$ valid for all $x, y, z \in R$. We use the word 'near-ring' to mean 'right near-ring'. We denote xy instead of $x\,.\,y$.

Definition 2.2 [9]

A *Γ-near-ring* is a triple $(M, +, \Gamma)$ where

(i) $(M\ , +)$ is a group.

(ii) Γ is a nonempty set of binary operators on M such that for each $\alpha \in \Gamma$, $(M\ , +, \alpha)$ is a near-ring,

(iii) $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Definition 2.3 [7]

A Γ-near-ring M is said to be *zero-symmetric* if $x\alpha 0 = 0$ for all $x \in M$ and $\alpha \in \Gamma$.

Throughout this paper, M denotes a zero-symmetric right Γ-near-ring with atleast two elements.

Definition 2.4 [9]

A subset A of a Γ-near-ring M called a *left (resp. right) ideal* of M if

(i) $(A, +)$ is a normal subgroup of $(M, +)$ (i.e.) $x - y \in A$ for all $x, y \in A$ and

$y + x - y \in A$ for $x \in A, y \in M$.

(ii) $u\,\alpha(x + v) - u\alpha v \in A$ (resp. $x\alpha u \in A$)

for all $x \in A, \alpha \in \Gamma$ and $u, v \in M$.

Definition 2.5 [6]

A fuzzy bi-ideal μ of M is called *c-prime* if for all $x, y \in M, \gamma \in \Gamma$, $\mu(x\gamma y) \leq \max\{\mu(x), \mu(y)\}$.

Definition 2.6 [10]

A subgroup B of $(M, +)$ is called a *bi-ideal* of M if and only if $B\Gamma M\Gamma B \subseteq B$.

Definition 2.7 [1]

A subgroup H of (M, +) is said to be a *weak bi-ideal* of M if $H\Gamma H\,\Gamma H \subseteq H$.

Definition 2.8 [16]

Let X be any set. A mapping $\tilde{\mu} : X \longrightarrow D[0,1]$ is called an *interval valued fuzzy subset* (briefly, an i.v. fuzzy subset) of X where $D[0,1]$ denotes the family of closed subintervals [0,1] and $\tilde{\mu}(x) = [\mu^-(x), \mu^+(x)]$ for all $x \in X$ where $\mu^-(x)$ and $\mu^+(x)$ are fuzzy subsets of X such that $\mu^-(x) \leq \mu^+(x)$ for all $x \in X$.

Definition 2.9 [11]

By an *interval number* $\tilde{a}$, we mean an interval $[a^-, a^+]$ such that $0 \leq a^- \leq a^+ \leq 1$ and where a^- and a^+ are the lower and upper limits of $\tilde{a}$ respectively. The set of all closed subintervals of [0, 1] is denoted by $D[0,1]$. We also identify the interval $[a, a]$ by the number $a \in [0,1]$. For any interval numbers $\widetilde{a_j} = [a_j^-, a_j^+], \widetilde{b_j} = [b_j^-, b_j^+] \in D[0,1], j \in \Omega$ we define $max^i\{\widetilde{a_j}, \widetilde{b_j}\} = [\max\{a_j^-, b_j^-\}, max\{a_j^+, b_j^+\}]$, $min^i\{\widetilde{a_j}, \widetilde{b_j}\} = [\min\{a_j^-, b_j^-\}, \max\{a_j^+, b_j^+\}]$, $inf^i\widetilde{a_j} = [\cap_{j\in\Omega} a_j^-, \cap_{j\in\Omega} a_j^+]$, $sup^i\widetilde{a_j} = [\cup_{j\in\Omega} a_j^-, \cup_{j\in\Omega} a_j^+]$, and let

(i) $\tilde{a} \leq \tilde{b} \Leftrightarrow a^- \leq b^-$ and $a^+ \leq b^+$,

(ii) $\tilde{a} = \tilde{b} \Leftrightarrow a^- = b^-$ and $a^+ = b^+$.

(iii) $\tilde{a} < \tilde{b} \Leftrightarrow \tilde{a} \leq \tilde{b}$ and $\tilde{a} \neq \tilde{b}$,

(iv) $k\tilde{a} = [ka^-, ka^+]$, whenever $0 \leq k \leq 1$.

Definition 2.10 [11]

Let $\tilde{\mu}$ be an i.v fuzzy subset of X *and* $[t_1, t_2] \in D[0,1]$. Then the set $\widetilde{U}(\tilde{\mu}: [t_1, t_2]) = \{x \in X | \tilde{\mu}(x) \geq [t_1, t_2]\}$ is called the *upper level subset* of $\tilde{\mu}$.

Definition 2.11 [1]

An i.v fuzzy subset $\tilde{\mu}$ in a Γ-near-ring M is called an *i.v fuzzy left (resp. right) ideal* if

(i) $\tilde{\mu}$ is an i.v fuzzy normal divisor with respect to the addition,

(ii) $\tilde{\mu}(u\alpha(x+v)-u\alpha v) \geq \tilde{\mu}(x)$, (resp. $\tilde{\mu}(x\alpha u) \geq \tilde{\mu}(x)$) for all $x,u,v \in M$ and $\alpha \in \Gamma$.

The condition (i) of definition 2.11 means that $\tilde{\mu}$ satisfies:

(i) $\tilde{\mu}(x-y) \geq min^i\{\tilde{\mu}(x),\tilde{\mu}(y)\}$,

(ii) $\tilde{\mu}(y+x-y) \geq \tilde{\mu}(x)$, for all $x,y \in M$

Note that $\tilde{\mu}$ is an i.v fuzzy left (resp. right) ideal of Γ-near-ring M, then $\tilde{\mu}(0) \geq \tilde{\mu}(x)$ for all $x \in M$, where 0 is the zero element of M.

Definition 2.12 [2]

An i.v fuzzy subset $\tilde{\mu}$ of M is called an *i.v fuzzy bi-ideal* of M if

(i) $\tilde{\mu}(x-y) \geq min^i\{\tilde{\mu}(x),\tilde{\mu}(y)\}$ for all $x,y \in M$.

(ii) $\tilde{\mu}(x\alpha y\beta z) \geq min^i\{\tilde{\mu}(x),\tilde{\mu}(z)\}$ for all $x,y,z \in M$ and $\alpha,\beta \in \Gamma$.

3. Intervalvalued c-prime fuzzy bi-ideals of Γ −near-rings

In this section, we introduce the notion of i.v.c-prime fuzzy bi-ideal of M and discuss some of its properties.

Definition3.1

An i.v. fuzzy set $\tilde{\mu}$ of M is called an *i.v. c-prime fuzzy bi-ideal* of M, if

(i) $\tilde{\mu}(x-y) \geq min^i\{\tilde{\mu}(x),\tilde{\mu}(y)\}$ for all $x,y \in M$.

(ii) $\tilde{\mu}(x\alpha y\beta z) \geq min^i\{\tilde{\mu}(x),\tilde{\mu}(z)\}$ for all $x,y,z \in M$ and $\alpha,\beta \in \Gamma$.

(iii) $\tilde{\mu}(x\gamma y) \leq max^i\{\tilde{\mu}(x),\tilde{\mu}(y)\}$ for all $\gamma \in \Gamma$ and $x,y \in M$.

Example 3.2

Let $M=\{0,a,b,c\}$ be a non-empty set with binary operation + and Γ= $\{\alpha,\beta\}$be a non-empty set of binary operations as shown in the following tables

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

α	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	b	b
c	0	a	b	c

β	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	a	c	b
c	0	a	b	c

Let $\tilde{\mu} : M \rightarrow [0,1]$ be an i.v fuzzy subset defined by

$\tilde{\mu}(0)= \tilde{\mu}(a)=[0.6,0.7], \tilde{\mu}(b)= \tilde{\mu}(c)=[0.2,0.3]$.

Then $\tilde{\mu}$ is an i.v c-prime fuzzy bi-ideal of M.

Theorem 3.3

Let $\tilde{\mu}$ be an i.v fuzzy subset of M. Then $\tilde{\mu}$ is an i.v c-prime fuzzy bi-ideal of M if and only if $\widetilde{U}(\tilde{\mu} : [t_1,t_2])$ is a c-prime bi-ideal of M for all $[t_1,t_2] \in D[0,1]$.

Proof: Assume that $\tilde{\mu}$ is an i.v c-prime fuzzy bi-ideal of M. Let $[t_1,t_2] \in D[0,1]$ such that $x,y \in \widetilde{U}(\tilde{\mu}:[t_1,t_2])$. Then, $\tilde{\mu}(x-y) = min^i\{\tilde{\mu}(x),\tilde{\mu}(y)\} \geq min^i\{[t_1,t_2],[t_1,t_2]\} = [t_1,t_2]$. Thus $x,y \in \widetilde{U}(\tilde{\mu}:[t_1,t_2])$ and $\alpha,\beta \in \Gamma$. We have $\tilde{\mu}(x\alpha y\beta z) = min^i\{\tilde{\mu}(x),\ \tilde{\mu}(z)\} \geq min^i\{[t_1,t_2],\ [t_1,t_2]\} = [t_1,t_2]$. Therefore, $x\alpha y\beta z \in \widetilde{U}(\tilde{\mu}:[t_1,t_2])$. Let $x,y \in \widetilde{U}(\tilde{\mu}:[t_1,t_2])$ and $\alpha,\beta \in \Gamma$. Then we have $\tilde{\mu}(x\alpha y) = max^i\{\tilde{\mu}(x),\tilde{\mu}(y)\} \leq max^i\{[t_1,t_2], [t_1,t_2]\} = [t_1,\ t_2]$. Therefore $x\alpha y \in \widetilde{U}(\tilde{\mu}:[t_1, t_2])$. Hence $\widetilde{U}(\tilde{\mu}:[t_1,\ t_2])$ is a c-prime fuzzy bi-ideal of M. Conversely, Assume that $\widetilde{U}(\tilde{\mu}:[t_1,t_2])$ is a c-prime fuzzy bi-ideal of M for all $[t_1,t_2] \in D[0,1]$. Let $x,y \in M$. Suppose $\tilde{\mu}(x-y) < min^i\{\tilde{\mu}(x),\ \tilde{\mu}(y)\}$. Choose $[0,0] < [t_1,t_2] < [1,1]$ such that $\tilde{\mu}(x-y) < [t_1,t_2] < min^i\{\tilde{\mu}(x),\ \tilde{\mu}(y)\}$. This implies that $\tilde{\mu}(x) > [t_1,t_2]$, $\tilde{\mu}(y) > [t_1,t_2]$ and $\tilde{\mu}(x-y) < [t_1,\ t_2]$. Then we have $x,y \in \widetilde{U}(\tilde{\mu}:[t_1,t_2])$ but $x-y \notin \widetilde{U}(\tilde{\mu}:[t_1,t_2])$ which is a contradiction. Thus we get $\tilde{\mu}(x-y) \geq min^i\{\tilde{\mu}(x),\tilde{\mu}(y)\}$. If there exist $x,y,z \in M$ and $\alpha,\beta \in \Gamma$ such that $\tilde{\mu}(x\alpha y\beta z) < [t_1,t_2] < min^i\{\tilde{\mu}(x),\ \tilde{\mu}(y)\}$. Then $\tilde{\mu}(x) > [t_1,t_2]$, $\tilde{\mu}(z) > [t_1,t_2]$ *and* $\tilde{\mu}(x\alpha y\beta z) < [t_1 t_2]$ so $x,y \in \widetilde{U}(\tilde{\mu}:[t_1 t_2])$ but $x\alpha y\beta z \notin \widetilde{U}(\tilde{\mu}:[t_1 t_2\ ])$ which is a contradiction. If there exists $x, y \in M$, $\alpha \in \Gamma$ such that $\tilde{\mu}(x\alpha y) > max^i\{\tilde{\mu}(x),\ \tilde{\mu}(y)\}$. Choose $[t_1,t_2]$ such that $\tilde{\mu}(x\alpha y) > [t_1,t_2] > max^i\{\tilde{\mu}(x),\ \tilde{\mu}(y)\}$. Then $\tilde{\mu}(x) < [t_1,t_2]$, $\tilde{\mu}(y) < [t_1,t_2]$ and $\tilde{\mu}(x\alpha y) > [t_!,t_2]$. Then $x\alpha y \in \widetilde{U}(\tilde{\mu}:[t_1,t_2])$ but $x,y \notin \widetilde{U}(\tilde{\mu}:[t_1,t_2])$ which is a contradiction. Hence $\tilde{\mu}(x\gamma y) \leq\ max^i\{\tilde{\mu}(x),\tilde{\mu}(y)\}$. Therefore $\tilde{\mu}$ is an i.v. c-prime fuzzy bi-ideal of M.

Theorem 3.4

Let $\tilde{\mu} = [\mu^-,\mu^+]$ be an i.v fuzzy subset of M. Then $\tilde{\mu}$ is an i.v c-prime fuzzy bi-ideal of M if and only if μ^-,μ^+ are c-prime fuzzy bi-ideals of M.

Proof: Assume that $\tilde{\mu}$ is ani.vc-prime fuzzy bi-ideal of M. For any $x,y,z \in M$ and $\alpha,\beta \in \Gamma$. Now,

$[\mu^-(x-y),\mu^+(x-y)] = \tilde{\mu}(x-y) \geq min^i\{\tilde{\mu}(x),\tilde{\mu}(y)\} = min^i\{[\mu^-(x),\mu^+(x)],\{\mu^-(y),\mu^+(y)]\} = min^i\{[\mu^-(x),\mu^-(y)],min^i[\mu^+(x),\mu^+(y)]\}$. It follows that $\mu^-(x-y) \geq \min\{\mu^-(x),\mu^-(y)\}$ and $\mu^+(x-y) \geq \min\{\mu^+(x),\mu^+(y)\}$. $[\mu^-(x\alpha y\beta z),\mu^+(x\alpha y\beta z)] = \tilde{\mu}(x\alpha y\beta z) \geq min^i\{\tilde{\mu}(x),\tilde{\mu}(z)\} = min^i\{[\mu^-(x),\mu^+(x)],[\mu^-(z),\mu^+(z)]\} = min^i\{[\mu^-(x),\mu^-(z)],min^i[\mu^+(x),\mu^+(z)]\}$. It follows that $\mu^-(x\alpha y\beta z) \geq \min\{\mu^-(x),\mu^-(z)\}$ and $\mu^+(x\alpha y\beta z) \geq \min\{\mu^+(x),\mu^+(z)\}$. For any $x,y \in M$ and $\gamma \in \Gamma$. Now, $[\mu^-(x\gamma y),\mu^+(x\gamma y)] = \tilde{\mu}(x\gamma y) \leq max^i\{\tilde{\mu}(x),\tilde{\mu}(y)\} = max^i\{[\mu^-(x),\mu^+(x)],[\mu^-(y),\mu^+(y)]\} = max^i\{[\mu^-(x),\mu^-(y)],[\mu^+(x),\mu^+(y)]\}$.It follows that $\mu^-(x\gamma y) \leq \max\{\mu^-(x),\mu^-(y)\}$ and $\mu^+(x\gamma y) \leq \max\{\mu^+(x),\mu^+(y)\}$. Conversely assume that μ^-,μ^+ are c-prime fuzzy bi-ideals of M. Let $x,y,z \in M$, $\alpha,\beta \in \Gamma$ .Then $\tilde{\mu}(x-y) = [\mu^-(x-y),\ \mu^+(x-y)] \geq \min\{[\mu^-(x),\ \mu^-(y)],min[\mu^+(x),\ \mu^+(y)]\} = min^i\{[\mu^-(x),\mu^+(x)]\,,\min[\mu^-(y),\ \mu^+(y)]\} = min^i\{\tilde{\mu}(x),\ \tilde{\mu}(y)\}$. Then we get $\tilde{\mu}(x\alpha y\beta z) = [\mu^-(x\alpha y\beta z),\mu^+(x\alpha y\beta z) \geq \min\{[\mu^-(x),\mu^-(z)], min[\mu^+(x),\mu^+(z)]\} = min^i\{[\mu^-(x),\mu^+(x)],[\mu^-(z), \mu^+(z)]\} = min^i\{\tilde{\mu}(x),\ \tilde{\mu}(z)\}$. Then we get $\tilde{\mu}(x\alpha y) = [\mu^-(x\alpha y),\mu^+(x\alpha y)] \leq$

$\max\{\mu^-(x), \mu^-(y)\}, \max\{\mu^+(x), \mu^+(y)\} = max^i\{[\mu^-(x), \mu^+(x)], [\mu^-(y), \mu^+(y)]\} = max^i\{\tilde{\mu}(x), \tilde{\mu}(z)\}$. Therefore $\tilde{\mu}$ is an i.v c-prime fuzzy bi-ideal of M.

Theorem 3.5

Let I be a c-prime fuzzy bi-ideal of M. Then for any $[t_1, t_2] \in D[0,1]$ with $[t_1, t_2] \neq [0,0]$ there exists an i.v c-prime fuzzy bi-ideal $\tilde{\mu}$ of M such that $\widetilde{U}(\tilde{\mu} : [t_1, t_2]) = I$.

Proof: Let I be a c-prime fuzzy bi-ideal of M. Let $\tilde{\mu}$ be an i.v fuzzy subset of M defined by $\tilde{\mu}(x) = \begin{cases} [t_1, t_2] \; if \; x \in I \\ \tilde{0} \; otherwise \end{cases}$ Then $\widetilde{U}(\tilde{\mu}: [t_1, t_2]) = I$. Assume that $\tilde{\mu}(x-y) < min^i\{\tilde{\mu}(x), \tilde{\mu}(y)\}$. This implies $\tilde{\mu}(x-y) = 0$ and $min^i\{\tilde{\mu}(x), \tilde{\mu}(y)\} = [t_1, t_2]$ so $x, y \in I$ and $\alpha, \beta \in \Gamma$ but $x - y \notin I$ which is a contradiction. Thus $\tilde{\mu}(x-y) \geq min^i\{\tilde{\mu}(x), \tilde{\mu}(y)\}$. Suppose that $\tilde{\mu}(x\alpha y\beta z) < min^i\{\tilde{\mu}(x), \tilde{\mu}(z)\}$. Then $\tilde{\mu}(x\alpha y\beta z) = \tilde{0}$, $min^i\{\tilde{\mu}(x), \tilde{\mu}(z)\} = [t_1, t_2]$. So $x, z \in I$ but $x\alpha y\beta z \notin I$ which is a contradiction. Hence $\tilde{\mu}(x\alpha y\beta z) \geq min^i\{\tilde{\mu}(x), \tilde{\mu}(z)\}$. Then $\tilde{\mu}(x\gamma y) > max^i\{\tilde{\mu}(x), \tilde{\mu}(y)\}$. Then $\tilde{\mu}(x\gamma y) = [t_1, t_2], max^i\{\tilde{\mu}(x), \tilde{\mu}(z)\} = \tilde{0}$ so $x\gamma y \in I$ but $x, y \notin I$ which is a contradiction. Hence $\tilde{\mu}(x\gamma y) \leq max^i\{\tilde{\mu}(x), \tilde{\mu}(y)\}$.

Theorem 3.6

Let H be a non-empty subset of M and $\tilde{\mu}$ be an i.v fuzzy subset of M defined by $\tilde{\mu}(x) = \begin{cases} \tilde{s} \; if \; x \in H \\ \tilde{t} \; otherwise \end{cases}$ for some $x \in M$, $\tilde{s}, \tilde{t} \in D[0,1]$ and $\tilde{s} > \tilde{t}$. Then H is a c-prime bi-ideal of M if and only if $\tilde{\mu}$ is an i.v c-prime fuzzy bi-ideal of M.

Proof: Assume that H is a c-prime bi-ideal of M. Let $x, y \in M$. We consider four cases.

1) $x \in H$ and $y \in H$.
2) $x \in H$ and $y \notin H$.
3) $x \notin H$ and $y \in H$.
4) $x \notin H$ and $y \notin H$.

Case (1): If $x \in H$ and $y \in H$. Then $\tilde{\mu}(x) = \tilde{s} = \tilde{\mu}(y)$. Since H is a c-prime bi-ideal of M, $x - y \in H$. Thus $\tilde{\mu}(x-y) = \tilde{s} = min^i\{\tilde{\mu}(x), \tilde{\mu}(y)\}$.

Case (2): If $x \in H$ and $y \notin H$. Then $\tilde{\mu}(x) = \tilde{s}, \tilde{\mu}(y) = \tilde{t}$ so $min^i\{\tilde{\mu}(x), \tilde{\mu}(y)\} = \tilde{t}$. Now $\tilde{\mu}(x-y) = \tilde{s} \; or \; \tilde{t}$ according as $x - y \in H \; or \; x - y \notin H$. By assumption $\tilde{s} > \tilde{t}$. We have $\tilde{\mu}(x-y) \geq min^i\{\tilde{\mu}(x), \tilde{\mu}(y)\}$. Similarly we can prove case (3).

Case (4): If $x \notin H$ and $y \notin H$.

Then we have $\tilde{\mu}(x) = \tilde{t} = \tilde{\mu}(y)$. So $min^i\{\tilde{\mu}(x), \tilde{\mu}(y)\} = \tilde{t}$. Next, $\tilde{\mu}(x-y) = \tilde{s} \; or \; \tilde{t}$ according as $x - y \in H \; or \; x - y \notin H$. So $\tilde{\mu}(x-y) \geq min^i\{\tilde{\mu}(x), \tilde{\mu}(y)\}$. Now let $x, y, z \in M \; and \; \alpha, \beta \in \Gamma$. Then we have the following eight cases:

(1) $x \in H, y \in H \; and \; z \in H$.
(2) $x \notin H, y \in H \; and \; z \in H$.
(3) $x \in H, y \notin H \; and \; z \in H$.
(4) $x \in H, y \in H \; and \; z \notin H$.
(5) $x \notin H, y \notin H \; and \; z \in H$
(6) $x \in H, y \notin H \; and \; z \notin H$.
(7) $x \notin H, y \in H \; and \; z \notin H$.
(8) $x \notin H, y \notin H \; and \; z \notin H$.

These cases can be proved by arguments similar to fuzzy cases above. Hence $\tilde{\mu}(x\alpha y\beta z) \geq$

$min^{i}\{\tilde{\mu}(x), \tilde{\mu}(z)\}$. Now let $x, y \in M, \gamma \in \Gamma$. We consider four cases

(1) $x \in H$ and $y \in H$.
(2) $x \in H$ and $y \notin H$.
(3) $x \notin H$ and $y \in H$.
(4) $x \notin H$ and $y \notin H$.

Case (1): If $x \in H$ and $y \in H$. Then $\tilde{\mu}(x) = \tilde{s} = \tilde{\mu}(y)$. Since H is a c-prime bi-ideal of M, we get $x\gamma y \in H$. Thus $\tilde{\mu}(x\gamma y) = \tilde{s} = max^{i}\{\tilde{s}, \tilde{s}\} = max^{i}\{\tilde{\mu}(x), \tilde{\mu}(y)\}$.

Case(2): If $x \in H$ and $y \notin H$. Then $\tilde{\mu}(x) = \tilde{s}$ and $\tilde{\mu}(y) = \tilde{t}$. So $max^{i}\{\tilde{\mu}(x), \tilde{\mu}(y)\} = \tilde{s}$. Now $\tilde{\mu}(x\gamma y) = \tilde{s}$ or $\tilde{t}$ according as $x\gamma y \in H$ or $x\gamma y \notin H$. By assumption $\tilde{t} < \tilde{s}$ we have $\tilde{\mu}(x\gamma y) \leq max^{i}\{\tilde{\mu}(x), \tilde{\mu}(y)\}$. Similarly we can prove case (3).

Case (4): If $x \notin H$ and $y \notin H$. We have $\tilde{\mu}(x) = \tilde{t} = \tilde{\mu}(y)$. So $max^{i}\{\tilde{\mu}(x), \tilde{\mu}(y)\} = \tilde{t}$. Next, $\tilde{\mu}(x\gamma y) = \tilde{s}$ or $\tilde{t}$ according as $x\gamma y \in H$ or $x\gamma y \notin H$. So $\tilde{\mu}(x\gamma y) \leq max^{i}\{\tilde{\mu}(x), \tilde{\mu}(y)\}$. Conversely Assume that $\tilde{\mu}$is an i.v c-prime fuzzy bi-ideal of M. Let $x, y, z \in M$, $\alpha, \beta \in \Gamma$ be such that $\tilde{\mu}(x) = \tilde{\mu}(y) = \tilde{\mu}(z) = \tilde{s}$. Since $\tilde{\mu}$ is an i.v c-prime fuzzy bi-ideal of M, we have $\tilde{\mu}(x-y) \geq min^{i}\{\tilde{\mu}(x), \tilde{\mu}(y)\} = \tilde{s}$ and $\tilde{\mu}(x\alpha y\beta z) \geq min^{i}\{\tilde{\mu}(x), \tilde{\mu}(z)\} = \tilde{s}$, $\tilde{\mu}(x\gamma y) \leq max^{i}\{\tilde{\mu}(x), \tilde{\mu}(y)\} < \tilde{t}$. So $x - y, x\alpha y\beta z, x\gamma y \in H$. Hence H is a c-prime bi-ideal of M.

Theorem 3.7

Let $\tilde{\mu}$be an i.v c-prime fuzzy bi-ideal of M. Then the set $M_{\tilde{\mu}} = \{x \in M | \tilde{\mu}(x) = \tilde{\mu}(0)\}$ is a c-prime bi-ideal of M.

Proof: Let $\tilde{\mu}$ be an i.v c-prime fuzzy bi-ideal of M. Let $x, y \in M$. Then, $\tilde{\mu}(x) = \tilde{\mu}(0)$, $\tilde{\mu}(y) = \tilde{\mu}(0)$ and $\tilde{\mu}(x-y) \geq min^{i}\{\tilde{\mu}(x), \tilde{\mu}(y)\} = min^{i}\{\tilde{\mu}(0), \tilde{\mu}(0)\} = \tilde{\mu}(0)$. So, $\tilde{\mu}(x-y) = \tilde{\mu}(0)$. Thus $x - y \in M_{\tilde{\mu}}$. Now for every $x, y, z \in M_{\tilde{\mu}}$ and $\alpha, \beta \in \Gamma$. We have $\tilde{\mu}(x\alpha y\beta z) \geq min^{i}\{\tilde{\mu}(x), \tilde{\mu}(z)\} = min^{i}\{\tilde{\mu}(0), \tilde{\mu}(0)\} = \tilde{\mu}(0)$. Thus $x\alpha y\beta z \in M_{\tilde{\mu}}$. Let $x, y \in M_{\tilde{\mu}}, \gamma \in \Gamma$. Then $\tilde{\mu}(x\alpha y) \leq max^{i}\{\tilde{\mu}(x), \tilde{\mu}(y)\} = max^{i}\{\tilde{\mu}(0), \tilde{\mu}(0)\} = \tilde{\mu}(0)$. Hence we get $x\alpha y \in M_{\tilde{\mu}}$.

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